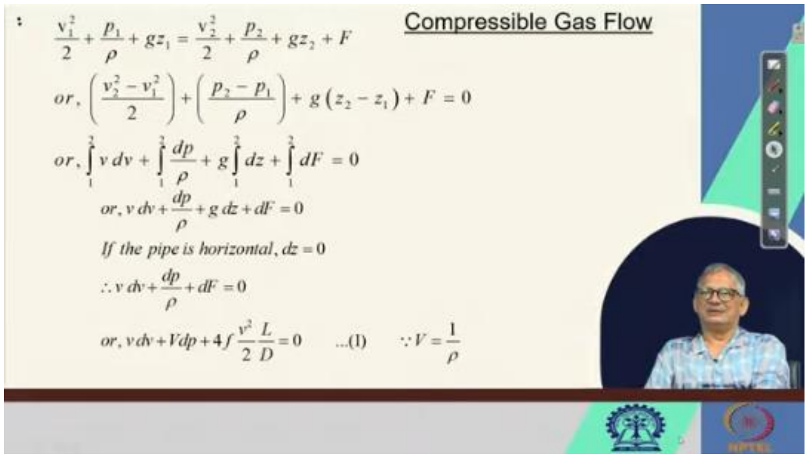
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture32

LECTURE 32 : COMPRESSIBLE GAS FLOW

Good morning, my dear boys and girls, students, and friends. We are not at the end, but we are almost two-thirds of our course, and now we will handle compressible gas flow, right? Whatever we have handled so far was incompressible, right? An incompressible fluid means the density is constant, right? But also, we had taken Newtonian fluid into account, right? Here, we are not saying that we will be handling non-Newtonian fluid as well. Because Newtonian fluid, or sorry, non-Newtonian fluid, is altogether different.

Its treatments are also different, and as I said earlier, one of my co-teachers, Professor Bhuhiya, will come and take care of the non-Newtonian fluid. Right? But now we are dealing with compressible gas flow, right? Definitely, the moment we say it is compressible, that means the fluid has a density that is susceptible to changes in pressure. Right? Unlike incompressible fluid, which we said earlier to be that. The density does not change appreciably, or the change in density is not significant, but here, the change in density is significant, and we call it compressible gas, where the

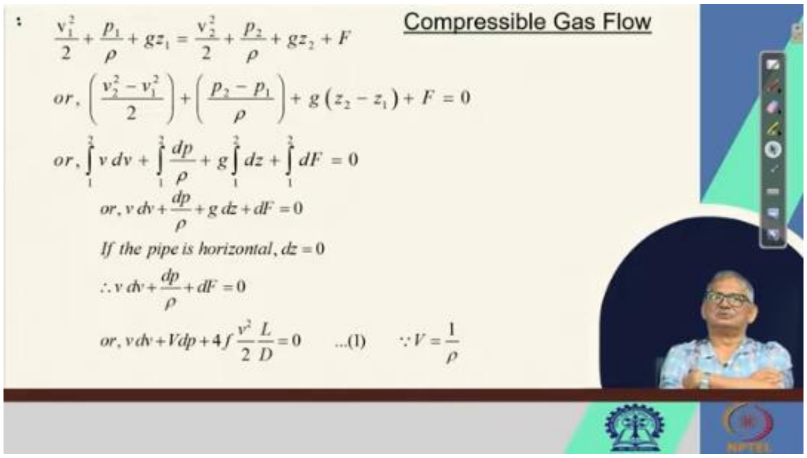


gas is compressed. The moment you compress it, it undergoes a change in density, right? So, here we start with an equation that is very well known to you. I give you this chance so that you can recollect where we are starting from, right? Yeah, between two points, one and two, if the velocities, pressures, and height.

So, the first term is the velocity gradient, right, and the second term is the pressure head, right, and the third term is the gravity gradient, right. So, this sum of the forces between the two points since the density is changing with pressure. So, we can say that the sum of the forces is equal to 0, right.

And under that, we can say that v squared by 2 at point 1 plus P_1 by rho plus g_{z1} equals v_2 squared by 2 plus P_2 by rho plus g_{z2} plus all other forces F, right. I hope by this time you have come to know what it is referring to. If you are not, then I tell you that this is the form of Bernoulli's equation for the fluid flowing, right. For compressible gas, we are using Bernoulli's equation between two points at a distance between z_2 and z_1.

Right. So, we have seen in Bernoulli's equation the sum of the forces equals 0. So, that means between two points, 1 and 2, we can write v_1 squared by 2 plus p_1 by rho plus g_{z1}. This is equal to v_2 squared by 2 plus p_2 by rho plus g_{z2} plus F, right.



