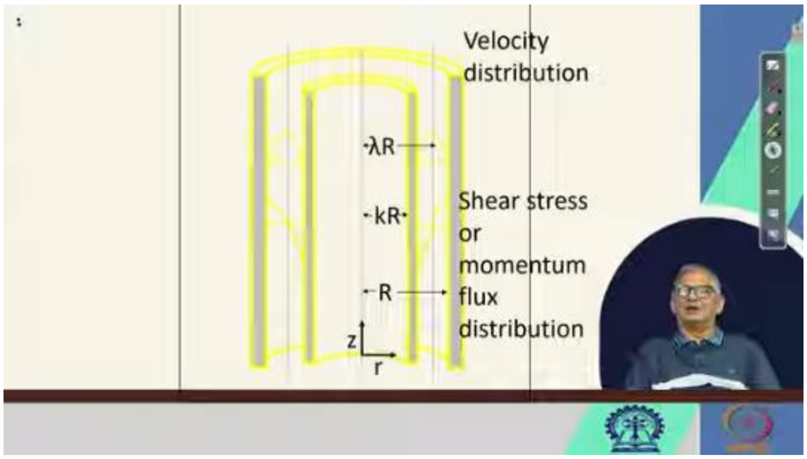
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture25

LECTURE 25 : FLOW OF FLUID THROUGH ANNULUR SPACE

Good morning, my dear friends, students, boys, and girls. We are almost midway through the course, midway in the sense that we have already done many things. Now, we will discuss flow through annuli, right? So, before going into this detail, the requirement of this, why it is so important, right? That we should know.



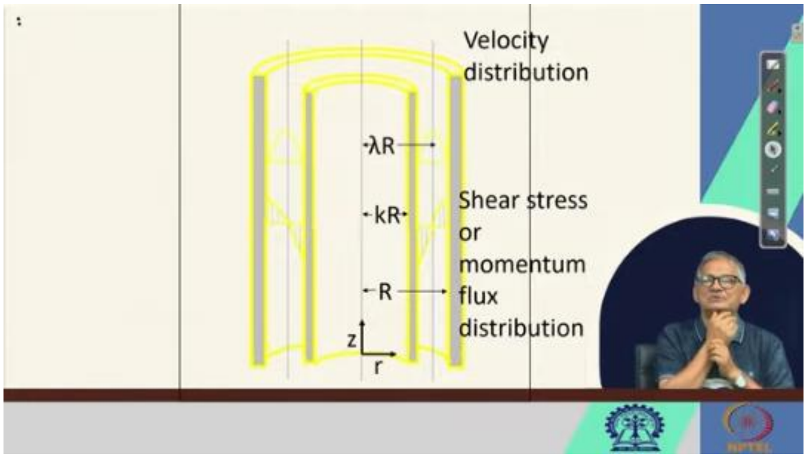
I hope you understand heat exchangers, right? There are many heat exchangers, like double tube, etc., where one tube has one fluid, another tube has another fluid, they may be in co-current or counter-current, and there is a heat exchanger. Meaning one is getting warm and the other is getting cold, an exchange of heat. So, under those situations, how the development of the flow characteristics is done, that is the topic today. Right, definitely, if we understand that, it will be very helpful for those who are doing heat transfer, mass transfer, or fluid flow, whatever.

In an annulus. Annulus means, obviously, one cylinder within another cylinder, and the liquid is flowing through the space between them. This inner one is nothing but like a flow through a pipe, which we have already done. But the outer one, which we have not done, where the fluid is flowing, and the flow behavior or the characteristics of the flow we do not know, we have to find it out, right. Then the difficulty is that in the pipe flow.

We knew that one fluid is moving, and we knew that within the boundary of the pipe, everything is happening, but that was within a domain. But in this case, it is not so. We have one inner cylinder and one outer cylinder through which the fluid is moving, right? If you remember, many classes before, we had shown one such problem where, in two concentric cylinders, one cylinder was constant and another cylinder was rotating, and there we wanted to find out the velocity distribution.

