

IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture35

LECTURE 35 : USE OF TRIAL AND ERROR METHOD TO FIND PRESSURE DROP

Good afternoon, my dear students, boys and girls, and friends. We are handling compressible gas flow, right? Fluids that are compressible, we are handling them, and we have developed one relation for finding out the pressure in a pipeline. We started with Bernoulli's equation.

Right. So, we also made one problem solution. We also said that solving problems is a very unique technique to grasp the subject, right? We have done one problem and its solution. Now, let us do another problem.



Now, for isothermal and adiabatic flow, we know that

$$pv = \frac{mRT}{M}, \text{ or, } \frac{pv}{m} = \frac{RT}{M}$$

$$\text{or, } pV = \frac{RT}{M}; \text{ or, } V = \frac{RT}{pM}$$

Substituting in eq. 2, we get,

$$G^2 \int_1^2 \frac{dV}{V} + \frac{M}{RT} \int_1^2 p dp + \frac{2fG^2}{D} \int_1^2 dL = 0$$

$$G^2 \ln \left(\frac{V_2}{V_1} \right) + \frac{M(p_2^2 - p_1^2)}{2RT} + 2fG^2 \frac{\Delta L}{D} = 0$$

It is saying that air at 290 Kelvin and 300 kilopascal enters a pipe and is flowing in isothermal compressible flow in the pipe with an ID of 0.1 meter. The length of the pipe is 50 meters, and the mass velocity at the entrance of the pipe is 170 kg per meter square second. Assuming the molecular weight of air to be 29 and the friction factor to be 0.004, and the viscosity of air to be 2×10^{-5} Pascal second, calculate the pressure at the exit.

Prob.: Air at 290K and 300 kPa enters a pipe and is flowing in isothermal compressible flow in the pipe having an ID of 0.1 m. The length of the pipe is 50 m. The mass velocity at the entrance of the pipe is 170 Kg/m²-s. Assuming, molecular weight of air to be 29, the friction factor to be 0.004, and the viscosity of air to be 2×10^{-5} Pa-s, calculate the pressure at the exit. **Solution:-**

$$N_{Re} = \frac{GD}{\mu} = \frac{170 \times 0.1}{2 \times 10^{-5}} = 850000, \text{ hence, the flow is turbulent}$$

Neglecting the logarithmic term

$$p_1^2 - p_2^2 \approx \frac{4f \Delta L G^2 RT}{DM}$$

$$= \frac{4 \times 0.004 \times 50 \times (170)^2 \times 8.314 \times 10^3 \times 290}{0.1 \times 29} = 1.922 \times 10^{10} \text{ Pa}^2$$

$$\text{or, } p_2^2 = p_1^2 - 1.922 \times 10^{10} = (300 \times 10^3)^2 - 1.922 \times 10^{10}$$

$$= 7.0778 \times 10^{10} \text{ Pa}^2; \therefore p_2 = 266.04 \text{ kPa}$$

It is a little different than the previous one in formulating the problem because, in the previous one, we had said that the gas is entering. And the rate was said as kg mole per second, right? So, we had converted that to capital G, that is, rho V or mass flux, whatever we call it. Here, it is the other way around. Here, it is given that the mass velocity, that is rho into v, that is, G, as 170 kg per meter square per second, which we found out in the previous problem by dividing with area also and molecular weight adjustment. You remember?

Now, apart from that, in the previous problem, molecular weight was not given because the gas was methane, and we know that its molecular weight is 16 because it is CH_4 . C carbon is 12, hydrogen is 1. So, 4 hydrogen, so 4. So, 12 plus 4 is 16, but air we cannot say just like that. Air, depending on its composition, what it is made of,

how much nitrogen, how much oxygen, how much others are there. So, it varies between 28, 29, like that. Here, for simplicity, we have taken the molecular weight of air to be 29. Another difference from the previous problem is that the friction factor here is directly given as 0.004, whereas, in the previous problem, we were given epsilon by D, that is relative roughness.

We had to find the Reynolds number. From the Reynolds number and epsilon by D for the pipe, which was given, we found the friction factor from Moody's chart, right? But here it is not required because it is directly given as 0.004. So, as usual, we say that before we solve the problem, we read the problem twice so that the understanding is proper. So, if we say that, then it is like this. Air at 290 Kelvin and 300 kilo Pascal.

So, the entire pressure, the entering air pressure, is known as 300 kilo Pascal, enters the pipe, and is flowing in isothermal compressible flow. In the pipe having an internal diameter of 0.1 meter. The length of the pipe is given as 50 meters; earlier, it was a little bit more, perhaps. It does not matter. However, the mass velocity at the entrance of the pipe is 170 kg per meter square second.

