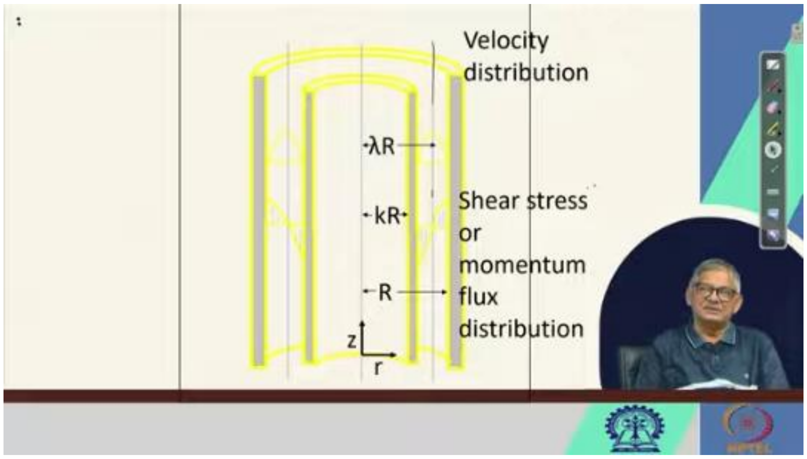
# IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture26

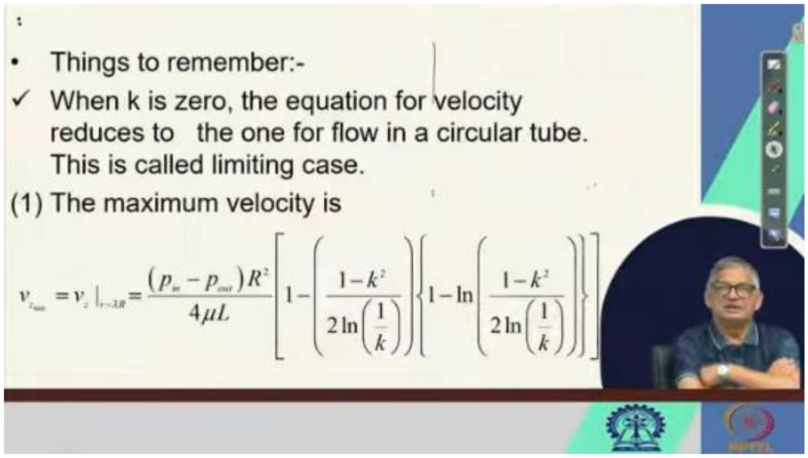
## LECTURE 26 : MEASUREMENT OF VISCOSITY WITH THE HELP OF DROP OF A BALL

Good morning, my dear friends, students, boys, and girls. We have seen in the previous class that it was not that easy, unlike the flow through pipes, which is much simpler than this. What is the distribution of velocity and the distribution of shear force? In the liquid which is moving through an annulus like this, right? So, we have found out the velocity distribution, and we have also found out the shear stress or momentum flux distribution under this condition.

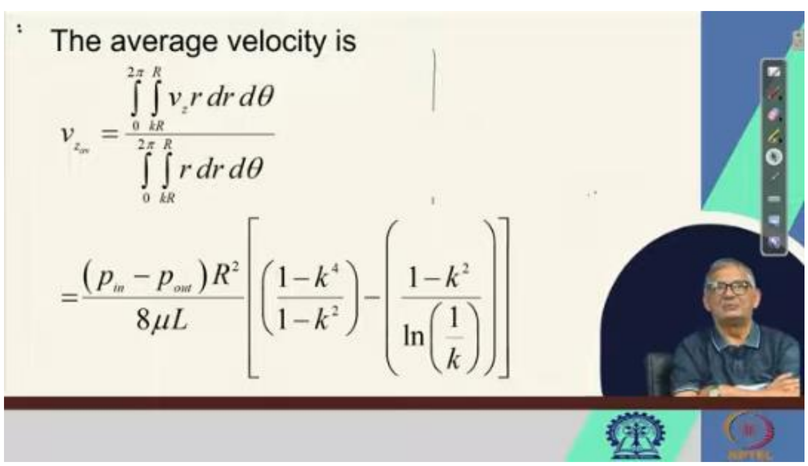
What is the condition? That is, at which point the velocity profile becomes maximum or the shear stress becomes minimum? That is not known. Yes, it is known that velocities are 0 at r is equal to K R and r is equal to R, capital R, right. Since we do not know, we have taken an arbitrary plane, like lambda R, which is like this, as it, and we found out, obviously, the shear stress and the velocity distribution $v_z$ in the shear momentum, right.