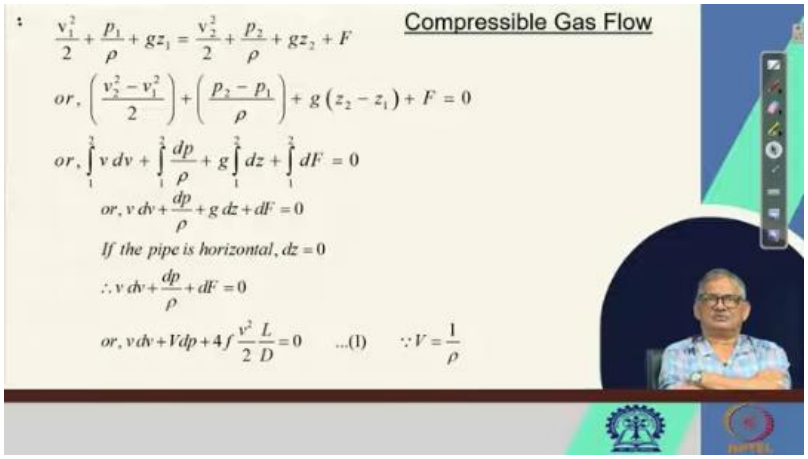
Now, upon rearrangement, we can write v_2 squared minus v_1 squared divided by 2 plus p_2 minus p_1 divided by rho plus g multiplied by z_2 minus z_1. This plus was there, but we have brought it here by taking all forces, all forces other than velocity head, pressure head, and gravitational head, we are saying all other forces equal to F, right? So, this means that this is v_2 squared minus v_1 squared divided by 2, which means it is the integral of v dv between point 1 and point 2, plus the integral of dp divided by rho between point 1 and point 2, plus g dz, the integral between point 1 and point 2. Plus again, the summation of all forces dF between point 1 and point 2 is equal to 0, right? This is actually the development starting from Bernoulli's equation.

Well, then by removing the limits, that is, point 1 and point 2. In general, we can write, in the differential form, Bernoulli's equation as v dv plus dp divided by rho plus g dz plus dF equals 0, right. Again, add another assumption if you put here, that is. Bernoulli's equation can be used anywhere, in any pipe, anywhere, whether it is horizontal, vertical, or zigzag, anything, right, provided you have to know all other forces properly.

So, if we assume that your pipe system is horizontal, then dz becomes 0 because there is no vertical height; z_2 minus z_1 is not the distance, but the height, ok. Then we can say for horizontal pipes, dz is 0. Therefore, we can write v dv plus dp divided by rho plus dF is equal to 0, right.

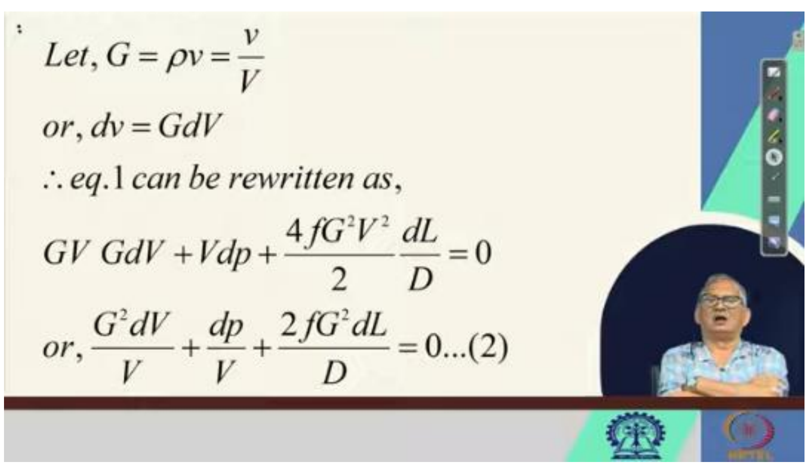
Now, you see, v dv, dp by rho, and dF, these are the three different terms. Now, we have to bring them under one umbrella. So, we can write v dv plus 1 by rho dp. Instead of that, we can write capital V, which is nothing but the inverse of density, that is specific volume. Right? The inverse of density, that is specific volume, we know, capital V is nothing but equal to 1 by rho.

