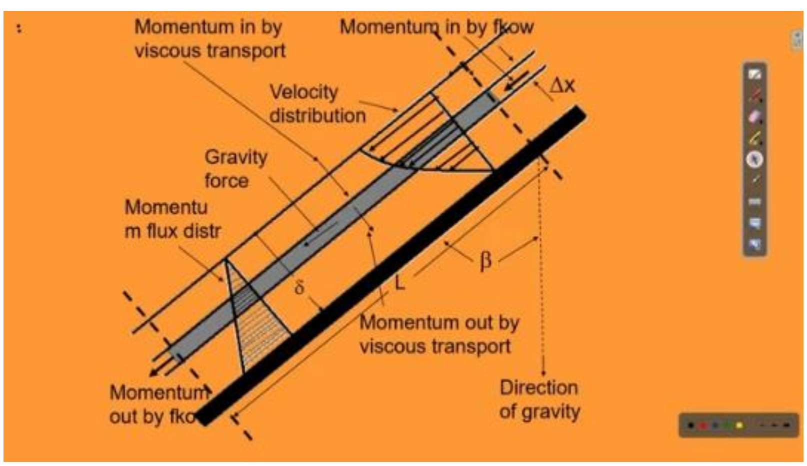
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

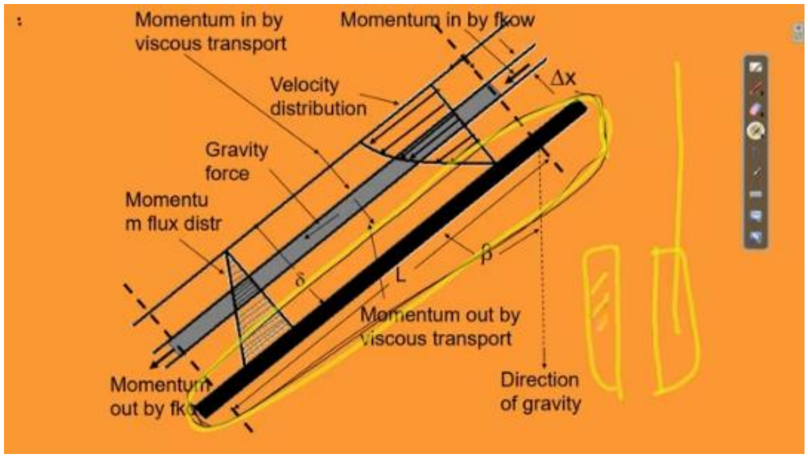
Lecture23

LECTURE 23 : FLOW OF FLUID THROUGH INCLINED OR HORIZONTAL SOLID SURFACE

Good morning, my dear boys and girls, students, and friends. So, we are in the process of fluid flow, right? Now, another new topic which is also very relevant is flow through a surface. For example, this is one, you see that the surface is here, that surface is this one, okay. This is the solid surface.

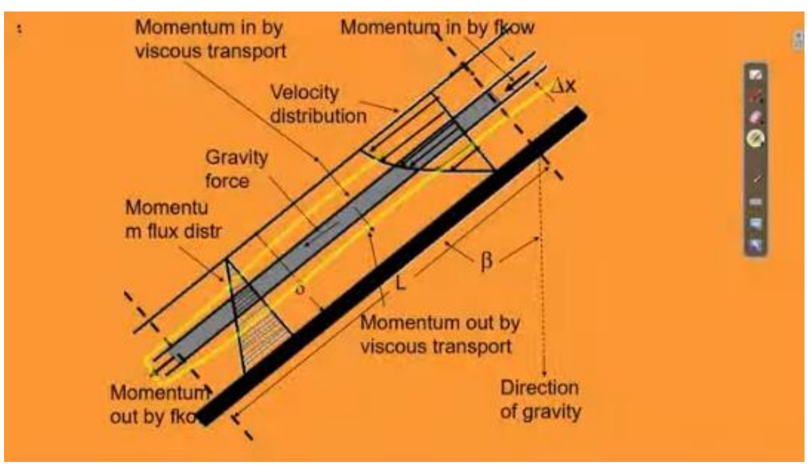


Okay, this is the solid surface. Now, here we are showing it to be inclined because if we are showing it to be inclined, it can also be vertical like this. We said in the beginning, in the preamble classes, do you remember, or you can search back, that things are getting concentrated through jacketed. This is jacketed. So, here heating is done.

