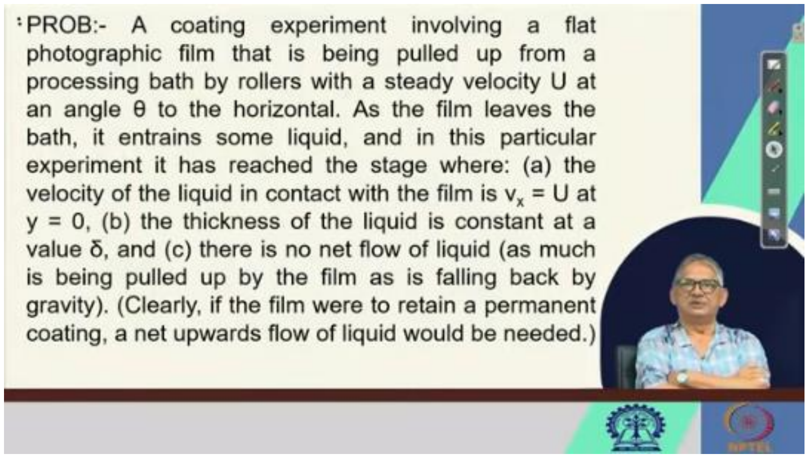
# IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture28

## LECTURE 28 : PROBLEMS AND SOLUTIONS OF MOVING SURFACE FLOW

Good afternoon, my dear students, friends, boys and girls. Now we have done flow through surfaces, right? Where the surface was maybe inclined or vertical, that depends on what angle you are providing. But there the surface was stationary. Now, let us see what happens if there is surface which is also moving.

So, we term it as problems and solutions on moving surface fluid flow ok. So let us take a problem. The problem is like this. A coating experiment involving a flat photographic film that is being pulled up from a processing butt by rollers with a steady velocity u at an angle theta to the horizontal.



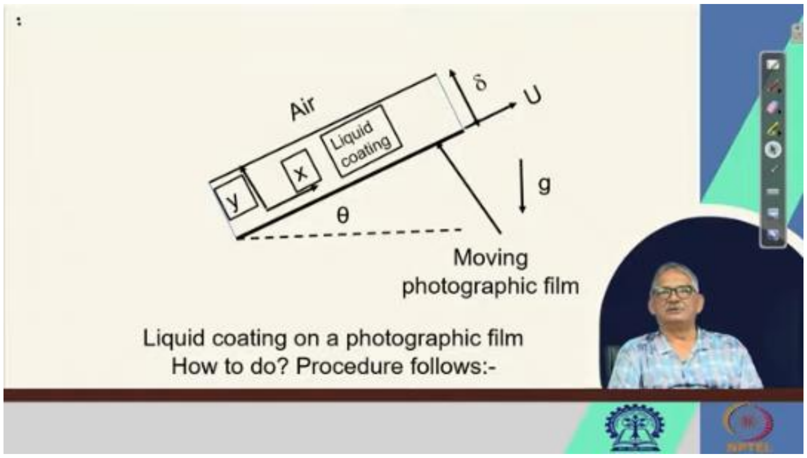

As the film leaves the bath, it entrains some liquid and in this particular experiment, it has reached the stage where number a, the velocity of the liquid In contact with the film is vx is equal to u at y is equals to 0. And b, the thickness of the liquid is constant at a value del (δ). And see, there is no net flow of liquid as such is being pulled up by the film as is falling back by gravity. This means it is clearly if the film were to retain a permanent coating, a net upwards flow of liquid would be needed.

Right? I hope it is a big problem. Obviously, Not as simple as earlier cases, but definitely we have to handle, isn't it? This is something like that I don't know whether you have seen or not, even in X-ray films and other photographic films, they need to be treated with some solution.

So that is immersed into that solution and then taken away. So based on that phenomena, this problem looks similar. It is saying that a coating experiment involving a flat photographic film that is being pulled up from a processing bath by rollers with a steady velocity u at an angle theta to the horizontal. As the film leaves the bath,

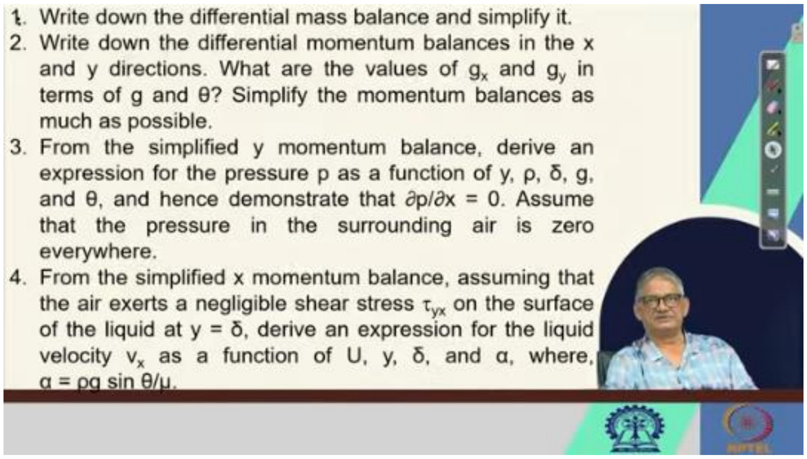
it entrains some liquid and in this particular experiment, it has reached the stage number A), the velocity of the liquid in contact with the film is $v_x$ is equals to u at y is equal to 0. Number B), the thickness of the liquid is constant at a value del. And number C), there is no net flow of liquid as much is being pulled up by the film is falling back by gravity. So this says clearly that if the film were to retain a permanent coating, a net upwards flow of liquid would be needed, right.

So, you have understood the problem. Now, let us see how this can be solved, ok. As you see, we have taken a volume element So, this is the liquid coating we have said that it is having thickness del and velocity is u in the x direction and the thickness is in the y direction del. And the coating of the film




and the upper layer which is air. So, there is if you remember earlier we had said gas-liquid interface, liquid-liquid interface, liquid-solid interface, right? And this surface solid is moving that is the photographic film is moving with an angle with the horizon as theta. Like liquid coating on a photographic film, how to do?

And this procedure it follows like this. Write down the differential mass balance and simplify it. Then write down the differential momentum balance in the x and y directions. What are the values of gx and gy in terms of g and theta? Simplify the momentum balances as much as possible.