So, instead of 1 by rho dp, we write capital V dp, right? And let us take this dF in the form of the Fanning friction factor or frictional force, because in pipe flow, it is mainly the frictional force that obstructs the flow. So, that is 4 f v square by 2 into L by D. So, we assume all these forces that dF, it includes all forces, mainly or primarily the frictional force, because that is the prime force that acts opposing the flow.



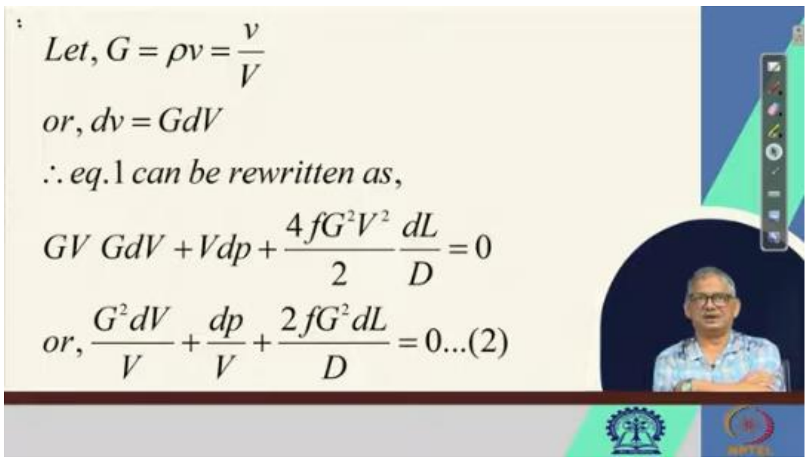
This is the force that acts as the drag force, right? And that we are taking as 4 f v square by 2 into L by D, right? Assuming again capital V equals to 1 by rho, right? If this is true, then we can write

a term called capital G, that is, rho into v, that is mass velocity, rho into v, capital G, right. Mass velocity, that is, rho is kg per meter cube, v is meter per second. So, again it is mass flux, kg per meter square per second, right? So, this we can write, small v by capital V. So, do not mix up small v is the velocity and capital V is the specific volume, right.



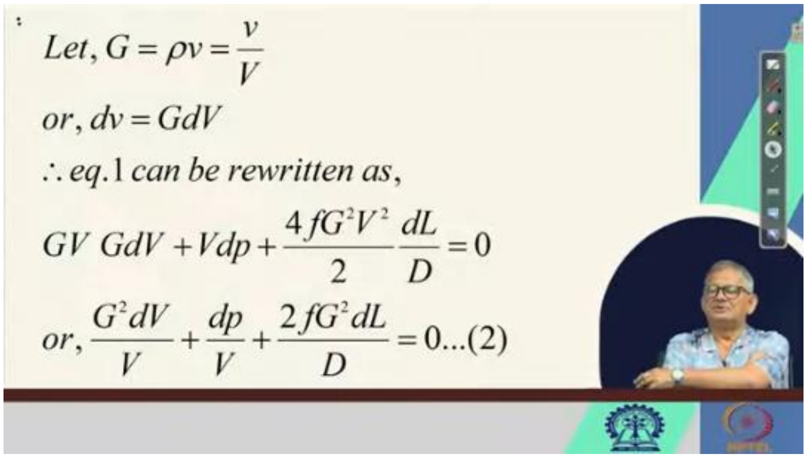
So, since we can write, capital G equals to rho v or capital G equals to v by d small v by capital V. So, in the differential form we can write dv is equal to capital G into dV. G, which is constant, because, that is mass per meter square per second, that is mass flux, right. So, that comes out of the integral or differential form. So, dv is equal to capital G dV, small dv is equals to capital G d capital V. Then, earlier, we had written this equation, which we have not given any name ok, maybe

that v dv plus dp by rho plus dF is 0 could be, no here, equation 1, ok, that v dv plus V dp plus 4 f v square by 2 into L by D is 0. So, this we can rewrite, with that new definition of or introduction of the new parameter, that is, capital G or called mass flux or mass velocity. right. So, that we can write as capital G V. You see the equation was, again, we go back, equation was v dv plus V dp plus 4 f v square by 2 L by D. So, small v, Right?



So, here we have seen that small v is nothing but capital G V. Okay? So, capital G, capital V. Right? And dv we have seen here is capital G d capital V. Right? So, the first term is okay. Then the second term was dp by rho.

So, that we have changed into rho into specific volume, capital V. So, capital V dp, right? Plus, we have seen that the frictional force is 4 f, if we go back, 4 f v square by 2 into L by D, right. So, here also we remain 4 f into v square, which is nothing but capital G square capital V square, right? v square by 2.