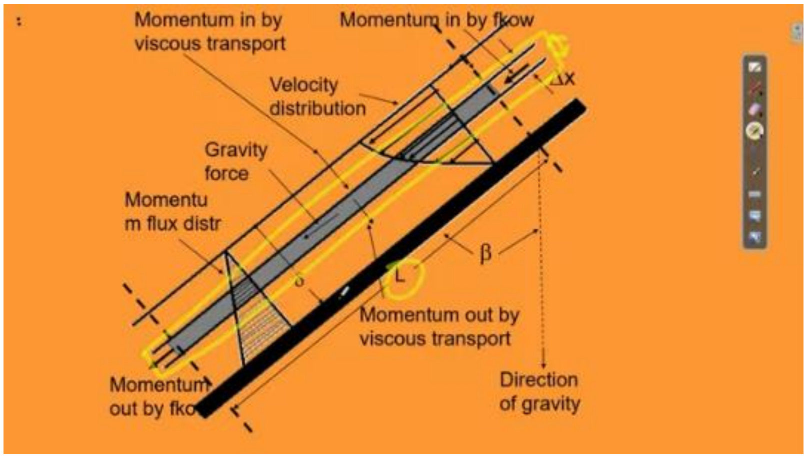


So, the gap between them is very, very narrow. So, a drop of slurry falls here, which gradually gets vaporized, and you get a concentrated outlet. Right? This we have shown, we have said in the beginning, right? Now, we have taken it inclined because we get cos beta, the angle of inclination, right?

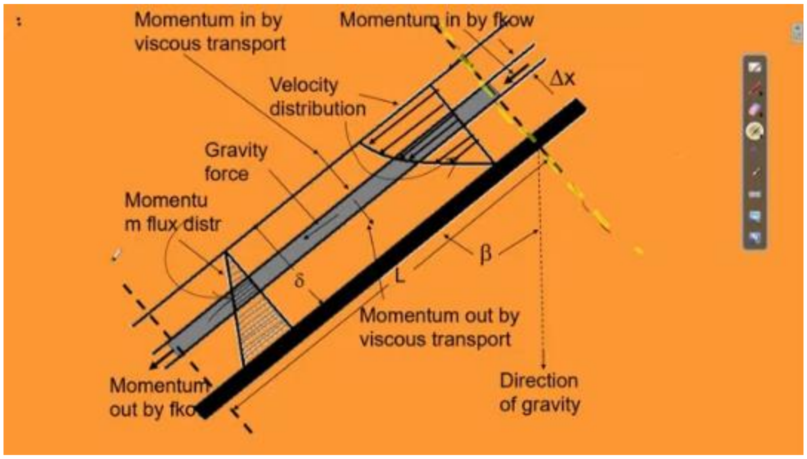
And And for this, we are taking a volume element. This volume element is this one, right? This volume element is this one where you see the one. One dimension is delta x, so small, right?