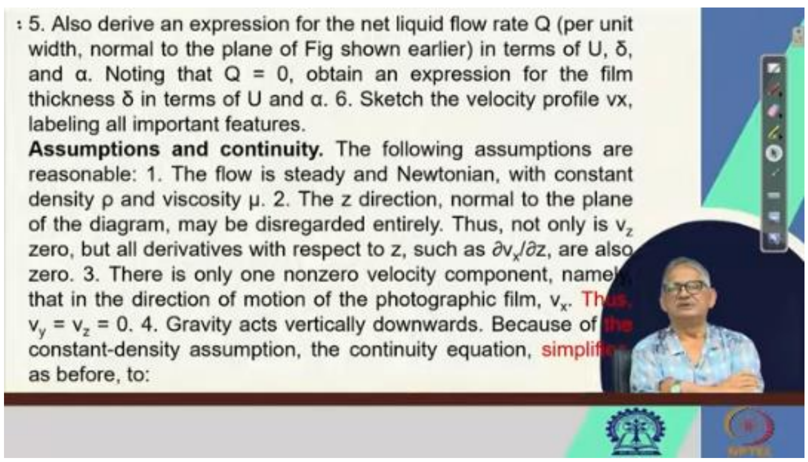



From the simplified y momentum balance, derive an expression for the pressure P. as a function of y, rho, del and g, g and rather theta. And hence demonstrate that del p del x is zero. Assume that the pressure in the surrounding air is zero everywhere. From the simplified x momentum balance assuming that the air exerts negligible shear stress $tau_{yx}$

on the surface of the liquid at y is del, derive an expression for the liquid velocity $v_x$ as a function of u, y, del and alpha.

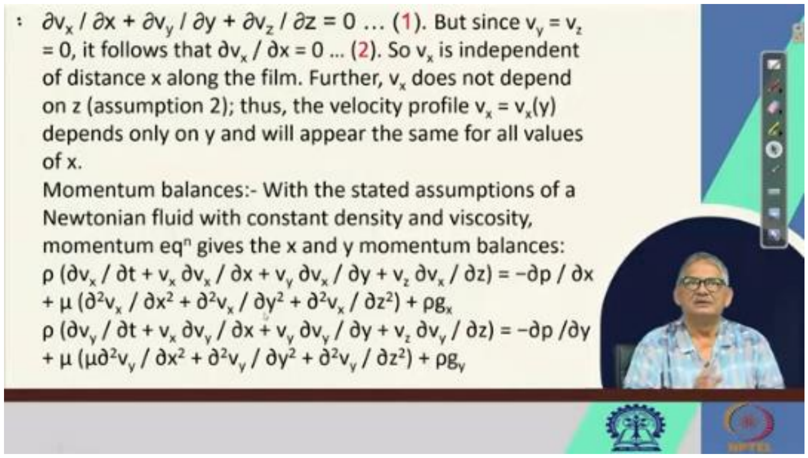
where alpha is rho into g into sin theta by mu, right. Alpha is rho into g into sin theta by mu. Then also derive an expression for the net liquid flow rate that is Q per unit width normal to the plane of the figure as we have shown it earlier. in terms of u, del and alpha nothing that sorry noting that q is equal to 0 obtain an expression for the film thickness del. in terms of u and alpha and last one is sketch the velocity profile $v_x$ leveling all important features right.

So, we have described how you should tackle the problem, what are the steps you could And do accordingly. Right? Some assumptions also may be required like the following assumptions are reasonable. Number one, the flow is steady and Newtonian with constant density rho and viscosity mu.



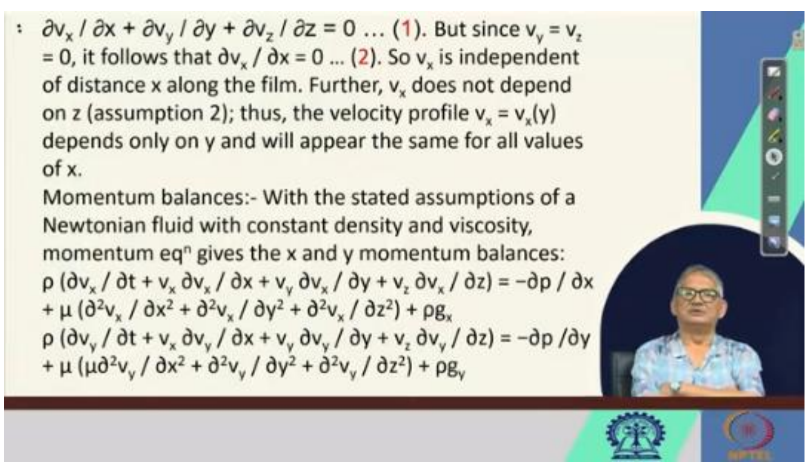

the z direction normal to the plane of the diagram may be disregarded entirely. Thus not only is $v_z$ is 0, but all derivatives with respect to z such as del $v_x$ / del z are also 0. There is only one non-zero velocity component and that can be named as in the direction of motion of the photographic film $v_x$. Thus $v_y$, $v_z$ is zero and gravity acts vertically downward because of the

constant density assumption, the continuity equation simplifies as like this. del $v_x$ del x plus del $v_y$ del y plus del $v_z$ del z is equal to 0. Since $v_y$ is $v_z$ is equal to 0, it follows that del $v_x$ / del x is equal to zero. So, $v_x$ is independent of distance x along the film.

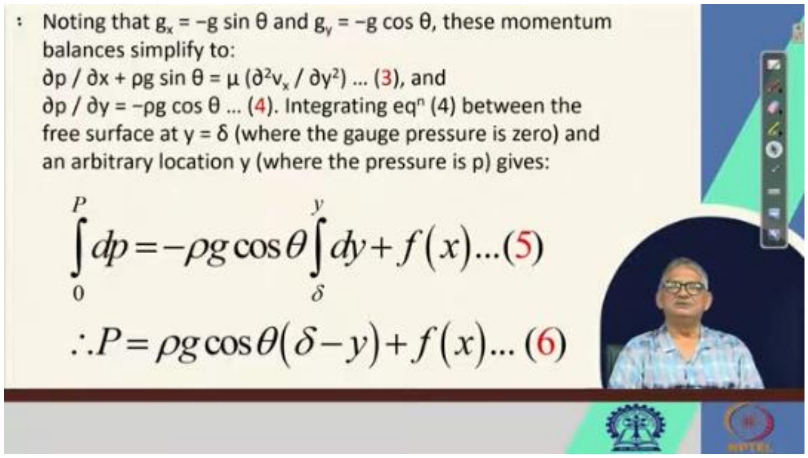



Further, $v_x$ does not depend on z, we have assumed earlier. Thus, the velocity profile $v_x$ is a function of $v_{xy}$ and this depends only on y and will appear the same for all values of x. Now, this was with the continuity equation. Now, with the momentum balance with the started assumptions of a Newtonian fluid With constant density and viscosity.