So, 4 f G square V square by 2 and L by D, we write L as dL, because in the differential form we are taking So, the small unit is dL over D is fixed. So, over D, the constant is equal to 0, right. So, this on rearrangement we can write that capital G square dV by V, right? capital G square dV by V plus dp by V plus 2 f G square dL by D is equal to 0.

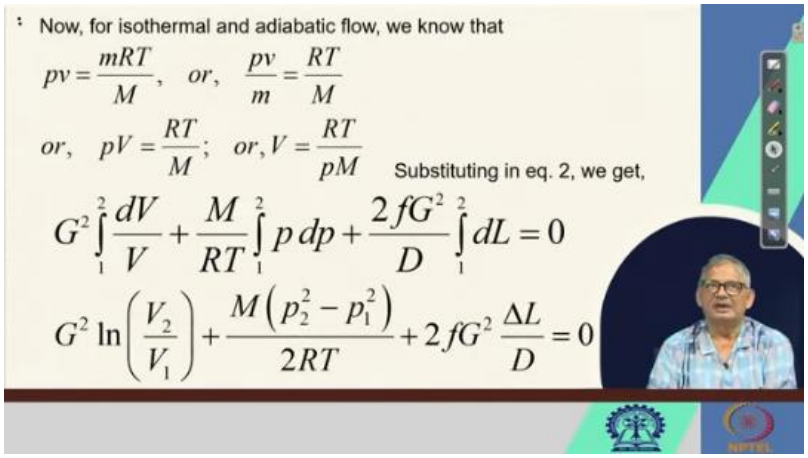
What did we do? We divided all with the term capital V square. We divided all the terms with capital V square. So, here we have one denominator V. So, G square dV by V.

are also V dp, it was. So, it becomes dp by V and in this all V square goes. So, 2 f G square d L by D equals to 0 and let us take the name of the equation or number of the equation it to be say 2. So, that we can identify as and when required.

So, what did we do for compressible gas flow? We started with Bernoulli's equation and we assumed that whatever forces are acting other than gravitational force, pressure and velocity force, ok. So, these three forces we have identified and rest of the sum of all forces we have assumed it to be nothing, but the frictional force, right.

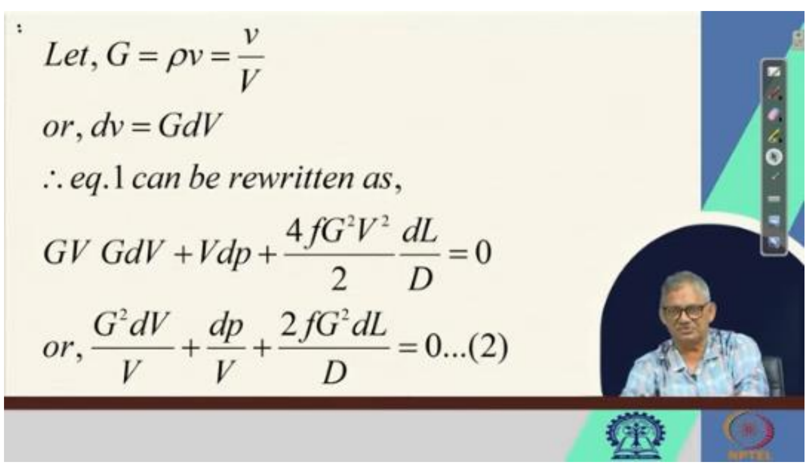
So, that is why we have taken 4 f v square by 2 L by D, right. And then we have defined a new term called capital G, which is nothing but mass velocity or we can say it to be mass flux, because we have said earlier also, a flux means anything per unit time per unit area. So, here also rho into v, rho is kg per meter cube and v is meter per second. So, it means kg per meter square per second, which is mass per unit area per unit time, right.

So, that we have introduced. And we have substituted all small v's in terms of capital V's and capital G, wherever it is possible, right? And by that, we have come to this point that we have an equation, say, we can name it as equation 2, as G square dV by V plus dp by V plus 2 G square dL by D, this is equal to 0, ok. Next, for isothermal and adiabatic flow.



