## IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture24

## **LECTURE 24 : REYNOLDS NUMBER OF FALLING FILM**

Good morning, my dear friends, students, boys, and girls. We are in the process of developing different correlations between velocities and others. So, it is now a flow through a surface. It is a continuous one. We have already developed the velocity at its maximum.

What is the maximum velocity that we have done, right? Obviously, while doing that, we said that there are certain assumptions. If you remember those assumptions, which we said that between the liquid layer and the above air or any gaseous thing that is non-liquid but gaseous. There is one condition we had given, and between the liquid layers also, we had given one condition, and the surface we said was inclined. So, there also the inclination is beta, if you remember.



So, we had given a condition. So, these three conditions prevail now also to find out the average velocity. We have found out the maximum velocity. Now, it is the average velocity. Now, by definition, average velocity earlier also we have said that  $v_{zaverage}$  is integration of.



0 to w in this case, again, another integration between 0 to del, the two ends, and  $v_z$ , area dx dy, right? dx dy and  $v_z$  is the velocity, right, divided by again that total area that is between 0 to w integral and between 0 to del integral dx dy. Ok. So, on integration of dx dy, so that dy is equal to w. And in the numerator, we have one w and del, but in the numerator w and denominator w cancels out.

So, in the denominator we have one del, right? This is nothing but simple mathematics, right? So, 1 by del, integration between 0 to del that integral  $v_x dx$ . Now,  $v_x$ , if we substitute, already, which we have done earlier, is rho g del square cos beta by 2 mu, right.



So, that is 0 to 1 or it could be said So, 0 to not 1, 0 to L, 1 minus x by del whole square into d of x by del, right. So, this on integration we get ultimately the final expression is rho g del square cos beta by 3 mu. So, the average velocity is rho g del square cos beta by 3 mu, right.

So, if it is average velocity, then the volume rate of flow, or volumetric rate of flow, Q, that can be found out from the relation that Q is equal to the integral from 0 to w of 0 to

del  $v_z$  into the area for flow, which is dx dy. So, this is nothing but capital W into del into  $v_{zaverage}$ . And that is equal to substituting  $v_{zaverage}$ , w, and del. So, it is rho g W del square cos beta divided by 3 mu.



That is the volumetric flow. And del, if you remember, we have taken del as the thickness, or whatever, of the film, right? If you remember, this was our solid surface, and we had liquid, right? This is the liquid surface, okay? This we termed as del,



that is the thickness, or film thickness, right. So, film thickness we can then say to be equal to del equal to 3 mu  $v_{average}$  over rho g cos beta, 3 mu  $v_{average}$  over rho g cos beta, rho g cos beta. This was giving some problem, right, as if some division symbol was coming. However, that was from the previous writing.



So, del is 3 mu  $v_{average}$  over rho g cos beta. This can also be rewritten in terms of volumetric flow rate as cube root of 3 mu Q over rho g W cos beta. Right. So, this can also be written in terms of mass flow rate as cube root of 3 mu m dot over rho square g cos beta.

3 mu m dot over rho square g cos beta. Obviously, m dot is the mass flow rate and is nothing but rho del  $v_{average}$ . Rho times del  $v_{average}$ . That is the mass flow rate. So, film thickness we can find out either directly from the average velocity, if it is known, and all other parameters like what is the angle cos beta, what is the density, what is the value of g, and the viscosity of the fluid.

Volume rate of flow Q,  

$$Q = \int_{0}^{W} \int_{0}^{\delta} v_z dx dy = W \delta v_{z_{av}} = \frac{\rho g W \delta^2 \cos \beta}{3\mu}$$
Film thickness,  $\delta$   

$$\delta = \sqrt{\frac{3\mu v_{z_{av}}}{\rho g \cos \beta}} = \sqrt[3]{\frac{3\mu Q}{\rho g W \cos \beta}} = \sqrt[3]{\frac{3\mu m}{\rho^2 g \cos \beta}}$$
where,  $\dot{m} = mass$  flow rate  $= \rho \delta v_{z_{av}}$ 

If we know the  $v_{average}$ , we can find out del. Then, it is also said to be in terms of volumetric flow rate, volumetric flow rate that del is equals to cube root of 3 mu Q over rho g W cos beta. and this is equals to again cube root of 3 mu m dot by rho square g cos beta where m dot means this is the mass flow rate that is rho del  $v_{average}$  right. Now z component of the force F of the fluid on the surface that is fz is equal to 0 to L, 0 to W, integral tauxz at x is equal to del into area dy dz.

This can be written. As 0 to 1 integral 0 to w integral of minus mu d  $v_z$  dx at x is equal to del times area dy dz, which can be simplified. As Lw into minus mu into minus rho g del cos beta by mu. So, two negatives go off, and mu, mu that goes off. So, it is rho g del Lw

cos beta. It is rho g del l w cos beta, which is the z component of the weight of the entire fluid in the film. Right? Now, for film flow, unlike the fluid flow in pipes and any the Reynolds numbers are defined differently.

The Reynolds number is 4 del  $v_{average}$  into rho by mu. So, here the Reynolds number is 4 del  $v_{average}$  into rho by mu. If you remember, the normal Reynolds number is D v rho by mu, right? D v rho by mu. Here it is 4 del  $v_{average}$  into rho by mu, instead of D for del.