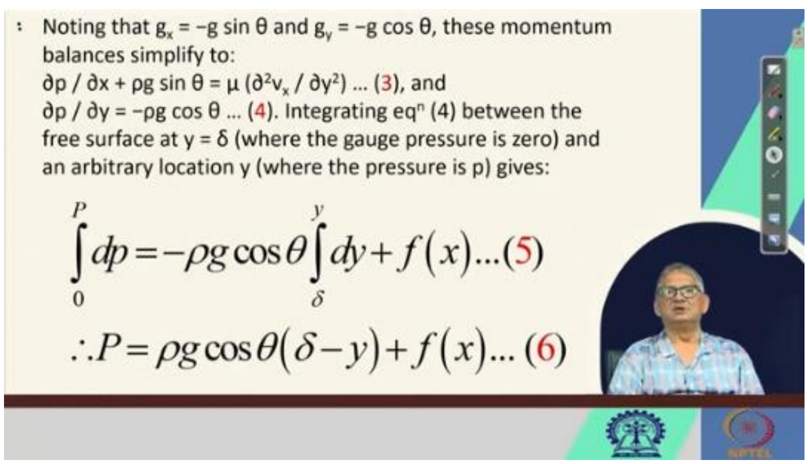
Momentum equation gives the x and y momentum balances. So we can write. rho times del $v_x$ / del t plus $v_x$ del $v_x$ / del x plus $v_y$ del $v_y$ / del y plus $v_z$ del $v_z$ / del z. This is equal to minus del p / del x plus mu del 2 $v_x$ / del x square plus del 2 $v_x$ / del y square plus del 2 $v_x$ / del z square plus rho $g_x$. right. This is one and another one is rho times del $v_y$ / del t plus $v_x$ del $v_y$ / del x plus $v_y$ del $v_y$ / del y plus $v_z$ del $v_z$ / del z del $v_y$ / del z rather is equal to minus del p / del y plus mu del 2 $v_x$ / del x square

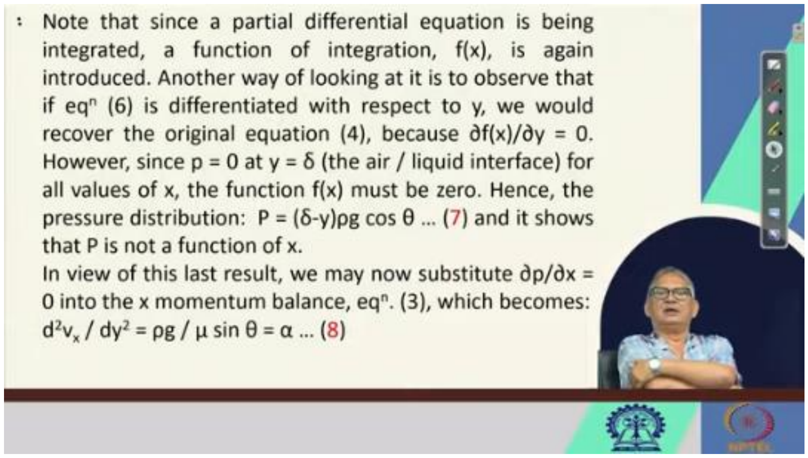



over yeah del 2 $v_x$ / del x square plus del 2 $v_y$ / del y square plus del 2 $v_y$ / del z square plus rho $g_y$. These are the two x and y momentum balances we get right. Then we note it here that $g_x$ is nothing but minus g sin theta and $g_y$ is nothing but minus g cos theta. So, these momentum balances simplify to del P / del x plus rho g sin theta. is equal to mu del 2 $v_x$ / del x square and del p / del y is equal to minus rho g cos theta.

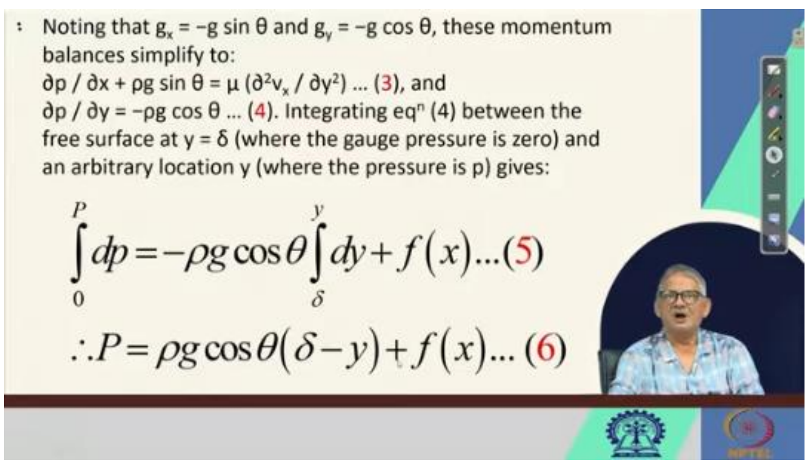


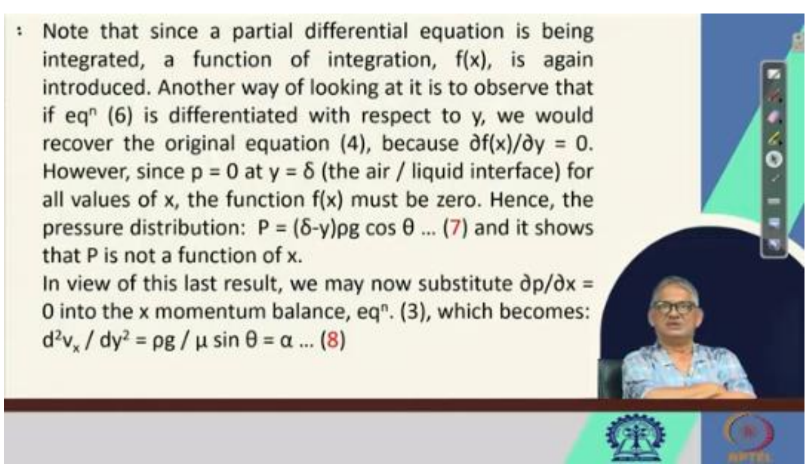

Now, integrating between, integrating 4 equations, this between the free surface at y is equal del ( ). where the gauge pressure is 0 and an arbitrary location y, where the pressure is p. This gives integral of dp between 0 to p. is equal to minus rho g cos theta, again between del and y, integral of dy plus function of x. Therefore, we can write P on integration, rho g cos theta into del minus y, we have taken care of minus, plus function of x, right? Again, let us look into that.



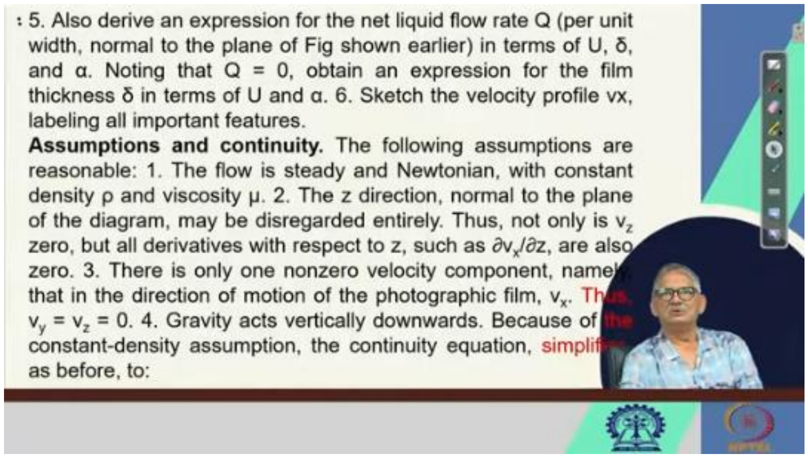



Since a partial differential equation is being integrated, a function of the integration, that is f (x) is again introduced. Another way of looking into it, as looking into it, as it is, rather to observe, at it is to observe that if the earlier equation number 6 which is this one, earlier equation number 6 is differentiated with respect to y. We would recover the original equation that is equation number 4.