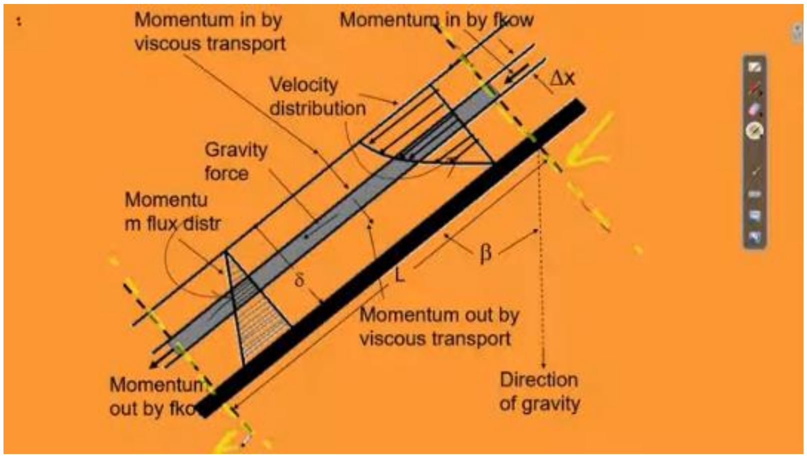


The third dimension is unit perpendicular to this, and the second dimension is the length of the surface, that is L, ok. Now, we are flowing a fluid through this. We have taken that shell or control volume like this; this is that, ok? This is that control volume, and we have to find out. The momentum flux distribution; we have to find out the momentum flux distribution, that is this. We have to find out the velocity distribution like this, and we have taken here; there is

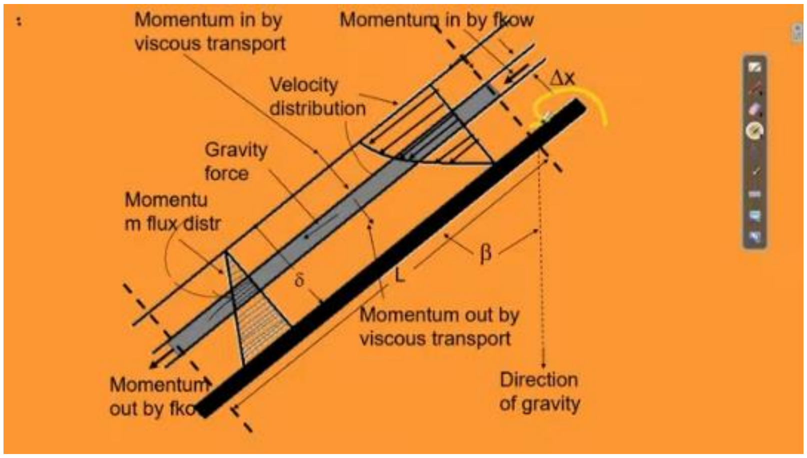


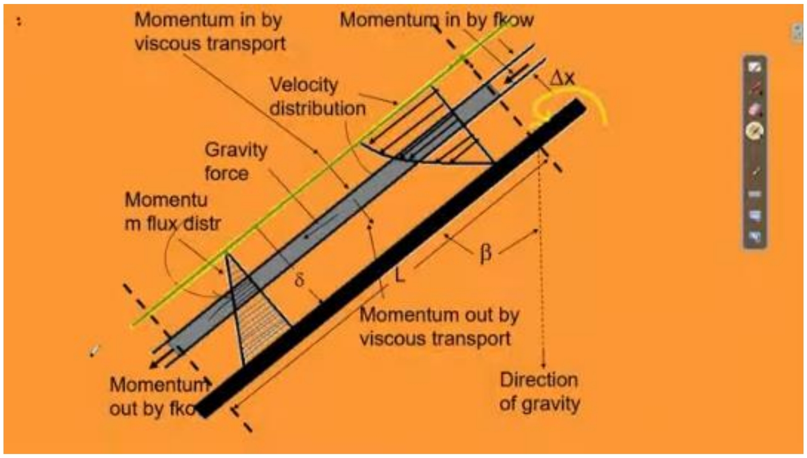
end effect, right? So, this is what means that there is no end effect; that is, whatever end effect is there, this is the perfect flow, fully developed flow, right? This is fully developed flow that we have considered, right? Now, another thing which is very important is that there are certain assumptions. You see, this is the surface, right?