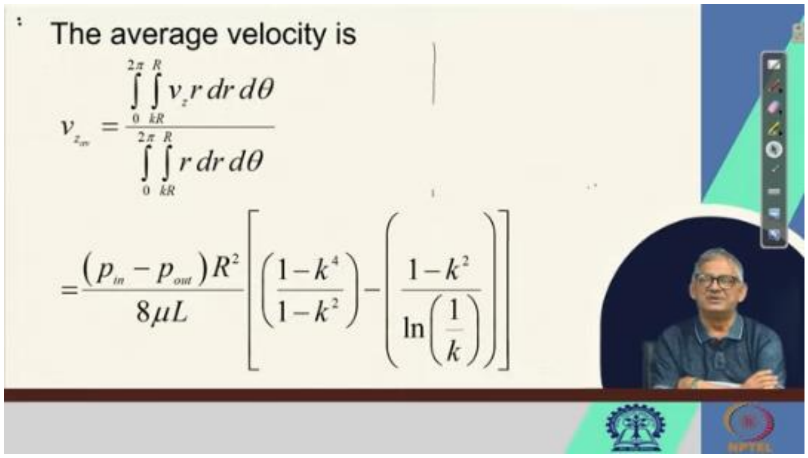

There are certain things to remember for this: that is, when k is 0, the equation for velocity reduces to the one for flow in a circular tube, and this is called the limiting case, right. Then, the maximum velocity is $v_{max}$ equals to $v_z$ at z is equal to lambda R. And that is on simplification and rearrangement. We can say that this maximum velocity can be said to be $P_{in}$ minus $P_{out}$ into R square by 4 mu L. So, this into 1 Minus 1 minus k square by 2 mu into 1 by k.

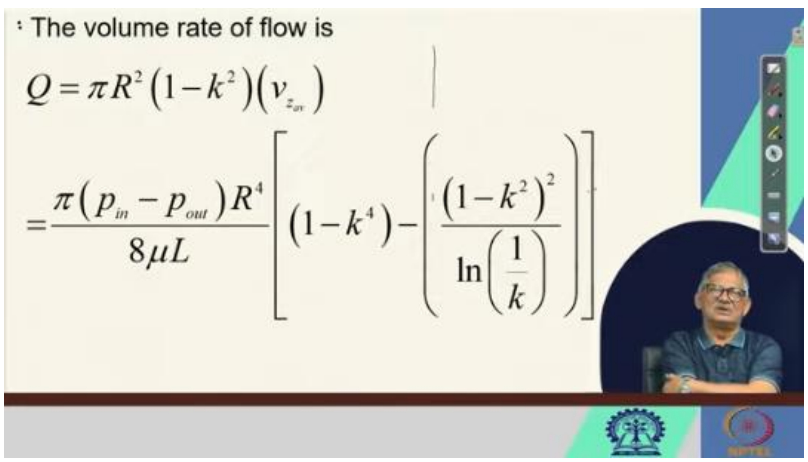



The maximum velocity, as we can see, is vzmax at vz when r equals $\lambda R$, which is ($P_{in}$ - $P_{out}$) * ($R^2$ / $4\mu L$) * (1 - (1 - $k^2$) / (2 * ln(1/k)) * (1 – ln (1 - $k^2$) / (2 * ln(1/k)))). This is the maximum velocity. The average velocity, as mentioned earlier many times, is nothing but the area-wise distribution, and the accumulation of those distributions is taken and expressed in the numerator between 0 to $2\pi$. And between kR to R, the integration of $v_z$ * r * dr * d$\theta$, this whole thing is the terminology, and r * $v_z$ * dr * d$\theta$, right. Between kR to R, $v_z$ * r * dr * d$\theta$, over 0 to $2\pi$, the entire area between 0 to $2\pi$, kR to R, where this integration is done between r * dr * d$\theta$.