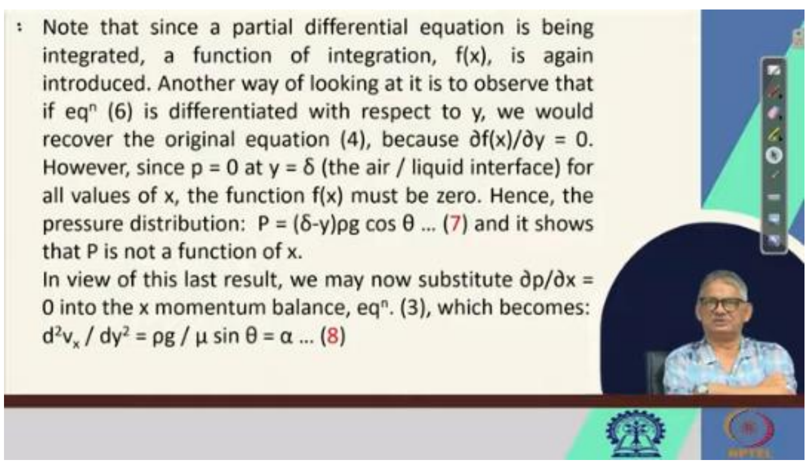


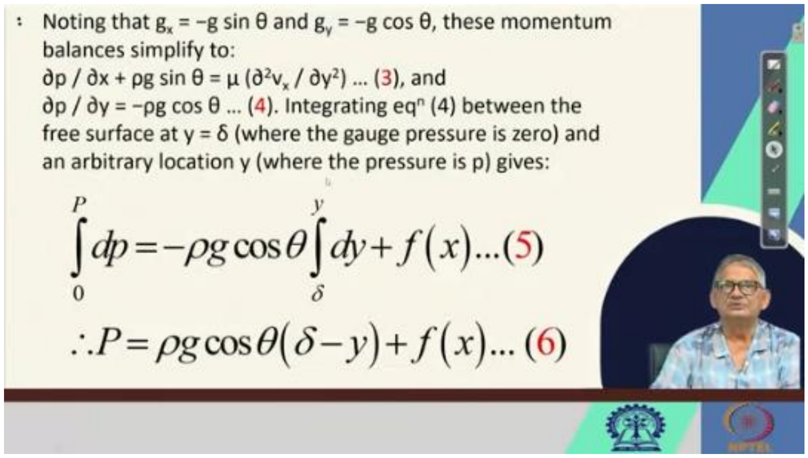
Where is that? Let us go back a little, equation number 4, 1, 2, 3, 4 yeah, this is the equation number 4 ok. Then This is why, because del function of x / del y is equal to 0. However, since P is 0 at y is del,



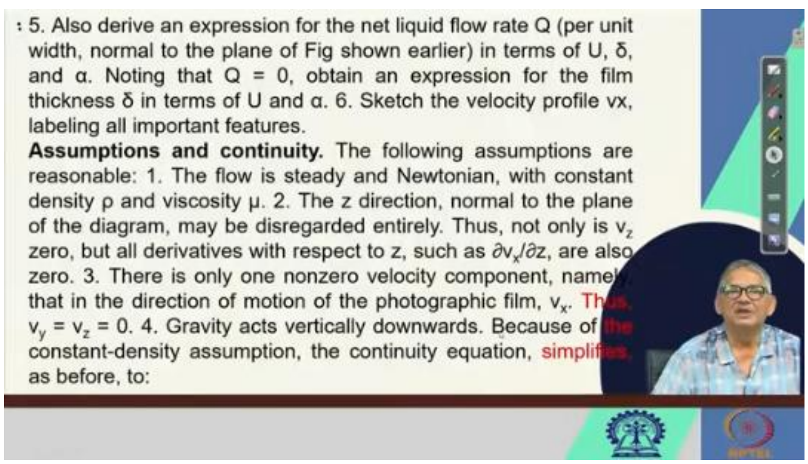

That is because the air-liquid interface for all values of x, the function f (x) must be 0. Hence, the pressure distribution P, that is, equals to del minus y rho g cos theta. And it shows that P is not a function of x. In view of this last result, we may now substitute del P / del x is equal to 0 into the x momentum balance, that is in equation 3, right. So, let us look into equation 3.

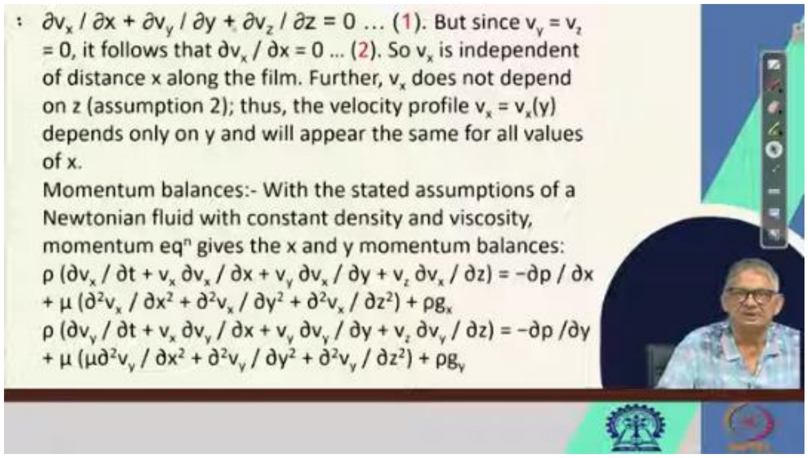


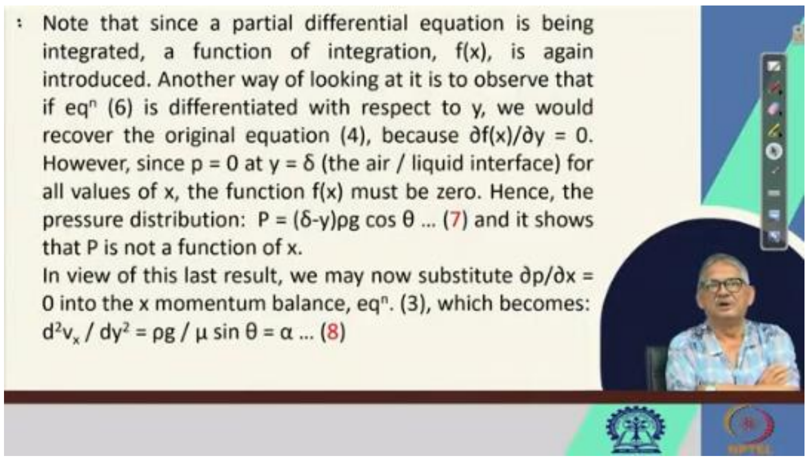

So, this was equation P del P / del x plus rho g sin theta is mu del 2 $v_x$ / del y square. right. So, if we put, hence, if we put, we may substitute del P / del x is equal to 0 into the x momentum balance which becomes del 2 $v_x$ / del y square. This is equal to rho g over mu sin theta that is alpha.