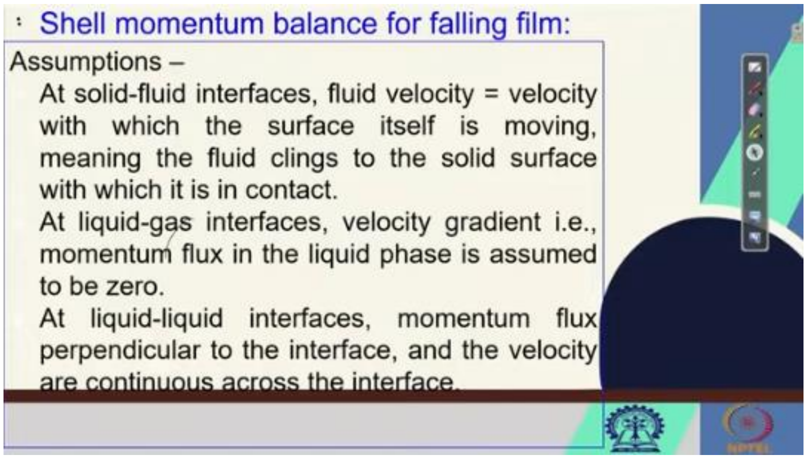
This is the surface, and this is the top layer of the fluid. Obviously, it is in the expanded form. So, this top layer is open to the atmosphere, right? This is open to the atmosphere. So, the boundary condition will vary accordingly. So, those we will discuss now.





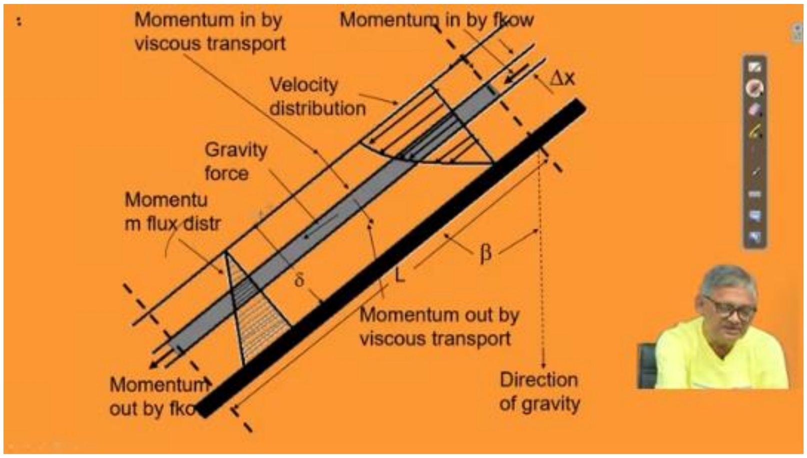
This is the direction of gravity. This is the direction of gravity, and the angle between the surface and the vertical is angle beta, right? The length, or thickness, whatever you call it, is del. So, the distance from the top to the surface is del, and one more thing: the layer which is in contact with the surface, that is, this layer, the liquid which is in contact with the surface, is assumed to be clinging to the surface. Right, it is clinging to the surface. That means if this surface is stationary, then the layer which is clinging to the surface is also stationary, right? There is no movement of that layer.

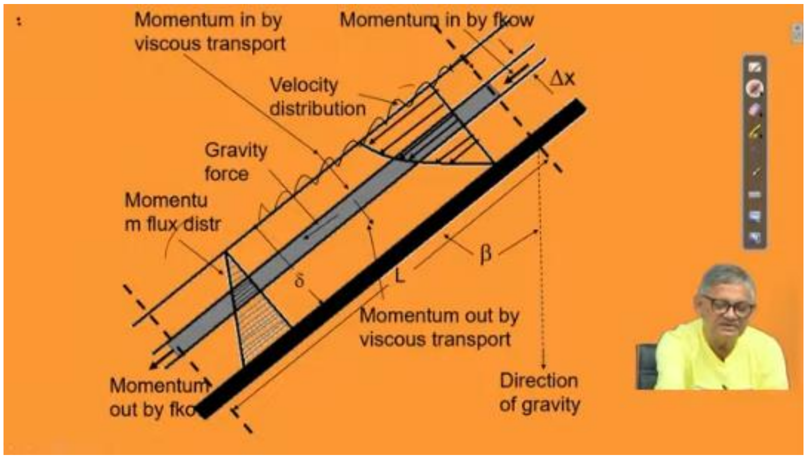
So, with this, let us go to the assumptions which are valid for this development. The assumptions are that we are doing a shell momentum balance, and this is called the falling film, right? As we have shown with an inclined surface. With an inclined surface, you have shown that the fluid is coming and falling into it, and there is a fully developed flow. There are no end effects; that is what we are putting into the assumptions. So, the assumptions say that at solid-fluid interfaces, the fluid velocity equals the velocity with which the surface itself is moving.