Now, for laminar flow without rippling, what do we mean by rippling? Rippling is like this: if you have a pond like this and if you drop a stone Then, surrounding the stone, a wave like this develops, right? It is not all along; maybe for a few waves, it develops. That is called rippling.



A similar situation is called rippling. Right? It is not that you have to throw a stone, but the condition that is generated is called rippling. So, that means in laminar flow, what we see is that the layers are flowing parallel to each other, right.



So, rippling means these layers have a little wave; they are not mixing, but still, there is a little wave, wavy. So, this condition is explicitly available when the Reynolds number is

between 4 to 25. If the Reynolds number is less than 4 to 25, then we say laminar flow without rippling. And now, if there is still rippling, then we say laminar flow with



Where 4 to 25 is less than the Reynolds number, which is less than 1000 to 2000. So, roughly from say 25 to 1000 or 2000, laminar flow with rippling exists. Then comes turbulent flow, if it occurs, when it is greater than 1000 or 2000. Obviously, 2000 is a very big number, 1000 is also a very big number, but with respect to the Reynolds number of the film flow. Compared to that in pipe flow, these numbers are obviously less, but we have to keep in mind how the Reynolds numbers for different situations are changing, right?

So, we can say turbulent flow has a Reynolds number. Greater than 1000 or 2000, then if the Reynolds number is more than 1000 or 2000, you can say it to be turbulent. And between 4 to 25 and 1000 to 2000, it is laminar flow with rippling. And obviously, less than 4 to 25 is laminar flow without rippling. Then we make a problem and its solution.



It's like that: an oil is flowing down a vertical wall as a film, 1.7 millimeters thick. The oil density is 820 kg per meter cubed, and the viscosity of that oil is 0.2 Pascal seconds. Then

you are asked to calculate the mass flow rate per unit width of the wall, the Reynolds number, and the average velocity. I repeat. An oil is flowing down a vertical wall as a film, 1.7 millimeters thick.

The oil density is 820 kg per meter cube, and the viscosity is 0.2 Pascal second. Calculate the mass flow rate per unit width of the wall. Reynolds number and also the average velocity. So, what is happening? We have a solid surface.

So, instead of water, some oil is given, and it is flowing down, right? We have given all the property values of the oil. Like the thickness of the film, 1.7 millimeters, the density of the oil, 820 kg per meter cube, and the viscosity as 0.2 Pascal seconds, right. We have to find out the mass flow rate per unit width of the wall. Reynolds number and the average velocity.

Then its solution could be like this: if the mass flow rate, earlier, if I had told you that we could not make m dot. In some place. So, this is the mass flow rate, m dot, from the relation we have, we had found out between the film thickness and the mass flow rate, if you remember. Then we had said m dot is delta cube rho g by 3 nu. m dot is delta cube rho g by 3 nu.



This is equal to 1.7 into 10 to the power minus 3 whole cube, 820 into 820 into 9.81 by 3 into 0.2 by 820, right? We have just substituted the values of the individual parameters given. Del, we have been given, 1.7 into rho minus 3, and it is in cubes so it is whole cube. Rho is given as 820 kg per meter cube. g, universal, is more or less 9.81.

3 nu, 3 is there, mu is 0.2, and its density is 820, so that is nu. So, this comes to equal to 0.054 kg per meter per second. We are asked what is the mass flow rate per unit width. This is per kg per meter per second, mass flow rate is kg per second, and unit width is per

meter, right. Similarly, the Reynolds number can also be calculated based on mass flow rate.

Again, unit width, we have said previously, and yeah, mass flow rate, we have already found out, 0.054 kg per meter per second. Reynolds number in terms of mass flow rate, we have said, 4 m dot. And that is 4 into 0.054 by 0.2, and that is equal to 1.08. Now, average velocity is rho g del square by 3 mu. Average velocity is rho g del square by 3 mu, that is, rho is 820, g is 9.81, and del is 1.7 into rho minus 3 whole square because del square. Divided by 3 mu, that is 3 into 0.2.



So, this became equal to 0.037 meters per second, 0.037 meters per second, ok. This is the average velocity. Before we conclude, here we can say that this is a very simple problem. This was oil flowing, and its property values were also given. There could be such points where you have two unknowns and one equation.

Then you may need the help of those supervisors or somebody so that the values, such as mass flow rate, Janos number, and average velocities, are correctly found out. Now, to do these corrections or to do these calculations, normally the scientific calculator is good enough. We do not need very complicated things. So, it can be from there directly, but at times if these

values are not properly given, then it is very difficult to find out. And in those cases, you may need to do some trial and error so that it can be easily established. So, the mass flow rate is very low, and as you know, the density of oil is also more or less, of course, less than water because oil generally flows on top of the water. And that is why it has been given 820. And the mu of that is also not very this or that.

That mu value of the fluid of the oil can be said to be around point 2, we have given it right. We have given the, of course, we have not given the mu directly, we have been given a point and That Reynolds number which we found out, 1.08, is also very very low. We said with or without rippling.

If it is between 4 to 25 without rippling, it can be said to be Reynolds number. And if it is between this 4 to 25 and 1000 to 2000, then it becomes something immortal. And obviously the average velocity which we have found out is perhaps the correct one where the relation we have used is rho g del square by 3 mu. If you follow the previous class also, then you will be able to relate to this.

So, we say that with this problem We thank you for listening to this class and yes, thank you all.