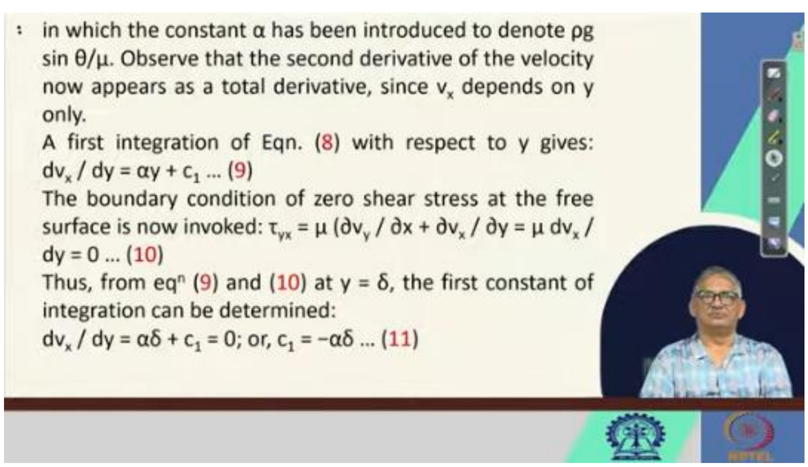

This alpha, we have defined earlier, just see, here we have redefined, rho g sin theta by mu, that is, alpha, right ok. Rho g by mu sin theta is alpha. Then we get that equation number 8 is, d 2 $v_x$ / d y square is equal to rho g over mu sin theta, that is alpha, ok. Now, this on simplification we note that the constant alpha has been introduced to denote rho g sin theta by mu and observe that the second derivative of the velocity now it appears as a total derivative.

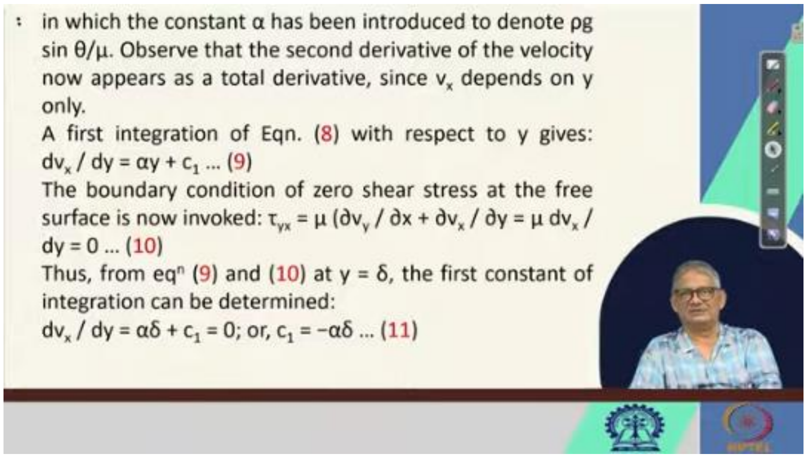




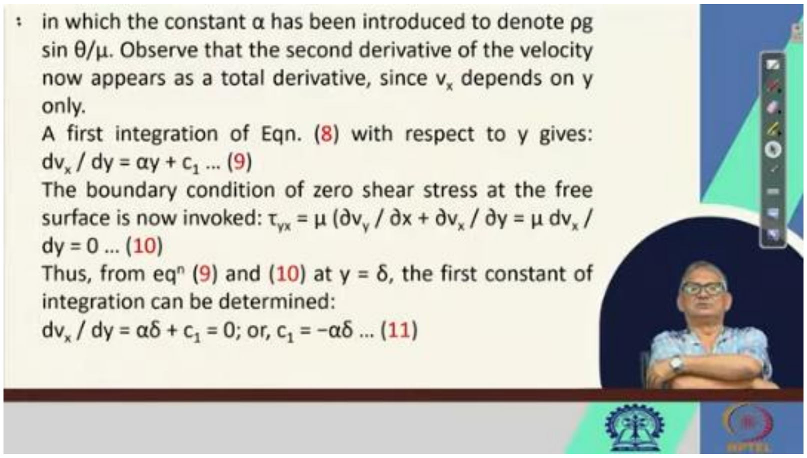




since $v_x$ depends only on y right. A first integration of the equation 8, just now we have seen with respect to y, it gives $dv_x / dy$ is equal to alpha y plus $C_1$ integration constant So, to find out this integration constant, we need to get a boundary so that this $C_1$ can be calculated. The boundary condition of zero shear stress at the free surface is now invoked. At the free surface, we also said earlier also, that the shear stress is zero.