So, mass velocity or mass flux is stated, assuming the molecular weight of air to be 29 and the friction factor to be 0.004. And the viscosity of air to be 2×10^{-5} Pascal, calculate the pressure at the exit. So, the inlet pressure is given, and the exit pressure we have to find out. Now, if you remember the relation which we had developed, it was in a quadratic form in terms of pressure. So, it was p_1^2 square, or p_2^2 square minus p_1^2 square, is equal to some constant, $\ln \sqrt{p_1}$ by p_2 , and a constant with all other terms, right?

Prob.: Air at 290K and 300 kPa enters a pipe and is flowing in isothermal compressible flow in the pipe having an ID of 0.1 m. The length of the pipe is 50 m. The mass velocity at the entrance of the pipe is 170 Kg/m²-s. Assuming, molecular weight of air to be 29, the friction factor to be 0.004, and the viscosity of air to be 2×10⁻⁵ Pa-s, calculate the pressure at the exit. **Solution:-**


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$$p_1^2 - p_2^2 = \frac{4f\Delta LG^2 RT}{DM}$$

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It was like $ax^2 + bx + c$, and we had to find out the solution of the equation, right. Earlier we did only a single iteration, but here you may need to do multiple iterations; that is how the problem is set. So, the Reynolds number is GD/μ , which is $170 \times 0.1 / 2 \times 10^{-5}$, that is equal to 850,000. Hence, the flow is turbulent; there is no ambiguity in that.

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Now, neglecting the logarithmic term, which is $\ln(p_1/p_2)$, we write $p_1^2 - p_2^2 = 4f\Delta LG^2 RT/DM$. So, that was the constant, and this constant we can find out; what is the value? It is 4×0.004 , which is given; ΔL is 50, and G is 170². R is 8.314×10^3 joules per kg, into temperature 290, divided by 0.1×29 . That is D and M ; D is 0.1, and M is 29.

This, on calculation, you can also calculate because I may do wrong, but it should be closer to 1.9 because rounding off or any other may lead to a different value, but again, different means where it is. So, here you have found it to be 1.922×10^{10} Pascal². Now, $p_2^2 = p_1^2 - 1.922$, rather 1.922×10^{10} , which is equal to 300×10^3 whole squared minus 3.33×10^{10} .

Prob.: Air at 290K and 300 kPa enters a pipe and is flowing in isothermal compressible flow in the pipe having an ID of 0.1 m. The length of the pipe is 50 m. The mass velocity at the entrance of the pipe is 170 Kg/m²-s. Assuming, molecular weight of air to be 29, the friction factor to be 0.004, and the viscosity of air to be 2×10⁻⁵ Pa-s, calculate the pressure at the exit. **Solution:-**


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Now, this 3.33 into 10 to the power of 10, we have done perhaps earlier, or simultaneously, okay. Now, then, we get P₂ square minus P₁ square minus 1.922 into 10 to the power of 10 is equal to 300 into 10 to the power of 3 whole square minus 3.33 into 10 to the power of 10. Or that is equal to 7.0778 into 10 Pascal. Now, we have received, we have obtained P₂ as 266.04 kilo Pascal. Right, because P₂ square was 7.077 into 10 to the power of 10 Pascal square.

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So, we see that a value of 266.04. If I remember roughly, then in the earlier problem also, we had the pressure somewhere around 265 kilo Pascal, 265 kilo Pascal. However, it does not mean that this is the final because whether P₂ is 266.04 or not, we are not sure, because what we have done, we have neglected the logarithmic term that is ln of P₁ by P₂. So, that term has to be included.

To do that, let us first take this value of P₁ as the first approximation, right. So, from the relation P₁ square minus P₂ square is equal to 2 G square RT by M ln of P₁ by P₂ plus 4 f G square RT delta L by D M. This shows that this is leading to P₁ square minus P₂ square

is equal to this $2 G^2 R T$ by M we have already found out. This was 1.922 into 10 to the power of $10 \ln$ of P_1 by P_2 . No, this was for the second term, that is $4 f G^2 R T \Delta L$ by DM , 1.922 into 10 to the power of 10 .