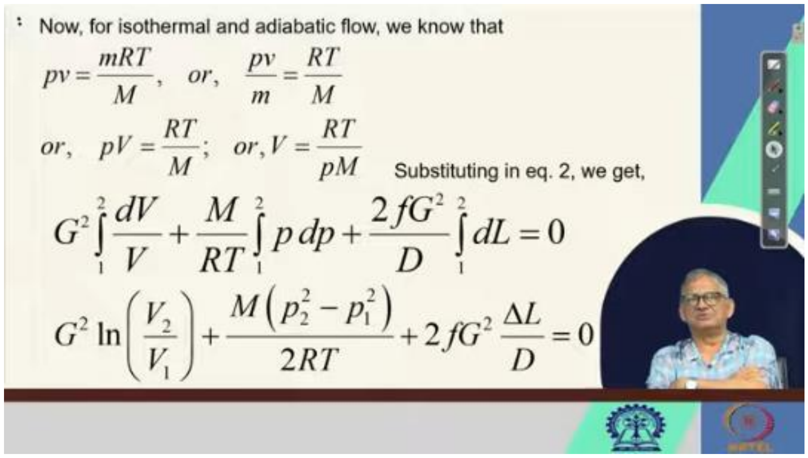
These two are different. I hope you know what is isothermal and what is adiabatic flow, though it is not part of the fluid flow, but yes it can be said it is part of fluid flow because isothermal means where the temperature remains constant, isothermal right, and adiabatic where the heat transfer is 0. Right, dq is 0. So, under these two situations, isothermal flow or adiabatic flow, we can write that P v equals to m R T by capital M or P v by m is equal to R T by M. So, small v by m is capital V, which is specific volume. So, P capital V equals to RT by M capital M, right.

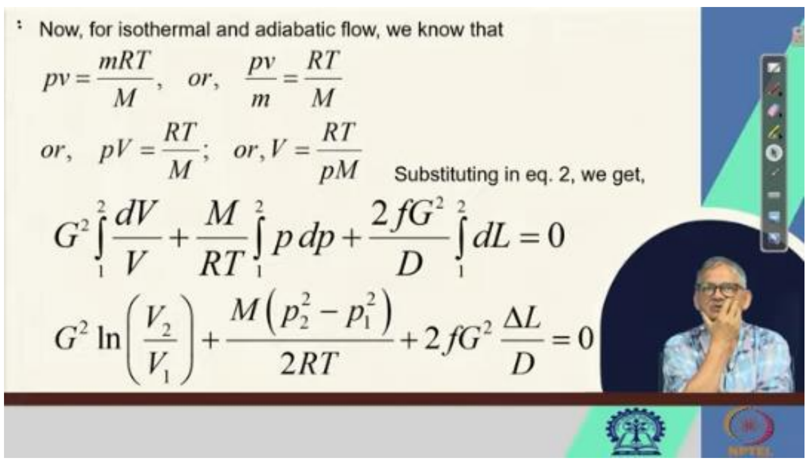
M is the molecular mass, or capital V, you can write to be equals to RT by PM. Right, RT by PM. Now, this RT by PM, if you substitute in the previous equation, which we named as equation number 2. What was it? It was G square dV by V plus dP by V plus 2F G square dL over D equals to 0. So, here we substitute capital V with RT by PM.



R is the universal gas constant, T is the temperature, P is the pressure, and M is the molecular weight. So, if we substitute and rearrange obviously, substitute means here you see capital V is RT by PM. So, we are replacing capital V with RT by PM. So, G square

RT by PM, PM by RT into dV, right. So, afterwards, what perhaps could have been that If you rearrange, means, you see, we have jumped steps, we have jumped a step here, G square dV by V. So, if you substitute V with RT by PM here, we substitute with RT by PM, but there is no V.