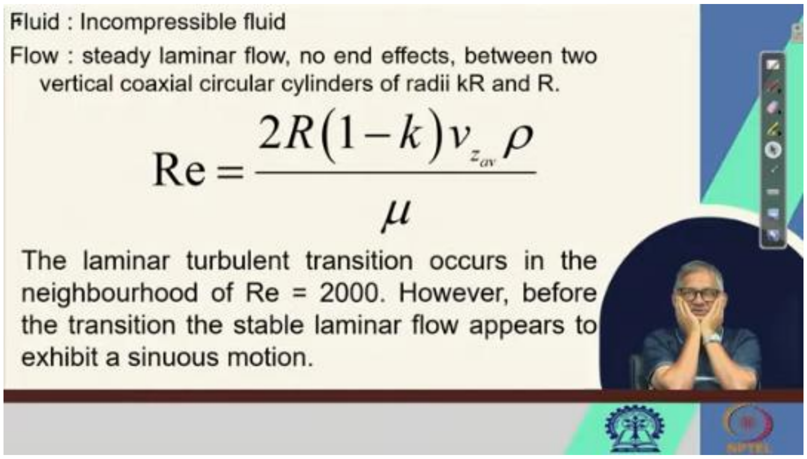
Right, but it is not. So, the movement of the cylinder is not there. It is two coaxial cylinders, true, but they are fixed, and the fluid is flowing through the annular space, the space between the two cylinders. We have to find out the flow behavior or flow characteristics of the fluid.

So, for that, as it is shown in the diagram, we take one direction, Z vertical, and another direction, R horizontal. And obviously, there is a third dimension like this; there would have been a third dimension which we are not considering, as we are considering it to be unity, right? So, in this, what we say is that we are forcing some liquid from the bottom to the top in the Z direction, and the radius R has an internal radius of kR, where k is a multiple of R. From the diagram, it appears that k is less than one. Otherwise, R will not be bigger, right? And the bigger diameter, the outer diameter, we are assuming to be R. So, between kR and R, a fluid is moving.

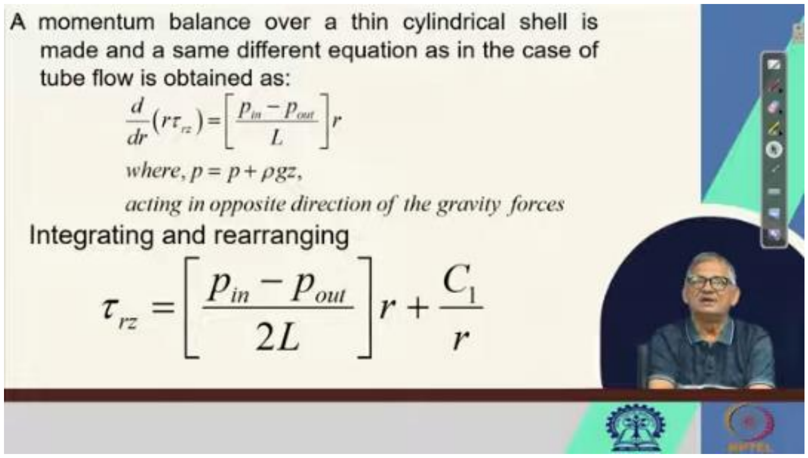


So, you have to find out either the shear stress or momentum flux distribution, as well as the velocity distribution. So, for that again, as earlier, some assumptions or some preambles are required. So, the first thing is that the fluid is incompressible, right? And the second thing is that we are saying the flow is steady and laminar. The third thing is that there are no end effects, which we have already described many times. And the fourth thing is that the fluid is moving between two vertical coaxial circular cylinders of radii kR and r, right?

So, for this, the Reynolds number we can define to be equal to 2R into 1 minus k into v_z into rho by mu. Unlike D v rho by mu, here it is 2R into 1 minus k, this unit as a substitution of D, and then vaverage rho by mu. Obviously, there are two zones: laminar and turbulent. So, the laminar-turbulent transition occurs in the neighborhood of around 2000 Reynolds number. So, before the transition, the stable laminar flow appears to exhibit a sinusoidal motion.