Now, from the equation (3) we can write

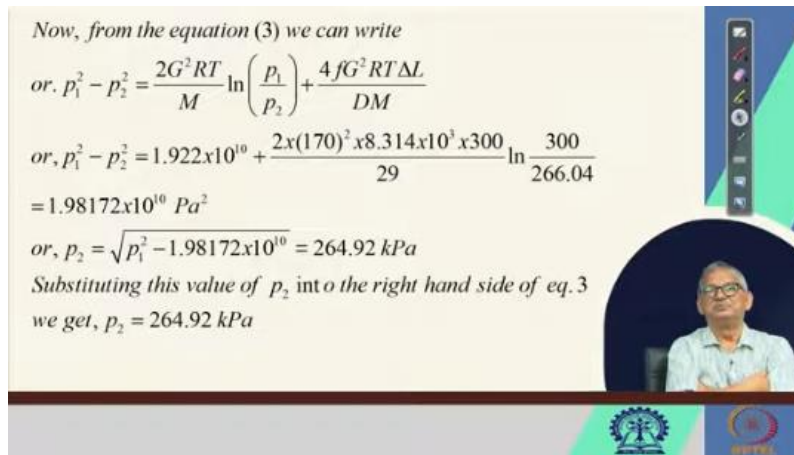
$$\text{or, } p_1^2 - p_2^2 = \frac{2G^2 RT}{M} \ln\left(\frac{p_1}{p_2}\right) + \frac{4fG^2 RT \Delta L}{DM}$$

$$\text{or, } p_1^2 - p_2^2 = 1.922 \times 10^{10} + \frac{2 \times (170)^2 \times 8.314 \times 10^3 \times 300}{29} \ln \frac{300}{266.04}$$

$$= 1.98172 \times 10^{10} \text{ Pa}^2$$

$$\text{or, } p_2 = \sqrt{p_1^2 - 1.98172 \times 10^{10}} = 264.92 \text{ kPa}$$

Substituting this value of p_2 into the right hand side of eq. 3 we get, $p_2 = 264.92 \text{ kPa}$



So, the remaining is $2 G^2 R T$ by M , which we have found out with the incorporation of values. And that has become 2 into 10 to the power 7 , sorry, 2 into 170 whole square into 8.314 into 10 to the power 3 into 300 by 29 . This times \ln of 300 by 266.04 , right. The third term or the second term, which we have written in the third place, is 2 into 170 whole square into 8.314 into 10 to the power 3 , that is the value of R . Into 300 by $29 \ln$ of 300 by 266.04 , 266.04 , we have found out this P_2 in the previous assumption, assuming one, here we have found out 266.04 neglecting the logarithmic term.

That we obtained. So, here we have taken that as the input value, right. I hope there are many ways of finding out roots, Runge Kutta, etc. By which we used to determine with the help of a computer, of course, not so small equations, very very big equations, during our PhD program. So, similarly, here it is a very small one, and many ways are there to find out the roots.

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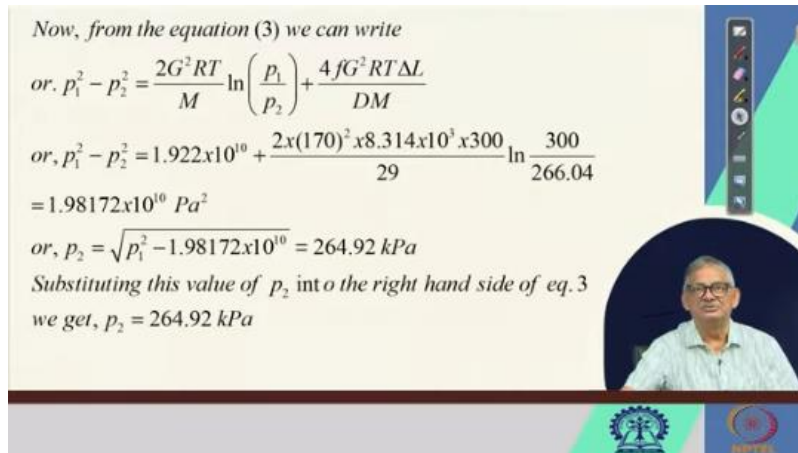
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Root means the solution of the equation, ok. So, here our first approximation, as we said, is We have taken the previous value, obtained value of P_2 , that is 266.04. By that, we get P_1 square minus P_2 square is equal to 1.98172 into 10 to the power 10 Pascal square, right. Therefore, P_2 can be obtained.

Under root p_1 square minus 1.98172 into 10 to the power 10, and by substituting the value of p_1 as well as calculating it properly, we get 264.92 kilopascal as the p_2 . You see, our first approximation when we did that, the logarithmic term we neglected and found out one value of P_2 . Now, in the next iteration, this value of P_2 , we have taken as the initial value, right. And we have found out the new value of P_2 , and the new value of P_2 has become 264.92; earlier, it was 266 point something. Here, 266.04 that $\ln 300$ by 266.04 in terms of the \ln term.