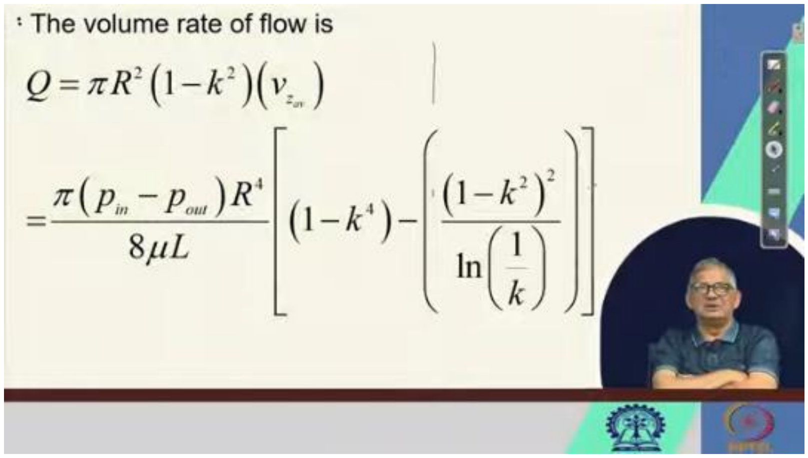


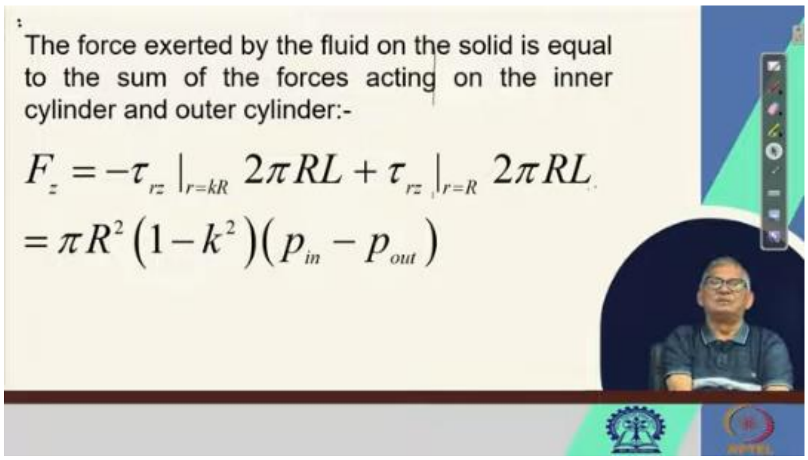

This, on simplification, we can write as ($P_{in}$ - $P_{out}$) / ($8\mu L$) * $R^2$ * ((1 - $k^4$) / (1 - $k^2$) - (1 - $k^2$) / ln(1/k)) is the $v_{zaverage}$. For the average, we have said, over the section, the entire section, the velocity distribution over the section, again divided by the area. So, that we have also taken into this, right. So, the average velocity we have found out to be $(P_{in} - P_{out}) / (8\mu L) * R^2 * ((1 - k^4) - (1/k^2) - (1 - k^2) / \ln(1/k))$. And the volumetric flow rate is that is, $Q = \pi R^2 * (1 - k^2) * v_{zaverage}$, where $Q = \pi * (P_{in} - P_{out}) / (8\mu L) * R^4$.