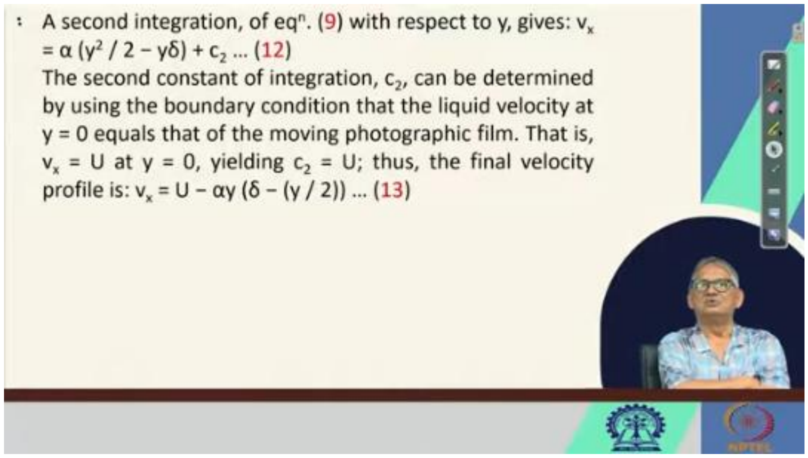


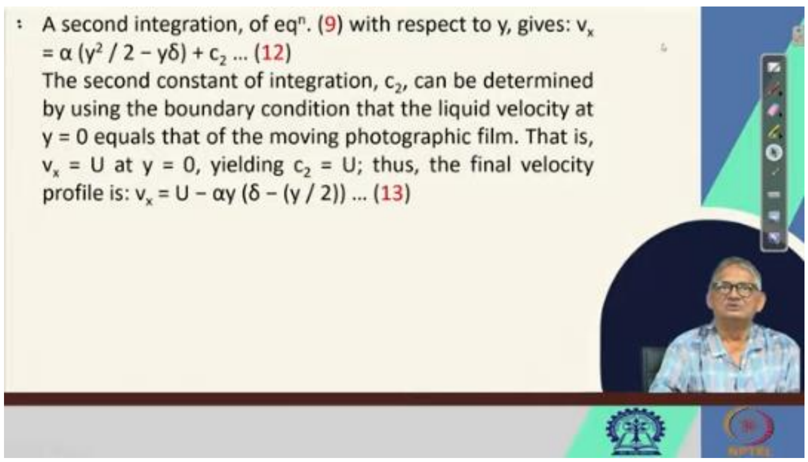



So, that can be brought into that $tau_{yx}$ is equal to mu del $v_y$ / del x del $v_x$ / del y. That is equal to mu d $v_x$ / dy is equal to 0. right. So, thus from this equation 9, and equation 10, at y is equal to del, the first constant of integration can be determined as d $v_x$ / d y is alpha del plus $C_1$, d $v_x$ / dy is alpha del plus $C_1$ is equal to 0. Therefore, $C_1$ is equal to alpha minus alpha del, $C_1$ is equal to minus alpha del.



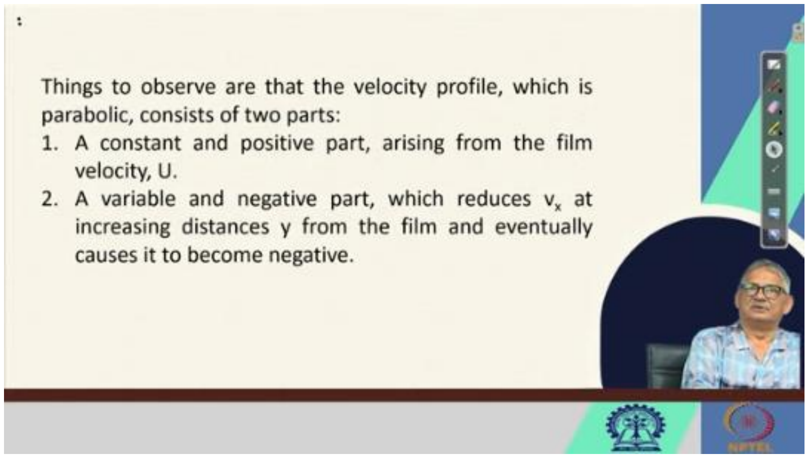

So, a second integration of equation 9 with respect to y this gives $v_x$ is equal to alpha into y square by 2 minus y del plus $C_2$. So, the second constant of integration $C_2$ this can be determined by using the boundary that the liquid velocity at y is equal to 0 equals that of the moving photographic film right. Here I repeat the second constant of integration $C_2$ that can be determined by using the boundary condition that the liquid velocity at

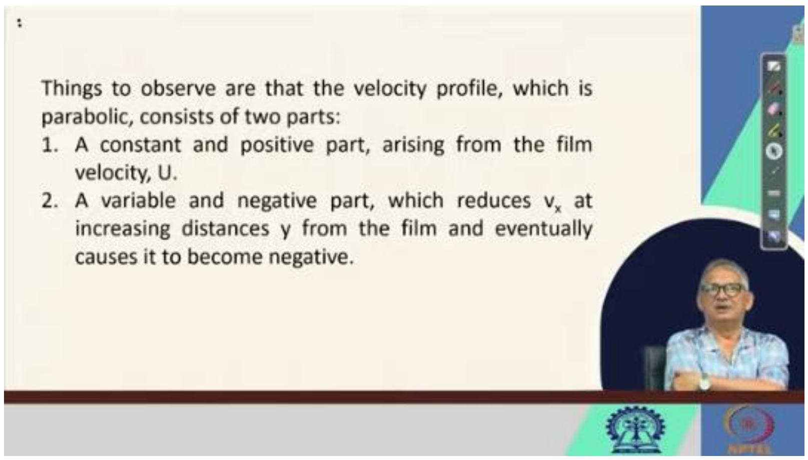



y is equal to 0 equals that of the moving photographic film that is $v_x$ is u at y is equal to 0. Because our photographic film that is the surface is also moving with a velocity u and that is the $v_x$ right. and that is at y is equal to 0 because y is the other side y is equal to del which is the free end. So, yielding $C_2$ is equals to u thus we can write the final velocity profile is $v_x$ is equal to u minus alpha y into del minus y by 2. right.

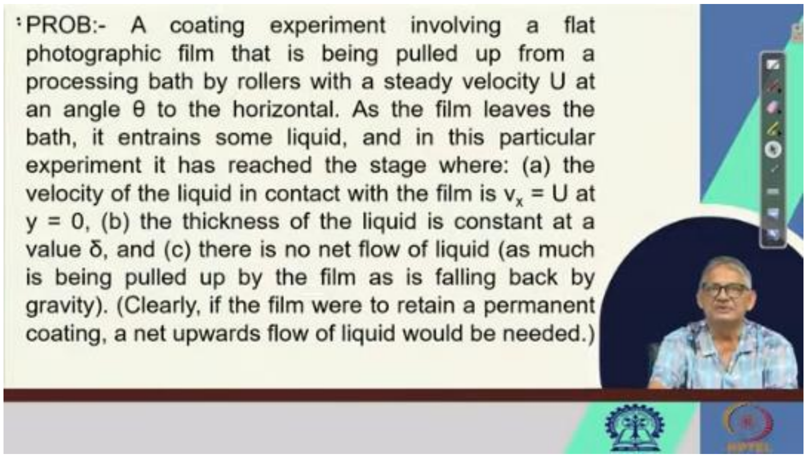
So, we found out the velocity profile that is whatever velocity was there u minus alpha y into del minus y by 2 is the $v_x$. So, here we have to keep in mind that The velocity profile which is parabolic consists of two parts. Number one, a constant positive part arising from the film velocity u and a variable and negative part which reduces $v_x$ at increasing distance y from the film and eventually causes it to be or to become negative right. I repeat that the things which we have to keep in mind here is that the velocity profile is parabolic.