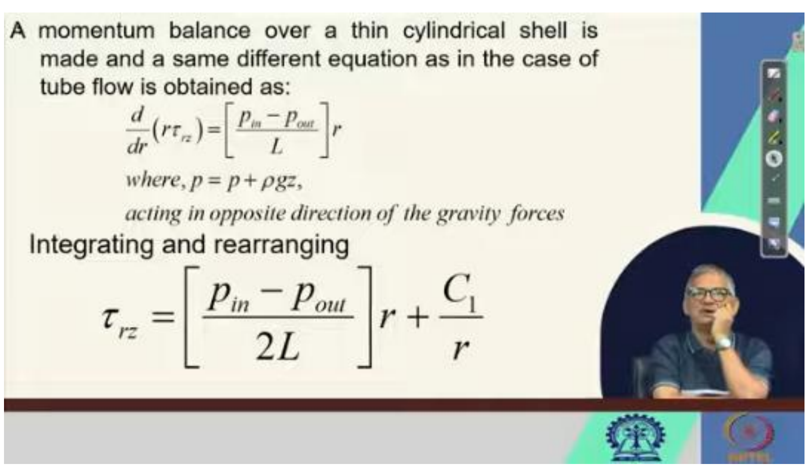


So, what do we do now? We take a momentum balance over a thin cylindrical shell and make the momentum balance. And the same differential equation as in the case of tube flow is obtained. Assuming similarity to the tube flow, we obtain that the relation between shear stress and pressure drop is like this. d dr of r tau$_{rz}$, as we have seen earlier, is equal to P_{in} minus P_{out} over L into r.



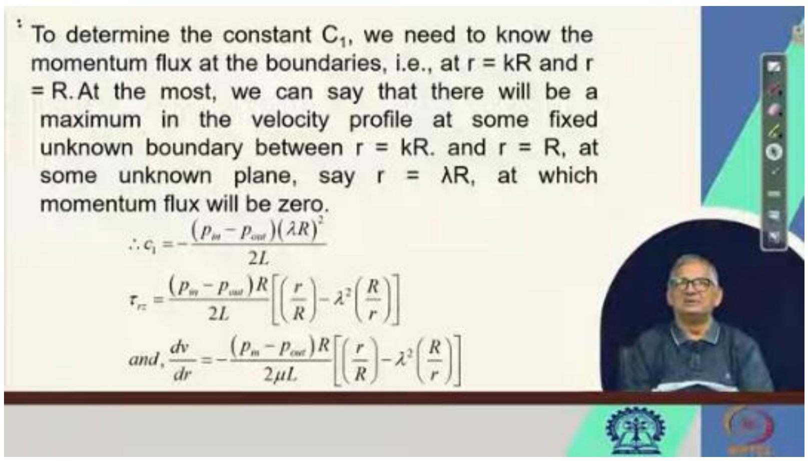
Here, capital P is P plus rho g_z because, if you remember from the pictorial view, we said that the fluid is flowing from the bottom to the top, right. So, we have the pressure difference, okay, that is P. That actual pressure is this P plus rho g_z, which is the height of the cylinder column, whatever you call it. So, that is capital P is P plus rho g_z. Now, this acts in the opposite direction of the gravitational forces, definitely, right.

And integrating this and subsequent rearrangement of this, d dr of r tau$_{rz}$ that is equal to P_{in} minus P_{out} over L into r. By rearranging and integrating again, by rearranging, we can write to be r tau$_{rz}$ is equal to P_{in} minus P_{out} over 2 L into r plus C_1 by r. Right. So, on integration and then rearranging, we get this: tau$_{rz}$ is equal to P_{in} minus P_{out} over 2 L into r plus C_1 by r. You see, in the previous relation or previous equation, we have d dr of r tau$_{rz}$ is equal to P_{in} minus P_{out} over L into r, right.



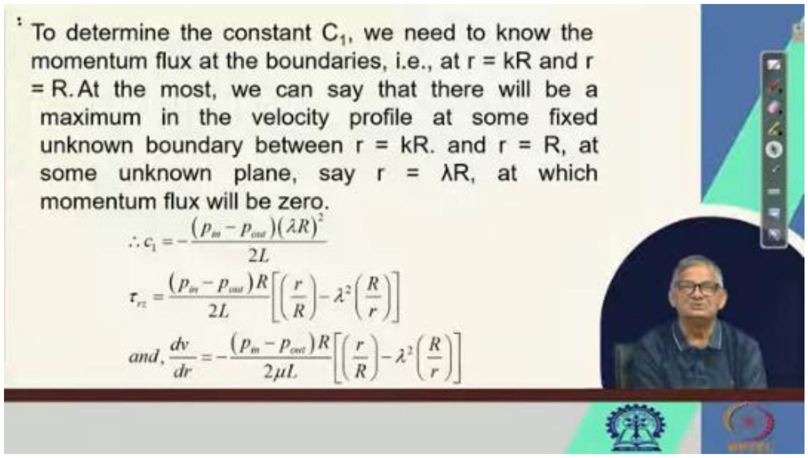
So, when we integrate this r, it becomes r squared by 2. So, that 2 has come to 2 L in the denominator. P_{in} minus P_{out} is constant. So, we have taken that there, right, and after this integration, we get r tau$_{rz}$ is P_{in} minus P_{ou}t over L. R squared by 2, which has been taken inside 2 L, plus C_1.