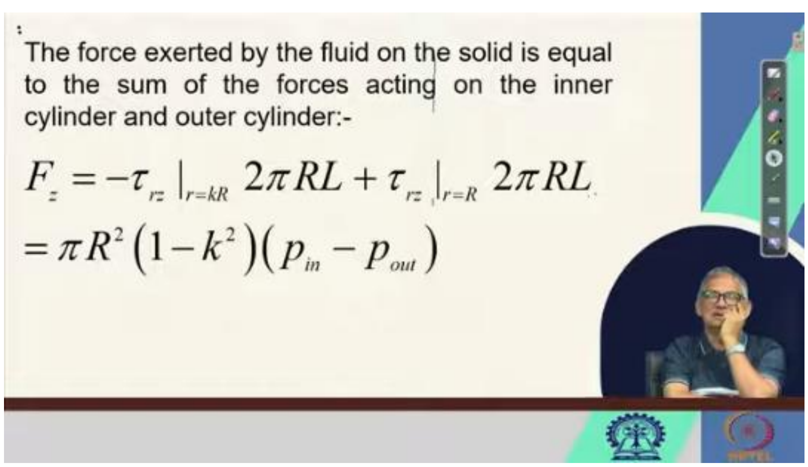



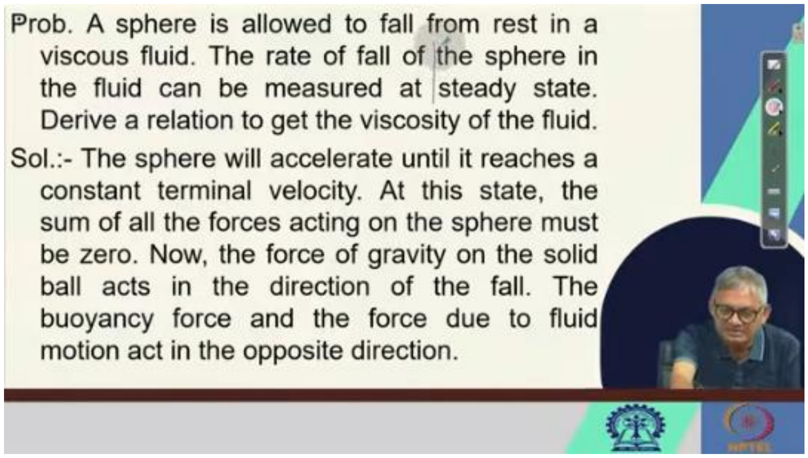

times $((1 - k^4) - (1 - k^2)^2) / \ln(1/k)$, right. The volumetric flow rate is like this: $Q = \pi R^2 * (1 - k^2) * v_{zaverage}$ is a diverge, where it is solved in this way that this is equal to $\pi * (P_{in} - P_{out}) * R^4 / (8\mu L) * ((1 - k^4) - (1 - k^2)^2) / \ln(1/k)$. So, this is the volumetric flow rate. The other terms, like the force exerted by the fluid on the solid, is equal to the sum of the forces acting on the inner cylinder and outer cylinder, and that can be found out as $F_z = -\tau_{rz}$ at $r = kR$.




into 2 pi R L plus tau r at r is equal to R into 2 pi R L. So, the whole thing is equal to pi R squared into 1 minus k squared into Pin minus Pout. This is how we can find out this force distribution. Now, as we say every time in every section, unless we have some problem solved, the grasping of the subject does not occur. So, we are bringing one problem for solution in this section, which is like this.






A sphere is allowed to fall from rest in a viscous fluid. The rate of fall of the sphere in the fluid can be measured at steady state. Derive a relation to get the viscosity of the fluid. Now the sphere will accelerate until it reaches a constant terminal velocity, but before that, let us again tell what the problem is. So, a sphere is allowed to fall from rest in a viscous fluid. A viscous fluid is there in which a sphere is allowed to fall.

The rate of fall of the sphere in the fluid can be measured at a steady state. So, derive a relation to get the viscosity of the fluid. So, this indicates that in the earlier case, which we said, if you remember that, coaxial cylinders rotating, one is fixed, another is rotating, and we said that this principle is useful for measuring viscosity. Measuring viscosity using Brookfield viscometer, that one, which is utilized to measure viscosity using Brookfield viscometer.

The similar technique that two coaxial cylinders, One is fixed and the other is rotating, right. You can use the inner one fixed, the outer one rotating, or the outer one fixed, & the inner one rotating, either way. Here also, this technique can give you a means of measuring the viscosity of a fluid.

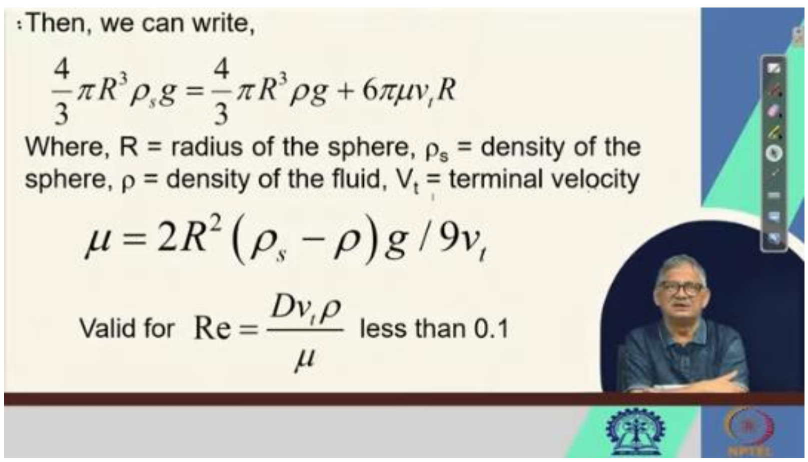
For that, what are you doing? You are dropping a ball or sphere. So, it is falling from rest in a viscous fluid. So, it was at rest here, now it drops, started through the viscous fluid.