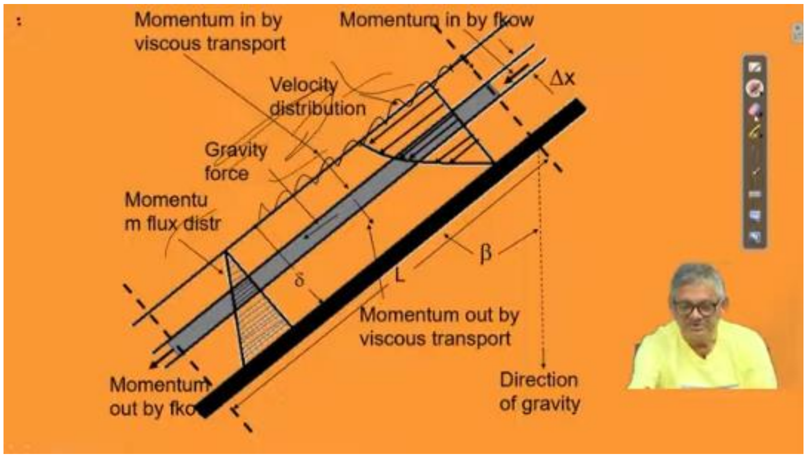


This means the fluid clings to the solid surface with which it is in contact. Right? At liquid-gas interfaces, which I have shown you and I told you that you remember. At liquid-gas interfaces, this is that, unless I take some pen it will not be.

This is that liquid-gas interface, right. This is gas or atmosphere or whatever, right. So, at these interfaces we The velocity gradient, that is momentum flux in the liquid phase, is assumed to be 0. That is, at liquid-gas interfaces, the velocity gradient, that is momentum flux in the liquid phase, is assumed to be 0.

