IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture33

LECTURE 33 : PRESSURE DROP RELATION FOR COMPRESSIBLE GAS

Good morning, my dear boys, girls, students, and friends. In the last class, you were discussing compressible gas flow, right? We said that the difference from earlier classes the fluid we had taken earlier was incompressible, right? But now we are dealing with compressible gas, which means the density is a function of pressure. Obviously, we said that we are starting with Bernoulli's equation. All other factors remain similar, like the flow being fully developed, steady flow, etcetera. Right, and we came to this. I said that after this, we will start because if repetition is more, then time is taken up.



So, our class periods are getting shortened. So, we came to this: G square dV by V dP by V 2 f G square dL by D equals to 0, right? Now, from here, we said that we define isothermal and adiabatic flow. I remember I also said in the last class that isothermal means where temperature is constant, and adiabatic means where dq is 0, meaning no heat transfer, right? So, under this situation, we know the ideal gas relation that is Pv equals to m R T.

Let,
$$G = \rho v = \frac{v}{V}$$

or, $dv = GdV$
 \therefore eq.1 can be rewritten as,
 $GV \ GdV + Vdp + \frac{4fG^2V^2}{2} \frac{dL}{D} = 0$
or, $\frac{G^2dV}{V} + \frac{dp}{V} + \frac{2fG^2dL}{D} = 0...(2)$
Now, for isothermal and adiabatic flow, we know that
 $pv = \frac{mRT}{M}$, or, $\frac{pv}{m} = \frac{RT}{M}$
or, $pV = \frac{RT}{M}$; or, $V = \frac{RT}{pM}$ Substituting in eq. 2, we get.
 $G^2 \int_1^2 \frac{dV}{V} + \frac{M}{RT} \int_1^2 p \ dp + \frac{2fG^2}{2D} \int_1^2 dL = 0$
 $G^2 \ln\left(\frac{V_2}{V_1}\right) + \frac{M(p_2^2 - p_1^2)}{2RT} + 2fG^2 \frac{\Delta L}{D} = 0$
 $Let, G = \rho v = \frac{v}{V}$
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by capital M. This small m rather we can rewrite. That Pv over m is equal to RT by M, or since v by m, small m is nothing but volume per unit mass, that is specific volume. So, we replace it with capital V. So, PV equals to RT by M. Therefore, we can write in terms of V, it is RT by PM, right? So, we just started with that equation number 2.

I can go back once more that G square dV by V plus dP by V plus 2 f G square dL by D equals 0. If we substitute V here, right, you see the first term we have not substituted it remains because dV by V is already there. So, if we integrate between two points between point 1 and 2, then we can say, and here also one more thing I said earlier if you remember, that we assume that z2 minus z1, that is, the height is not there because it is horizontal, right, that also we said if you remember, or if you do not remember then take it again that the pipe is horizontal.

Now, for isothermal and adiabatic flow, we know that

$$pv = \frac{mRT}{M}, \quad or, \quad \frac{pv}{m} = \frac{RT}{M}$$

$$or, \quad pV = \frac{RT}{M}; \quad or, V = \frac{RT}{pM}$$
Substituting in eq. 2, we get,

$$G^{2} \int_{1}^{2} \frac{dV}{V} + \frac{M}{RT} \int_{1}^{2} p \, dp + \frac{2fG^{2}}{D} \int_{1}^{2} dL = 0$$

$$G^{2} \ln\left(\frac{V_{2}}{V_{1}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2} \frac{\Delta L}{D} = 0$$

So, we are integrating it between two points 1 and 2. So, dV over V plus M by RT here we are replacing that V, right, dP over V, it was the second term, right, dP over V was the second term. So, this capital V, that is specific value, we have replaced with RT by M or RT by P M and the P has gone inside because P is here variable.

So, M by RT, which are constants, are out of integration, and integration between point 1 and 2 of P dP, right, plus 2 f G square by D, here there was no V capital. So, we take that length from 0 to 1 point to another point, right. So, L_1 to L_2 or whatever we say that is between point 1 and 2 dL, this equals 0. So, on integration, we get G square since it is dV by V, it is nothing but ln V and by putting the boundary which we have given between point 1 and 2.