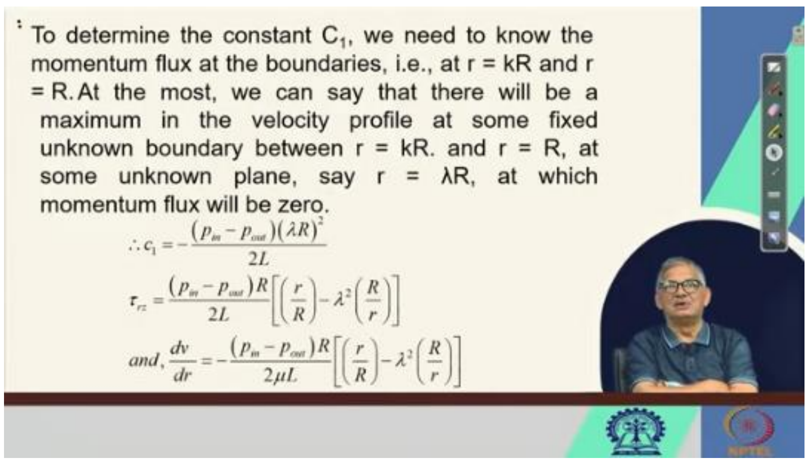
Now, if we divide r tau$_{rz}$, which is from the left side, by r, then it remains tau$_{rz}$ is equal to (P_in minus P_out) by 2L into r plus C_1 by r. Now, we have to find out the value of C, that is C_1, in this case, then we need to know the momentum flux at the boundaries. What is the boundary? Boundaries are either at r is equal to kR or r is equal to capital R. But at r is equal to kR or at r is equal to capital R, we do not know what the boundary is, because we have no idea about the velocity profile or the stress profile.



So, we cannot say that at r is equal to R, at r is equal to kR and at r is equal to capital R, the boundary will be like this. So, we can say that there will be a maximum in the velocity profile at some fixed point, which is unknown, and beyond the normal boundary between r is equal to kR and r is equal to R. Obviously, it has to remain within that annular space. So, it should be between r is equal to kR and r is equal to capital R. Obviously, this is not

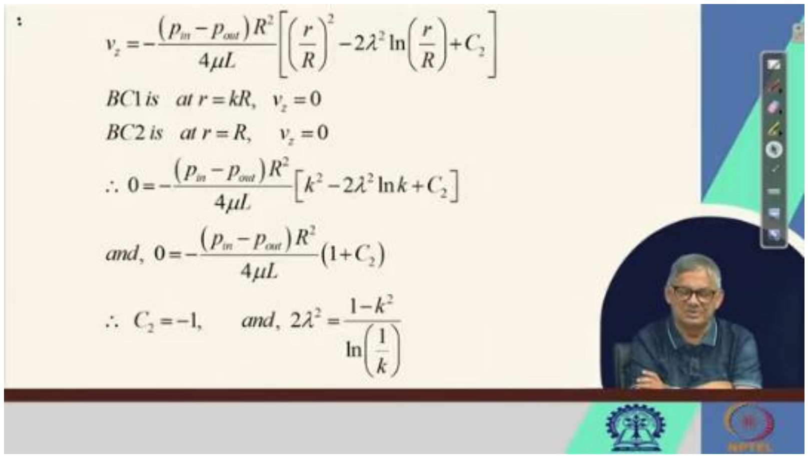
certain or not known that this point is becoming the maximum, right. So, we assume that point to be equivalent to lambda R. Obviously, as you see that r is equal to kR with one and r is equal to lambda R another thing comes up. So, this brings up momentum flux, and that can be obtained as C1 is equal to (P_in minus P_out) over 2L into (lambda R) whole square or (p0 minus p_in) by 2L, (p0 minus p_in) by 2L into R, r by R minus lambda square into R by r, and d v / d r equals to minus (p_in minus p_out) into R into r by R minus lambda square into R by r.





tau$_{rz}$ equals to p$_{in}$ minus p$_{out}$ by 2L into R into r by R, that is small r by capital R minus lambda square R by r, capital R by small r. And from here, we can write dv dr is equal to minus p$_{in}$. minus pout times R by 2 mu L into r by R minus lambda square into R by r. And dv / dr we get P$_{in}$ minus P$_{out}$ or P$_{in}$ minus Pout over 2 mu L into R times small r by capital R minus lambda square R by r. Now, this is another equation which we have arrived at, and now we have to solve it. So, we get v$_z$ is p$_{in}$ minus P$_{out}$ R square by 4 mu L r by R whole square minus 2 lambda square into ln r by R plus C$_2$.