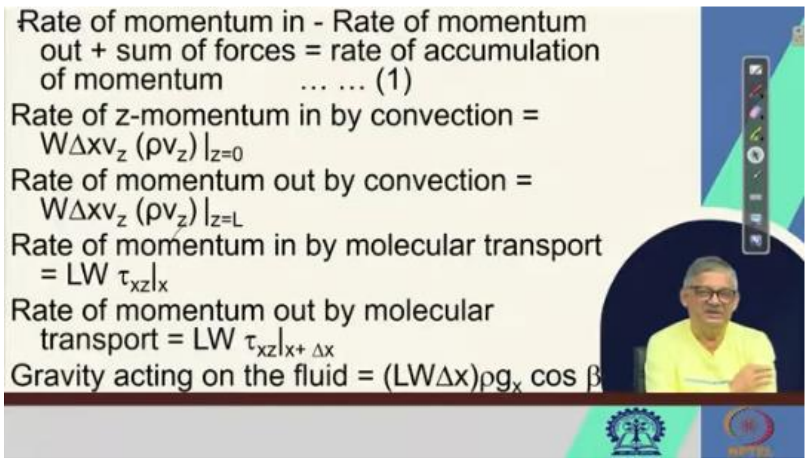






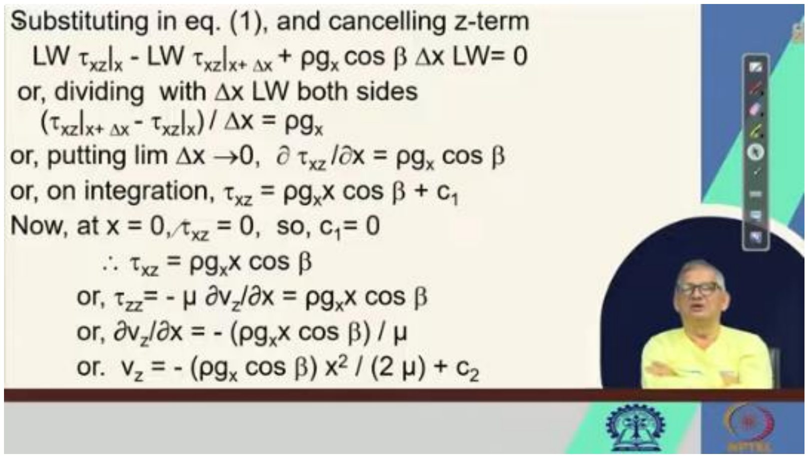
At liquid-liquid interfaces, momentum flux perpendicular to the interface and the velocity are continuous across the interface, right. With these three assumptions, if you want, you can repeat quickly that at solid-fluid interfaces, that is the bottom part Velocity, fluid velocity is the velocity with which the surface itself is moving, meaning the fluid clings to the solid surface with which it is in contact, right? And at liquid-gas interfaces, the velocity gradient, that is momentum flux in the liquid phase, is assumed to be 0. And the third point is very vital, all are vital of course, at liquid-liquid interfaces, momentum flux perpendicular to the interface and the velocity are continuous across the interface.

Then the boundary or governing equation we can write that the rate of momentum in minus the rate of momentum out plus the sum of the forces acting on the control volume is equal to the rate of accumulation of momentum, right? Similar to the earlier also. The rate of momentum in minus the rate of momentum out plus the sum of the forces acting on the volume element is equal to the rate of accumulation of momentum. Now, the rate of z momentum in by convection is the area that is w into delta x v_z into rho v_z at z is equal to 0. Similarly, the rate of momentum out by convection is also w delta x v_x into rho v_z is equal at x is equal to L is 0.



It is the rate of momentum out by convection. Then, once convection is over, it becomes molecular transport, that is, the rate of momentum in by molecular transport is L w tauxz at x. And the rate of momentum out by molecular transport is Lw τ_{xz} at x plus delta x. Gravity acting on the fluid is Lw delta x into rho g_x cos beta. Lw delta x is the volume element. Rho g_x is the

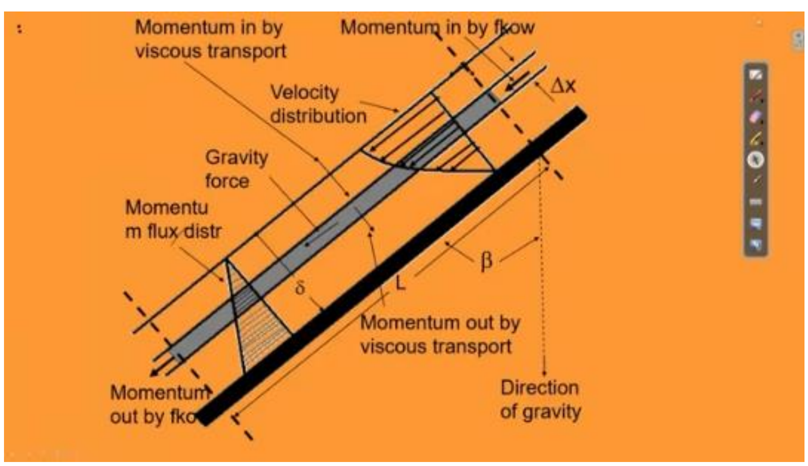
gravity, and this is at an angle of beta, so cos beta. So, individual terms we have found out. Now, if we substitute this in the governing equation, as earlier we have said, then we can cancel the rate of momentum in earlier, the rate of momentum in by convection, and the rate of momentum out by convection because it is under steady state. So, whatever is coming, is going out.