The rate of fall of the sphere in the fluid can be measured at steady state. So, you are asked to derive a relation to get the viscosity of the fluid. It is not complicated, so it is very easy. Only the thing you need to know is that whenever you are depending on some pictorial view, that view must be taken very properly. Similarly, here also, we are saying that it is falling freely or from rest.

So, that has to be maintained explicitly, and the conditions of the falling of the ball must be maintained, okay. Now, for its solution, if we take. the sphere will accelerate until it reaches a constant terminal velocity. So, its velocity will continue to increase until it reaches a constant terminal velocity, right. So, on their side,

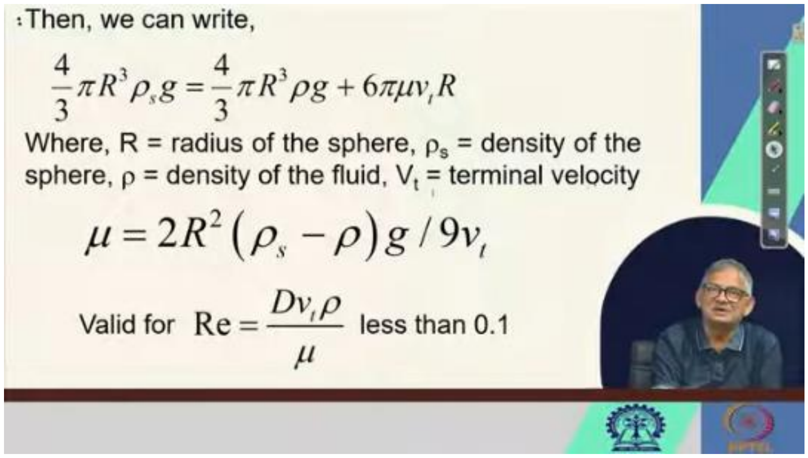
the sum of all forces acting on the solid ball includes gravity, which acts in the direction of the fall. The buoyancy force and the force due to fluid motion act in the opposite direction. Obviously, buoyancy acts like this, and the fluid moves like this. So, they are acting in the opposite direction, right. So, the force of gravity on the solid ball acts in the direction of the fall.

And the buoyancy force and the force due to fluid motion act in the opposite direction. Next is that we can write, that four-thirds pi R cube into rho g, rho solid into g, right, s is for solid. So, four-thirds pi R cube into rho g, is equal to four-thirds pi R cube into rho into g, plus 6 pi mu $v_t$ R, so, this takes care of all the terms.




I repeat, $4/3\ \pi R^3 \rho_s g$, right, that is equal to $4/3\ \pi R^3 \rho g$ plus $6 \mu \pi v_t R$. So, R is the radius of the sphere. and $\rho_s$ is the density of the sphere, mind it, density of this sphere. If you put a plastic ball, it will not go; it will float. So, you have to take accordingly the density,

right. R is the radius of the sphere, $\rho_s$ is the density of the sphere, and $\rho$ is the density of the liquid or fluid, $v_t$ is the terminal velocity. So, therefore, we can write $\mu$ is equal to $2 R^2 (\rho_s - \rho) g / 9 v_t$. I repeat, So, from the earlier relation we have, we have equated that $4/3\ \pi R^3 \rho_s g$