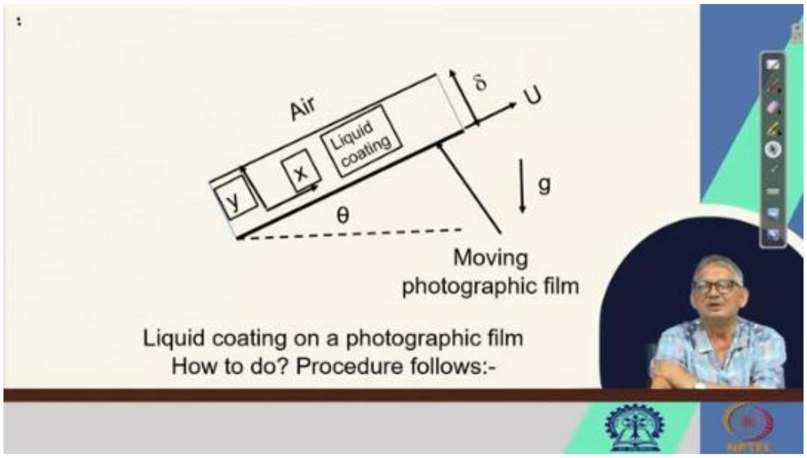



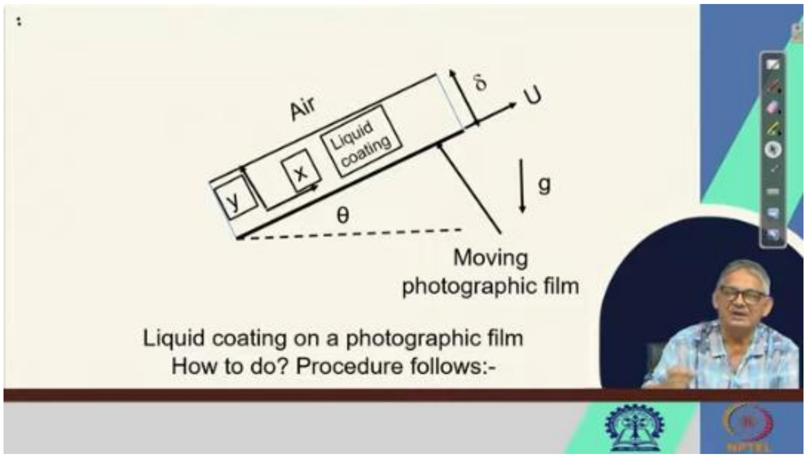

And it consists of two parts. First part is constant and a positive part arising from the film velocity u. A variable and negative part which reduces $v_x$ at increasing distance y from the film and eventually causes it eventually causes it to become 0. So, let us look into $v_x$ again that $v_x$ is u minus alpha y into del minus y by 2 right. As y is increasing

del minus y by 2 is also decreasing right. So, that is why we have said we can say that eventually causing it to become negative ok. before going further because now the time is also not so much. So, let us recapitulate again so that it becomes easier for us to understand. First, the problem we have understood.

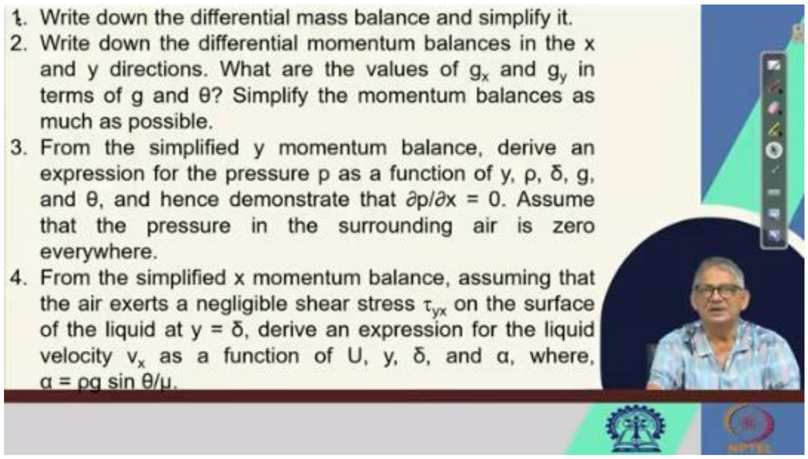


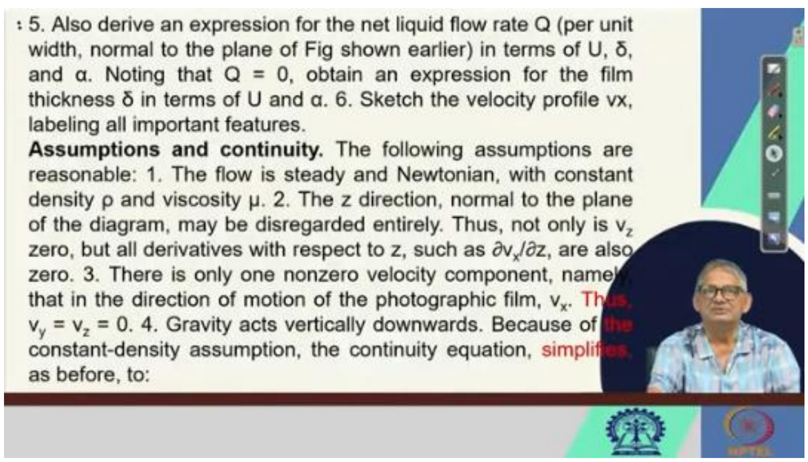

Second, we have drawn that film which is moving right with an angle of theta with the horizon and the upper part is air interface. So, we have taken that two dimensions Y and X. Y is up to del that is the film and X is the direction of the flow. having a velocity already u, right? And another is the gravitational pull, right?



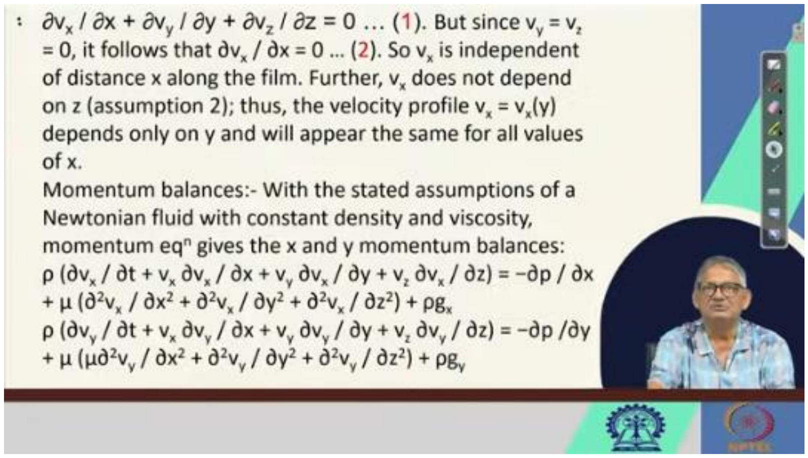



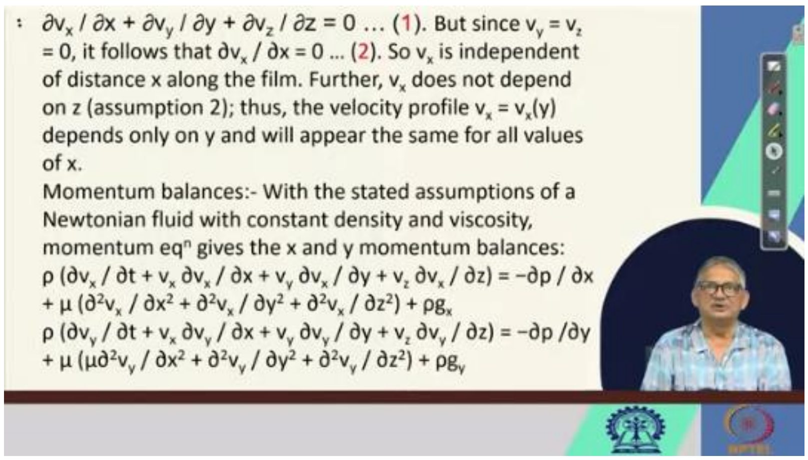
We have discussed with the how it should be done. Then we have found out Some assumptions which is also very valid assumptions, flow is steady and Newtonian, constant density and viscosity and z direction normal to the plane that has nothing to do. So, all z components are taken off. one non-zero velocity component that the direction of motion of the photographic film that is $v_x$.