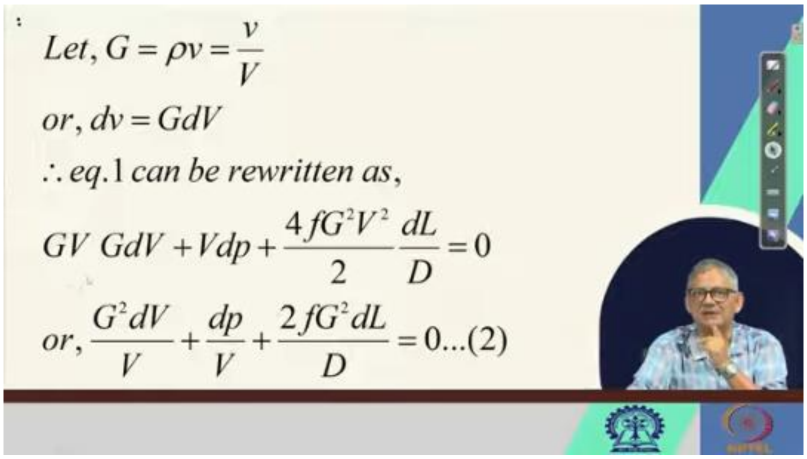




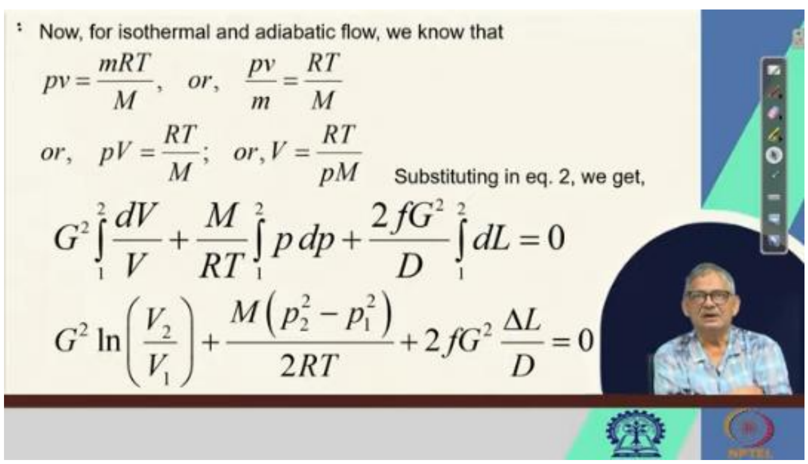
So, and G square is out of the differential or integral part, right. So, we can say that, this RT by PM, this RT by PM, these two if we take care, then The equation comes to G square between point 1 to 2, dV by V, right, and M by RT is this. This was dP by V, right. So, that V, we have substituted with M by RT, and P we have taken into inside.

So, it is P dP. Integral of p dp between point 1 to 2, integral dV by V between point 1 to 2 into G square plus 2 f G square by D, again between point 1 to 2, dL, this is equal to 0, right. Now, this we have not in the first place we have not here, dV by V we have not substituted. We only substituted dV by V there, that V. Right. In terms of R T by P M, and that P has gone inside, right.

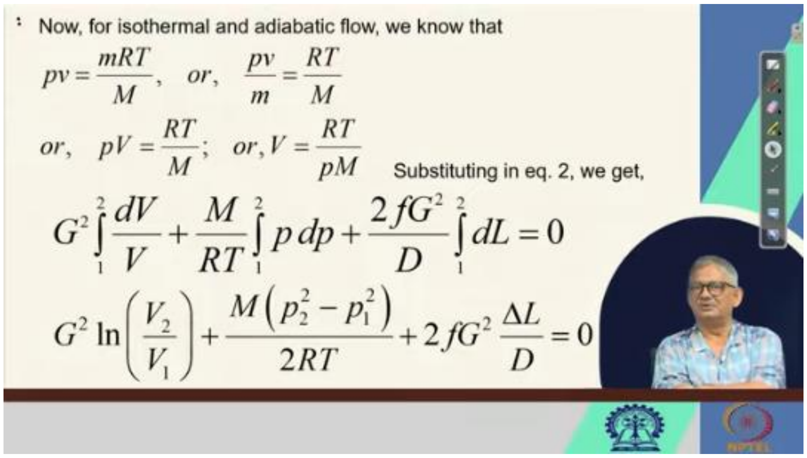
So, R T by P M that is it has become M by R T and integral of P dP, right. Here it is integration between point 1 to 2, dV by V, right. Here it is R T by, sorry, M by R T. And p dp between point 1 to 2, and this dL, we are taking as integral between point 1 to 2 dL 2 f G square by D, right. So, that is what we have come, ok.



Now, on integration it gives G square. ln V_2 by V_1, by substituting the domain that G square ln V_2 by V_1 plus M now p dp. So, it is p square by 2. So, p_2 square minus p_1 square by 2 R T. Right, plus 2 f G square by D and this dL can be written as delta L or ultimately it is capital L, right.



So, 2 f G square delta L by D or L by D, which normally we write as dimensionless length or any non-dimensional parameter, right. So, after integration, we have G square ln V_2 by V_1 plus M by RT P_2 square minus P_1 square plus 2 f G square delta L by D equals 0. So, with this, perhaps, before proceeding further, our time for this session is up, and obviously, in the next class, we will start with this as G square after, because we will not repeat, we have explained clearly and unambiguously that G square ln V_2 by V_1 M P_2 square minus P_1 square by 2 R T plus 2 f G square delta L by D equals 0.



This from here will move forward with compressible fluid flow, ok. So, thank you for listening.

Thank you so much.

Thank you.