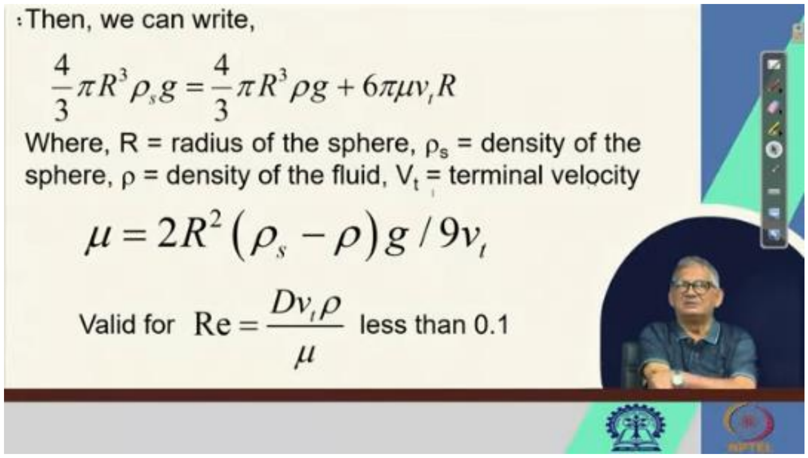



is equal to 4/3 $\pi$ $R^3$ $\rho$ g plus 6 $\pi$ $\mu$ $v_t$ R. Obviously, R is the radius of this sphere, $\rho_s$ is the density of this sphere. $\rho$ is the density of the liquid or fluid, and $v_t$ is the terminal velocity. Therefore, $\mu$ is equal to 2 $R^2$ ($\rho_s - \rho$) g / 9 $v_t$, which is valid for Reynolds number, $N_{Re}$, less than or equal to D $v_t$ $\rho$ / $\mu$, 0.1. So, valid Reynolds number is D $v_t$ $\rho$ / $\mu$, where it will be less than 0.1.

That is the primary thing, which you have to maintain: that the Reynolds number, the variation of the Reynolds number, is less than 0.1. Only then can you write mu is equal to 2 R square. rhos minus rho, where rhos is the density of the solid, and rho is that of the fluid, then this is D $v_t$ rho by mu, which is less than 0.1. So, what I would like to highlight here is that we are getting a technique by which the viscosity of a fluid can be determined.

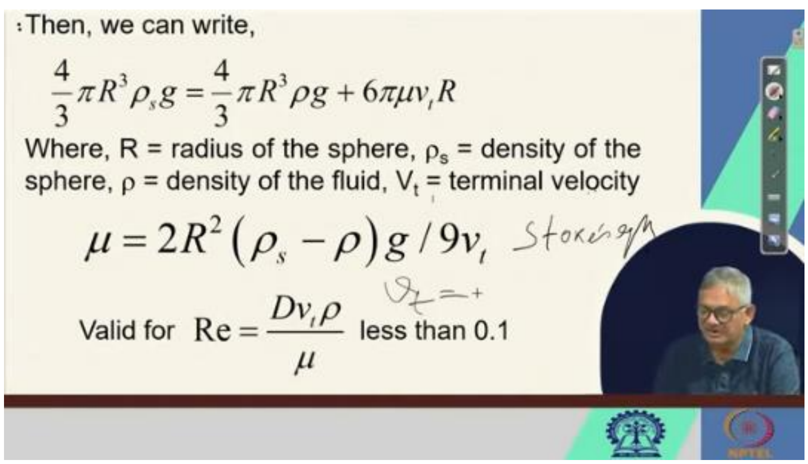
And it is a very, very easy process, as you have seen that you do not need many things to do, right, you do not need many things to do. What do you need? You need the property values, both for the solid and the liquid, and then you need to know what the terminal velocity is, which is appearing, right. Only then can you find out the viscosity of the fluid, right.