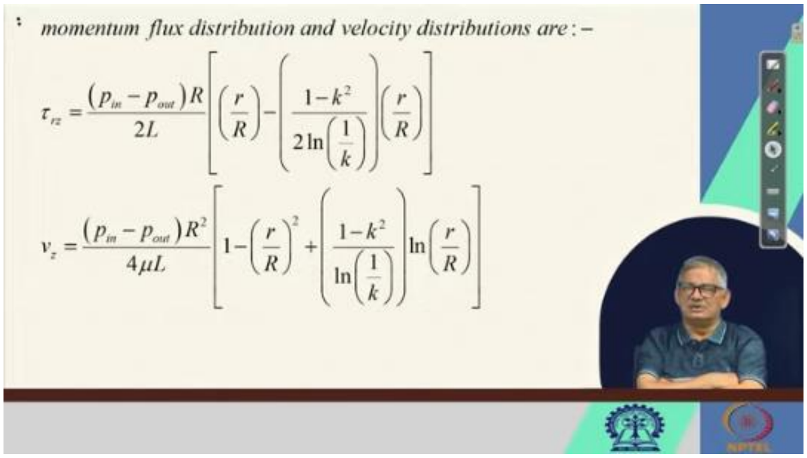
The boundary condition, at r is equal to kR, is v$_z$ is 0, and the boundary condition at r is equal to R, r is equal to r rather capital R, v$_z$ also equal to 0. So, we can write 0 equals to p$_{in}$ minus p$_{out}$ by 4 mu L into R square into bracket k square minus 2 lambda square into ln k plus C$_2$, bracket closed, ok. So, the boundary condition is like this, at r is equals to kR, v$_z$ is 0. And the second boundary is that r is equal to R, v$_z$ is also 0, why? Because both are solid surfaces.



So, the liquid, inner liquid in the inner tank, in the inner cylinder, that is also getting cling with the surface, and the outer cylinder that is also clinging with the surface. So, this means that if you put the boundary v_z is 0 at r is equal to KR and at r is equal to capital R. So, it is 0 equals to P_{in} minus P_{out} over 4 mu L into R square into k square minus 2 lambda square ln k plus C_2. This is for one boundary, and the other boundary is 0 equals to minus P_{in} minus P_{out}, there also it was minus P_{in} minus P_{out} into R square by 4 mu L into bracket one plus C_2 bracket closed. So, we can write.

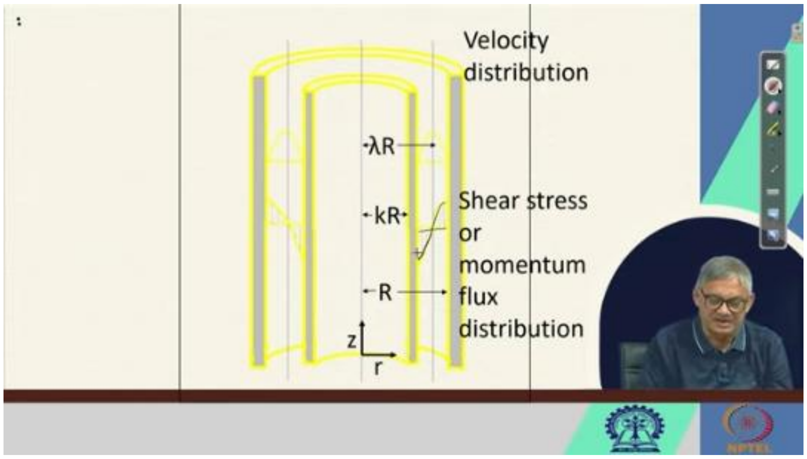
Since C_2 is equal to minus 1 and 2 lambda square is equal to 1 minus k square by ln 1 by k, then we can write: momentum flux distribution and velocity distributions are like this. C_2 we have taken as minus 1, not that C_2 we have taken minus 1. C_2 has come to be minus 1, and 2 lambda square is 1 minus k square by ln 1 by k. So, we found out both C_2 and 2 lambda square. Therefore, we can find out the momentum flux distribution and velocity distribution like this: tau_{rz} is P_{in} minus P_{out}