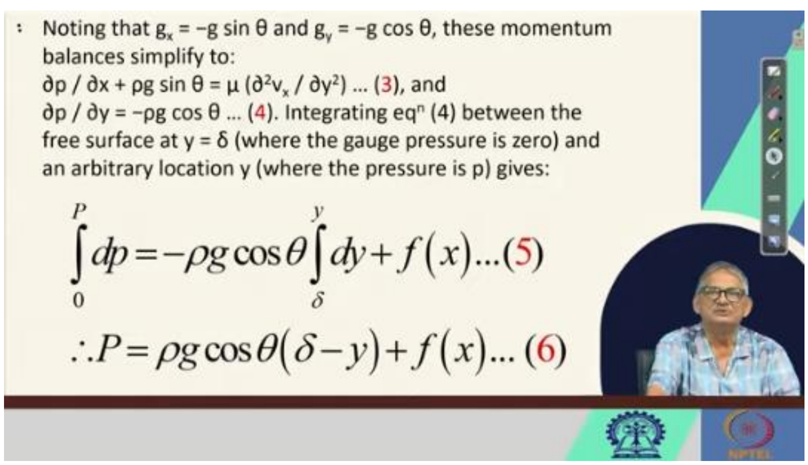


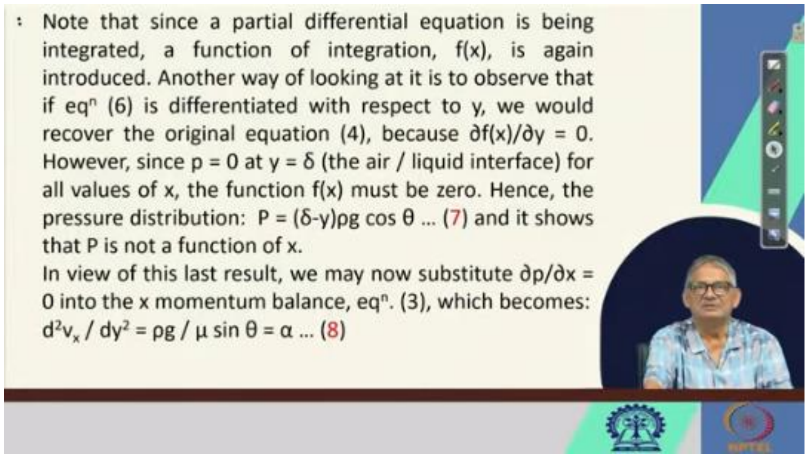

So, $v_y$ if $v_z$ all are equal to 0 right. So, like this simplifies we started with finding out from the continuity equation and then momentum balance. We found out some relations and then integrating them with the limit once for pressure for 0 to P and the y direction that is del 2 y. So, you got pressure relation P is also with a function of x, right?

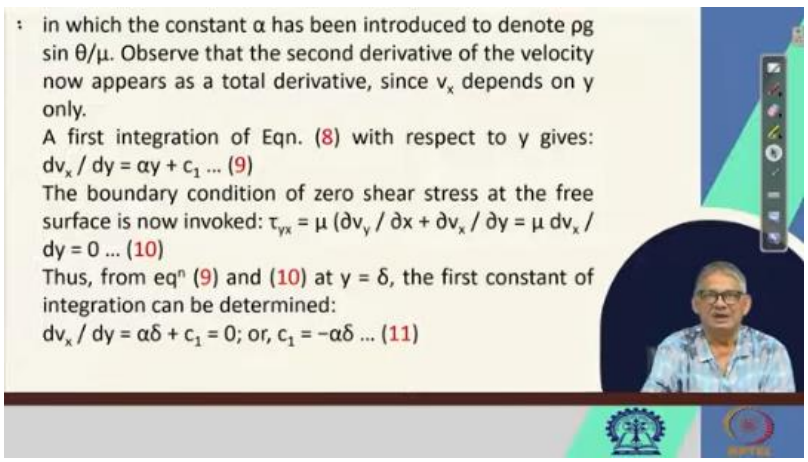


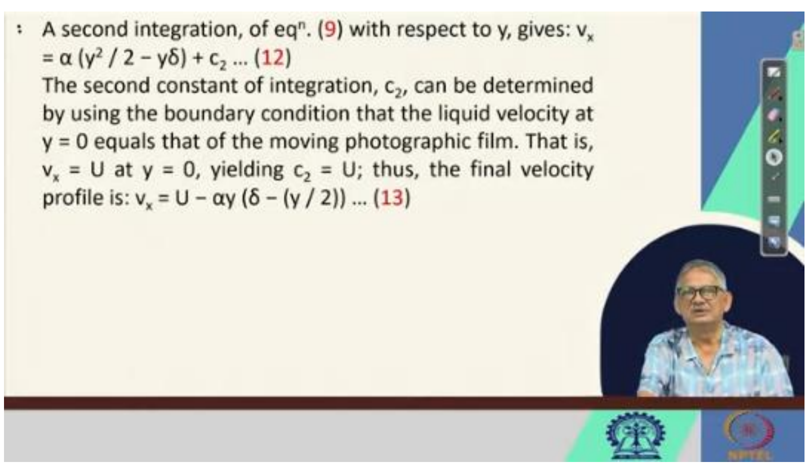




And then we integrated and shown that the del 2 $v_x$ / del y square is rho g by mu sin theta and we named this rho g by mu sin theta as alpha. Then we found out that $dv_x$ d y is alpha del plus $C_1$ and we found out $C_1$ to be minus alpha del. Like that we found out the other equation by integrating and we found out the integration constant and that is, $C_2$ is u. So, we got the velocity profile $v_x$ is u minus alpha y into del minus y by 2 that is 1 and from there we interpreted that this profile is parabolic in nature, but it has two components.




One is the positive part arising from the film velocity u and another one is variable and negative part which reduces $v_x$ at increasing distance y from the film and eventually causes it to become negative ok. Now, the time is up. Thank you for listening and we will continue with it in the next class. Okay.

Thank you very much.