So, we can write $\ln(V_2/V_1)$, ok, plus, Since it is the integration of P dP, it will be P²/2 on integration, right. So, that P²/2, if we replace with the boundary that is point 1 and point 2, then we can write $(M/2RT)(P_2^2 - P_1^2)$. Of course, this we are arranging as $2fG^2\Delta L/D$ or L/D. Normally we know that L/D is a non-dimensional, dimensionless parameter, right. So, we come to G² ln(P₁/P₂),

Because here, you see, it was $G^2 \ln(V_2/V_1)$, ok. And we have shown earlier that V = RT/PM, right. So, we can write of V_2 and V_1 , you see, from this relation. So, we see that V = RT/PM, right, and from there we can write here itself that P = RT/(VM),

right. So, that means, P is inversely proportional to V, right. So, RT/PM remains constant for the two points 1 and 2. So, it is P_1 and P_2 . So, that can be shown as

Now, for isothermal and adiabatic flow, we know that

$$pv = \frac{mRT}{M}, \quad or, \quad \frac{pv}{m} = \frac{RT}{M} \qquad \mathcal{R} = \mathcal{R} + \mathcal{R}$$

that, P_1 is inversely proportional to V_1 and P_2 is inversely proportional to V_2 , right. If that is true, then what we can write, what we can say is that what we can say is that this R T by P M, that one, then V_2 , this G square ln V_2 by V_1 we can easily replace with P_1 by P_2 , right. So, from that relation of P inversely proportional to V, we can write P_2 by P_1 equals to V_1 by V_2 , right. So, V_2 by V_1 , we can write that is equal to P_1 by P_2 , right. So, if you substitute this in this part, then we can we can say that G square ln P_1 by P_2 plus M by 2 RT into p_2 square minus p_1 square, which is normal, because here it was already shown that p_2 square minus p_1 square, right, here it was normally shown.



$$G^{2}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)

So, p_2 square minus p_1 square by 2 RT M by 2 RT plus 2 f G square delta L by D. So, this is equal to 0. Right, or we can say that p_1 square minus p_2 square in terms of that is 2 G square RT by M ln p_1 by p_2 plus 4, we have replaced we have replaced some. So, it is becoming 4 f G square R T delta L by D M, right. How did you get it?



We got it, you see, from the term. From the term M by 2 R T p_2 square minus p_1 square. So, that p_2 square minus p_1 square we have taken here, right? We remain M by 2 R T. Now, we have taken that p_2 square minus p_1 square. Okay, out. So, we have M by 2 RT here, okay? Here we have G square ln P₁ by P₂, right?

$$G^{2}\ln\left(\frac{p_{1}}{p_{2}}\right) + \underbrace{\frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT}}_{QRT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)

So, if we divide with M by 2 RT, then it becomes P_1 square minus P_2 square. G square remains as it is. G square was there. So, M by 2 RT, this 2 and this RT remains there, and it was ln P_1 by P_2 , right? Since we have divided with M by 2 RT here also, then we have. Now this 2 and this 2 is making 4, f was there, G square was there, f and G square remains there, delta L by D this was there, it remains there.

So, the additional term is M by RT, right? So, this is what by dividing with M by 2 RT all the three terms we arrive. This equation or this relation that is p_1 square minus p_2 square. Why we have taken p_1 square minus p_2 square? Because you see here, it was here, it was.

$$G_{2}^{3}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} - \frac{2G^{2}RT}{M}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM_{+}}$...(3)

$$G_{2}^{3}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

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 p_2 square minus p_1 square, right? And this all equals 0. So, we moved it to the other side. So, the sign changes. So, it becomes p_1 square minus p_2 square. Hopefully, this is understandable, and we have understood how the relation has been built up between pressure and all other terms.