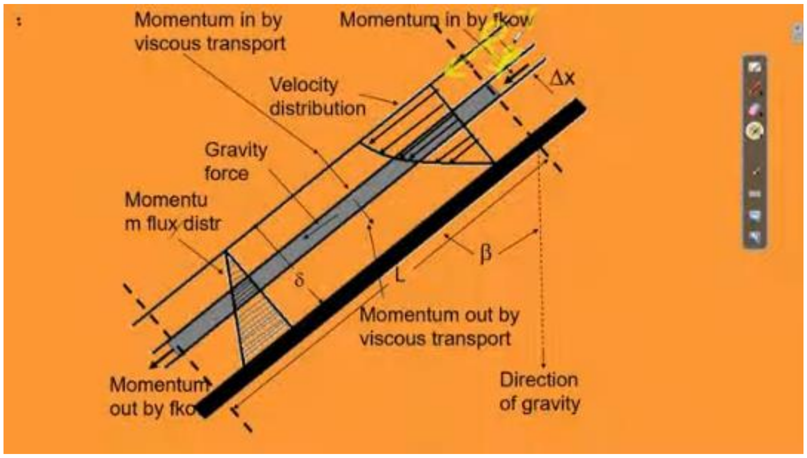


So, we can write that equation, which is the governing equation, eliminating the bulk transport, as L w tau$_{xz}$ at x minus L w tau$_{xz}$ plus delta x plus rho g$_x$ cos beta delta x into L w. So, this is equal to 0. Now, if we divide both sides by delta x L w, then the left side becomes tau$_{xz}$ at x plus delta x minus tau$_{xz}$ at x divided by delta x, because here also we had L w, L w. So, L w goes out, delta x remains and rho g$_x$ because delta x L w goes out. So, rho g$_x$ cos beta, right?

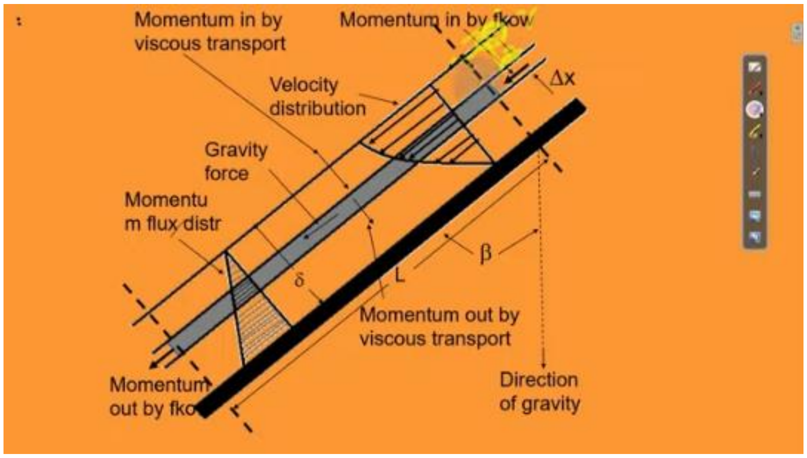
Now, putting the limit x tends to 0, we can write it as del del x of tau$_{xz}$ that is equal to rho g$_x$ cos beta or, on integration, we can write tau$_{xz}$ is equal to rho g$_x$ x into cos beta plus C$_1$. Now, at x is equal to 0, tau$_{xz}$ is also 0; that is the boundary. At x is equal to 0, tau$_{xz}$ is also 0. Mind it, we have taken our axis from the open end. I go back to that diagram again.

Here is our axis, okay. Here is our axis, here. So, X goes like this, and Z goes like this, and the third dimension is Y. Right? So, it starts from here.





So, that means if that is the truth, then we can say that At x equals to 0, the boundary is tauxz is equal to 0, that was our, this is in accordance with the assumptions we have made, right. Then if delta x tends to 0, tauxz del x is equal to rho g_x cos beta. So, again on integration, this gives tauxz is equal to rho g_x x cos beta plus C_1. So, the C_1 is the integration constant.