So, from there, we got 264.92 kilopascal. Now, if we again take 264.92 kilopascal as the input, then obviously We have to redo the calculation. Since I did it on my own, and since there was no one else to calculate, so what did I do? I simply took that equation above, where, we found out p_1 square minus p_2 square is equal to 1.922 10 to the power 10 plus 2 into 170 square into

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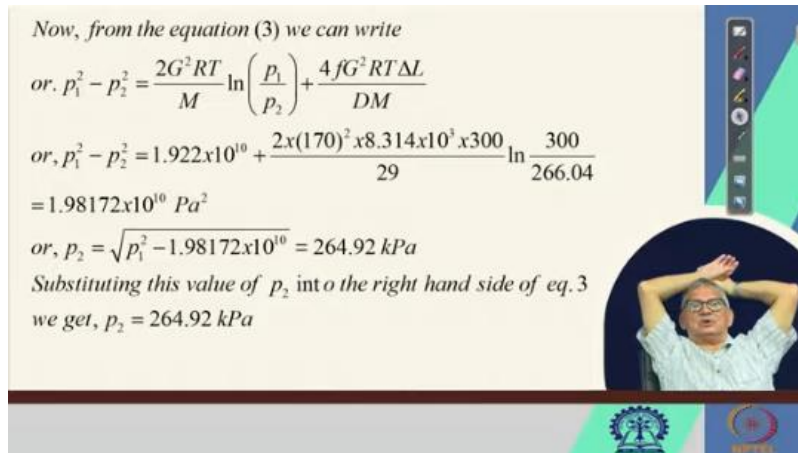
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8.314 into 10 to the 3 into 300 by 29, ln of 300 by 266.04; instead of that, now our thing is 264.92, right? All other remains the same. So, we got with this kind of change in the values of pressures, instead of putting 266.04, we put 264.92 and we obtain P_2 as 264.92 kilopascal. So, that means, you see, we have obtained two successive values of P_2 identical. Now, what I would like to highlight here to you is that it is not necessary that all the time it will be so easy that by one or two trials you are able to.

Find out. It may so happen that in the first trial you got 266.04, in the second trial you got 262.52, whereas your root or the value of the pressure, which you are determining, lies between this 266 and 262. So, again you have to do it and see where it is coming. It may or may not come exactly the same value, as in this case, 264.92 kilopascal.

Earlier also, we got 264.92 kilopascal. But you may get 265 point something or 264 point something. So, then you have to be very cautious and see whether two successive values are coming the same or not. At least the constant values, like 264, will remain the same, and now, how many how many digits after the constant 264 you want in your answer.

So, what do we do? We take and with some other technique, we also find out parallelly and see whether the value is the same or not, right? So, with this, obviously, I hope we have come to the end of the end of this session, but it is that this topic of compressible fluid flow is quite different from that of the incompressible fluid.

Now, from the equation (3) we can write

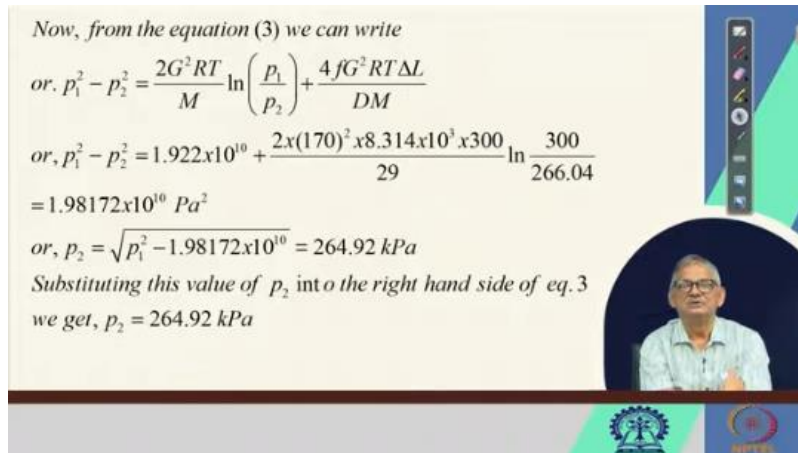
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right. We started with the Hagen Paisley's equation, sorry we started with the Bernoulli's equation, right. Found out the relation between pressure in terms of many other known parameters like velocity, like pressure, like temperature, like pressure, temperature, then molecular weight of the gas etc. These are many many variables which we need to know beforehand.

Only the good thing is that earlier we have not directly found out the value of the pressures because for any flow pressure is a must because inlet pressure the more the inlet pressure the more is your supply easy, easy to supply. for finding out what easiness it has, you will be in definitely problem. So, we conclude with this that the relation of pressures developed in compressible fluid is a typically quadratic type of equation, where the constants and many other variables or pseudo constant variables, they have found out that what is the value of the pressure from the given relation, right.

So, I wish you do some more problems and grasp it as if you can solve some problems and also the Bernoulli's concept of this. gas that is your compressible gas. Okay. Thank you very much.

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It is nice of you.

Thank you.