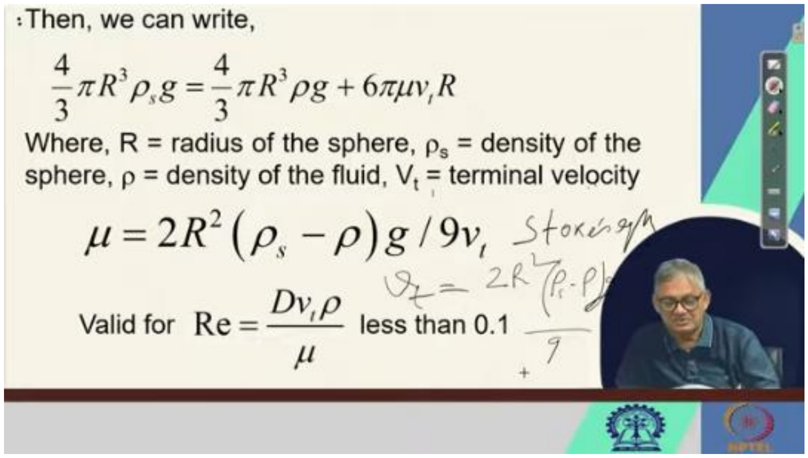
As you can see, we have very simply equated buoyancy and the forces acting, and this we said is valid for $N_{Re}$ less than 0.1, this is very primary, that the Reynolds number, which is defined as D $v_t$, where vt is the terminal velocity, into rho by mu, is less than 0.1. right. So, therefore, we rearrange this mu, sorry, we rearrange this mu as, mu is equal to 2 R square rhos minus rho, where rhos is the density of the solid and rho is the density of the liquid, right. So, obviously, the density of the solid is more, that is why it is penetrating into




Liquid right into g. The value of g is known for everyone divided by 9 $v_t$, right. So, I think from here a very unique relation is found out that is called Stokes equation. If you can find out from any book or any source, right? You see that Stokes equation is what? v $_{terminal}$, if you know the density, if you know the viscosity

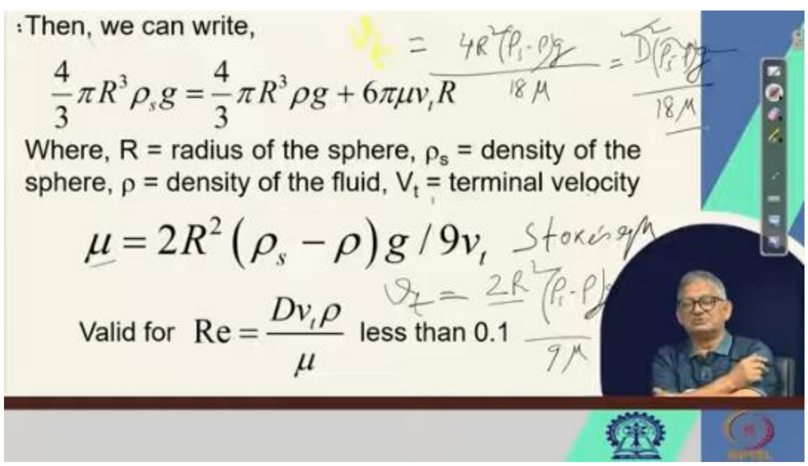
then v terminal, it goes there, then that becomes 2 R square into rhos minus rho into g of course, into g divided by this 9 into 2 that divided by 9. right. So, this I am writing again on the top. This can be rewritten as $v_t$, I do not know why the other color is not coming. So, $v_t$ is equal to



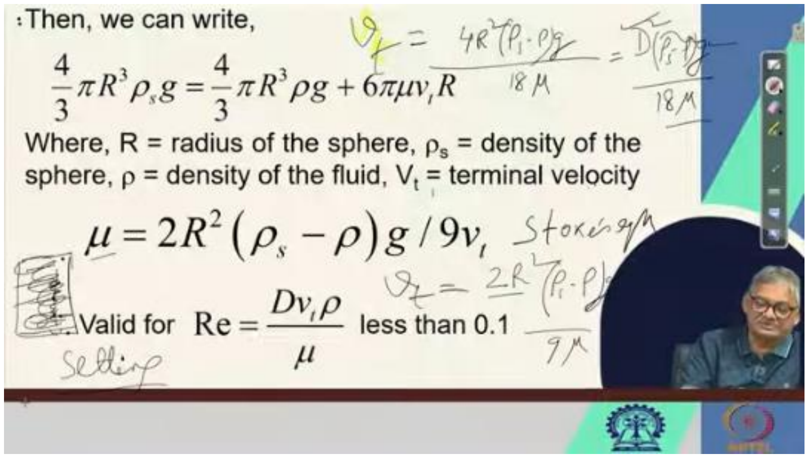



this is 2 R square, right. So, if it is 4 R, then 4 R square into rhos minus rho into g divided by, this we have made 4. So, it can be 18 into this mu which, we have not written, right 18 mu. So, that can be written again as 4 R square means D square,