Obviously, here you see everything is expressed in terms of pressure, right? Everything is expressed in terms of pressure. p_1 square minus p_2 square, then $\ln p_1$ by p_2 , right, and all other terms are more or less known or constant. For example, we know the term G, capital G, which we said to be nothing, but which we said to be nothing, but rho v, right?

$$G^{2} \ln \left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M} \ln \left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)
...(3)
 $G^{2} \ln \left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$
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...(3)

That is the mass flux or mass velocity, right? So, you can take it; R is the universal constant, known; T is the temperature of working, which is also known; M is the molecular weight of the fluid, right? Here, f is the friction factor. It can be determined or obtained, as we have seen earlier. G, we have already said; R, T, known; delta L, how much between point 1 and 2, what is the gap that we know.

$$G^{2} \ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M} \ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)

The diameter of the pipe through which this compressible fluid is flowing is also known, but here you see one difficulty is present. That difficulty, in the sense that whenever you are handling this with pressure, then you have a quadratic equation: P_1 squared minus P_2 squared is equal to something in terms of $\ln P_1$ by P_2 plus a constant, right? It is a quadratic equation. So, the solution may not be that easy. Right, because what is going to happen?

We have the pipe; we have the pipe; this diameter is known, right? We have the fluid flowing through this. Right, we know the length of the pipe. So, that is delta L. So, delta L and D are known. We know at what velocity it is coming. So, the mass velocity G is known.

$$G^{2}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)

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Because the density of the fluid at that temperature is also known, since we are giving a definite temperature for the operation, right. We know the universal gas constant R, and since we know the fluid which is flowing. So, the molecular weight is also known. So, all these parameters are known.



So, but still, we have one equation like, Still, we have one equation like x squared plus a b x plus c is equal to 0, right. In earlier days, I do not know if you have seen that under this kind of situation, if we do not take b also, if we take only a x, then in earlier days, we have seen that this is a. So, we can find out with minus, say, ok, that will be easier instead of a if we write it to be b x, right. So, we used to say a x squared plus b x plus c. So, by Sridharacharya



solution, we can say the root of this is minus b plus minus b square minus 4ac, b square minus 4ac by 2a. So, you can find out the solution, what is the root means, what is the value of x, right. That we could find out. But here it is not exactly so because we have another very uncommon term which is ln of p_1 by p_2 which is ln of p_1 by p_2 . Now, out of these two, as we said that in the pipe, we know the length, we know the diameter, fine, but we do not know what is the pressure at the inlet which is P_1 higher.

$$G^{2}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M}\ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)
 $M + bM + C = 0$

And what is the outlet pressure P_2 ? These two are not known. Even if one is known out of the two, either exit pressure or the inlet pressure, if they are known, then we can substitute say P_1 is known, this we can find out P_2 , provided we know $\ln P_1$ by P_2 . $\ln P_1$ by P_2 is not known because we do not know P_2 . If P_1 is known, so we have to find out P_2 .



So, for that reason, it may be required that You may have to do trial and error, right, because here both are not there, p_1 and p_2 both are not there. So, what to do? So, we assume one p_1 , if p_2 is given, p_1 is some assumed P_2 is given, then by this assumed value P_1 and P_2 , we know ln P_1 by P_2 , we know this



We also know this term, say this is equal to C, some number because all 4 f G R T delta L D M are known. So, a constant value we also know this, say 2 G square R T by M. So, this is say, C right. So, we have this p_1 square minus p_2 square B ln p_1 by p_2 plus c, right. Out of that, we assumed p_1 and p_2 is given, we have to find out p_1 .





So, we know the value of p_1 by knowing that value of P_1 , by assumed value of P_1 by P_2 , you find out the value of P_1 here, right. If the value is too much by trial and error, you have to do if the value is too much high or low. So, you have to take another value, then this value, you can take, whatever you have obtained, as assumed value.



Again, do that finding. Find out if you already have the same B, you already have the same C. So, you know $\ln P_1$ by P_2 because the new P_1 is the obtained P_1 . So, you again find out the P_1 value. You will see by this trial and error, maybe within a couple of trials and errors, you can determine the value within your acceptable limit of error, right. That limit, obviously, you have to fix whether you are happy with a single digit, double digit, or triple digit. Triple means, I, what I mean after the decimal.



So, this way, this can be done, okay. Now, the time is up for this class. Let me thank you all for listening to the class.

$$G^{2} \ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{M\left(p_{2}^{2} - p_{1}^{2}\right)}{2RT} + 2fG^{2}\frac{\Delta L}{D} = 0$$

or. $p_{1}^{2} - p_{2}^{2} = \frac{2G^{2}RT}{M} \ln\left(\frac{p_{1}}{p_{2}}\right) + \frac{4fG^{2}RT\Delta L}{DM}$...(3)

Thank you.