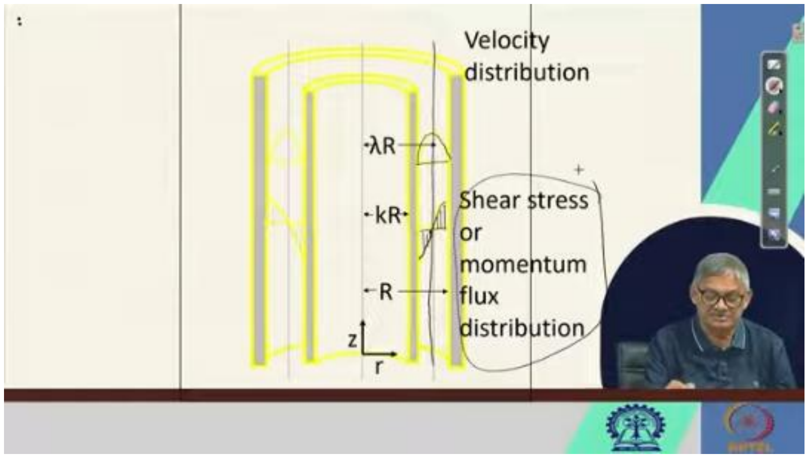
into capital R by 2 L whole into r by R minus 1 minus k square by 2 ln 1 by k into r by capital R, and v_z is equal to p_{in} minus p_{out} into R square by 4 mu L. into 1 minus small r by capital R whole square plus 1 minus k square by ln 1 by k ln of r small r by capital R. So, these are the two distributions, one for momentum and another for the velocity distribution. This we have to keep in mind that yes, we have not mathematically solved everything. At some points, we have taken into consideration that through mathematics it should come like this. Where the exact point at which the velocity will be maximum or the shear stress will be minimum is not known. So, it is not that easy to find out what is the



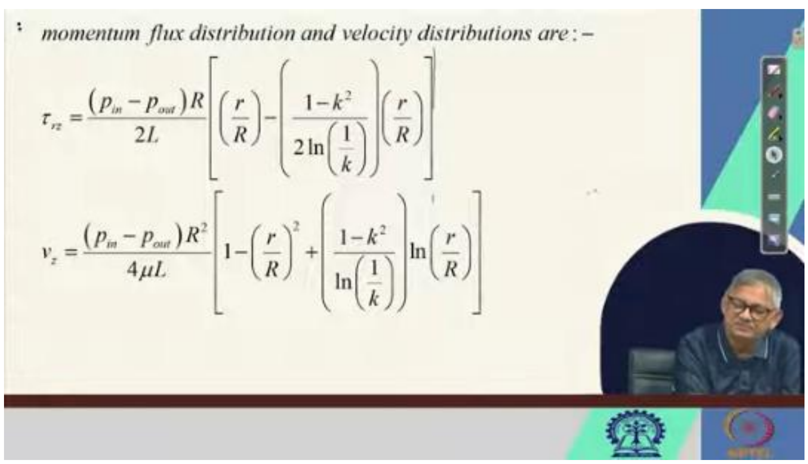
velocity profile and momentum flux or shear stress profile, right? What we have done? We have taken an arbitrary plane, lambda in between R and kR, and see that how much tau_{rz} and v_z are getting affected. If we go back to the diagram, here you see that I do not know whether it is visible or not that this is the

Diagram for the stress profile, where stress profiles are like this, right? And this is the velocity profile. Which is like this, and this is known as that arbitrary velocity profile determination or to find out the maximum velocity, where it is, or to find out the minimum stress of momentum. So, this we have now made that, yes. This can be obtained. And we have obtained the mass flow rate, sorry, we have obtained,





So, we have obtained the stress distribution, and we have obtained the velocity distribution, and they are like this: tau$_{rz}$ is equal to P_{in} minus P_{out} over 2 L into R into r smaller by capital R minus 1 minus k square over 2 Ln of 1 by k. Into r by capital R, this is for shear stress. Similarly, for the velocity distribution or velocity profile, we have obtained v_z that is equal to some common that is P_{in} minus P_{out} or P_{in} minus P_{out} or P_{in} minus P_{out}. Into R square by 4 mu L into 1 minus small r by capital R whole square plus 1 minus k square ln 1 by k into ln of r by R, right?



So, with this, the time is up. Let us stop this class, and we shall continue in subsequent classes to find out the other parameters, okay.

Thank you all.