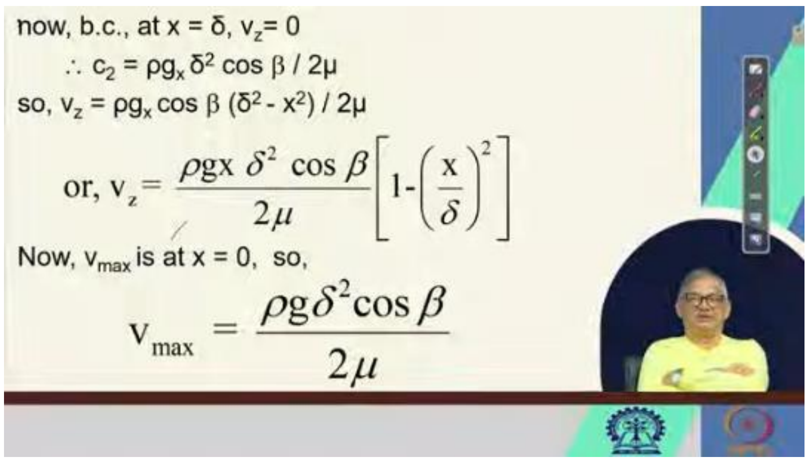


Now, we need to know the boundaries again. At x equals to 0, tau_{xz} is also equal to 0. At x is equal to x, at x equal to 0, tauxz is 0. So, C_1 is also 0 and therefore, we can write tau_{xz} is rho g_x into x cos beta and del Ok, we define tau_{zz} as no, this tau_{zx} only, not z is mu minus mu del vz / del x and this is equal to rho g_x into x into cos beta.

Therefore, there is a negative term, minus mu, minus mu del v_z / del x. So, that negative term to replace it, we are putting it to the left-hand side, ok. And I should do this otherwise it will be keeping in your eye. That this, should be xz instead of zz, this is xz, ok. So, tau_{xz} is minus mu from this definition del vz / del x that is rho g_x x into cos beta. Or del v_z del x we can write is equal to minus rho g_x into cos beta divided by mu.

Therefore, v_z on integration we get v_z is equal to minus rho g_x cos beta x square by 2 mu plus C_2, right. Another integration constant has come, and we have to find out with boundary. So, the boundary condition as we have stated the problem earlier is that at x equals to v_z is equal to 0.

At x equal to del, v_z is equal to 0. Therefore, C_2 becomes equal to rho g_x into del square into cos beta by 2 mu, and v_z is rho g_x cos beta into del del 2 square minus del square cos beta by 2 mu into 1 minus x by delta, right. So, by putting in the boundary, v_z is equal to del becomes rho g_x del square cos beta by 2 mu, and v_z becomes rho g_x cos beta into del



Therefore, we can write v_z is equal to rho g_x square rho g is rather x into del square cos beta by 2 mu into 1 minus x by del whole square. So, as the value is increasing from x_0 to x del, that second term x by del becomes 0. Right, or at x equals to 0, it also becomes 0. So, accordingly, we get the v_z. So, v_{zmax} is at x equals to 0. Hence, we can say that v_{max} or v_{zmax}, whichever is suitable for you, B max or v_{zmax} is rho g del square cos beta by 2 mu. So, we find out the maximum velocity when a fluid is flowing through an inclined surface. Right. We have to find out also the average velocity and some force, whether it can be taken later or whether the force is acting. We will come next class with the average velocity, right.

So, maximum velocity is rho g del square cos beta by 2 mu, average velocity we will find out, and instantaneous velocity is is rho g del square into del square by 2 cos 2 del 2 mu, right. So, it becomes rho g_x del square cos beta by 2 mu, that is the instantaneous velocity into 1 minus x by del whole square. It is parabolic in nature, and the maximum velocity we have seen it to be rho g del square by cos beta rho g del square cos beta by 2 mu, ok. Thank you.

now, b.c., at $x = \delta$, $v_z = 0$

$\therefore c_2 = \rho g_x \delta^2 \cos\beta / 2\mu$

so, $v_z = \rho g_x \cos\beta\,(\delta^2 - x^2) / 2\mu$

$$\text{or, } v_z = \frac{\rho g x\ \delta^2\ \cos\beta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

Now, v_{max} is at $x = 0$, so,

$$v_{max} = \frac{\rho g \delta^2 \cos\beta}{2\mu}$$