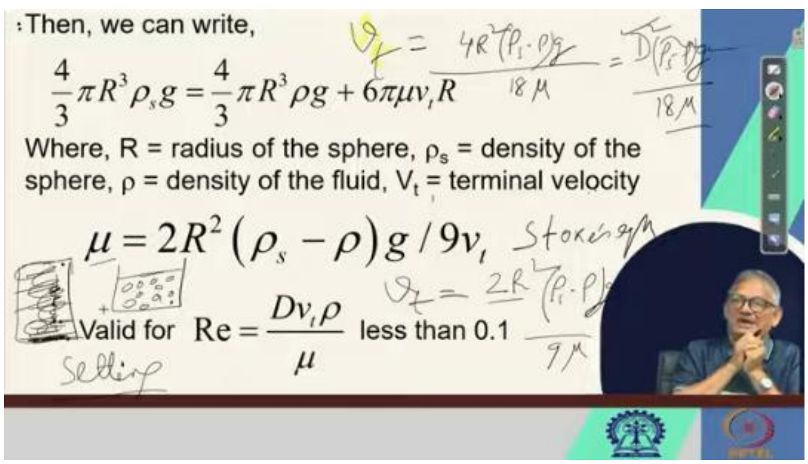
Right, into (solid minus liquid) density into g by 18 mu. I think this is the equation, which is very, very helpful, and widely used all over. To find out the terminal velocity, provided you know the density, provided you know the difference between density, and diameter, g is known, and viscosity mu is also known, right. So, if they are known, then you can find out vterminal, terminal velocity. This is very, very useful when you just think you have one container, right, and you have a slurry. This is liquid, and there are some solids, right.



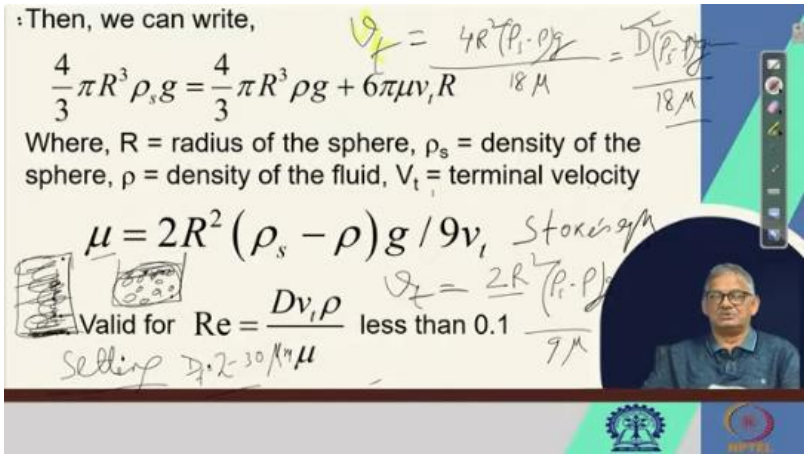

What will happen? These solids will settle down. So, in settling, you can find it out. Because these solids are heavier. So, they will fall and settle at the point.




So, what is the terminal velocity required for settling? You can find out. You know the densities, you know the diameter, you know gravitational force, you know viscosity of the fluid, Right, this is for settling. The other one also can be done here, that you have milk. Milk contains a lot of fat, right? And at any moment at home, if you see milk, it is having a crust on the top, right, and that crust is nothing but a layer of fat, right.




Generally, these fat globules have a diameter, $D_{fat}$, roughly equal to between 2 and 30 micrometers in size. So, here also, knowing the density of fat, knowing the density of liquid water, or in this case milk, you can determine, and also knowing the diameter of the container, you can find out what the terminal velocity, $v_t$, is. Right, to make this fat rise to the top, or separate, right. How much percentage of it is getting separated can also be determined. So, this is unique; it is not only that




it is giving you a solution for finding out the viscosity, but also it is giving you an opportunity to find out the settling velocity of solids or some dust. slurry, and also you can find out what is the time required, or how much, yes, how much time you may require to get the fat globules separated within or from the milk, right. So, these two are very, very unique. So, what I suggest is that with the help of this, you try to solve some problems; then it becomes more clear to you. Then, we thank you for listening to the class.

Thank you all, okay. Good day.