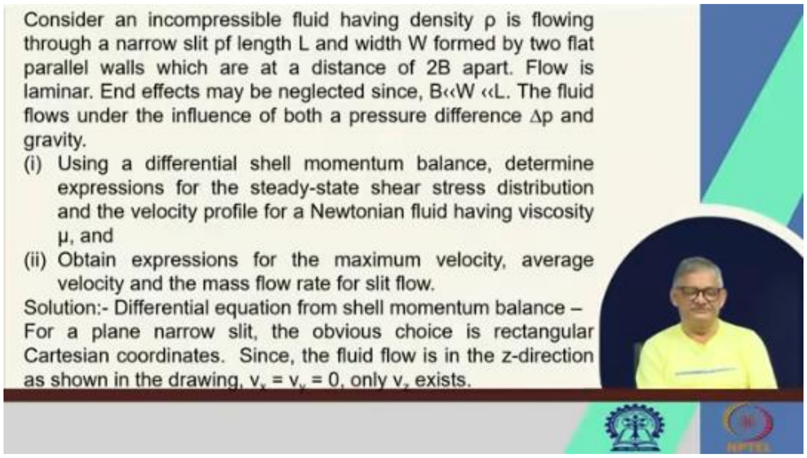
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture22

LECTURE 22 : VERTICAL FLOW THROUGH PARALLEL PLATES

Good morning, my dear friends and students. We are now in the process of fluid flow through pipes, right? We have done many courses, many classes, right? And now we would like to cover some more topics which are relevant to

pipe flow, right? We have covered flow through pipes, and we have covered flow through two parallel flat plates. So, we have done many like that. Now, one more is very important, and this, if you see, is a fluid which is incompressible, has a density of rho, and is flowing through a narrow slit. This is of'



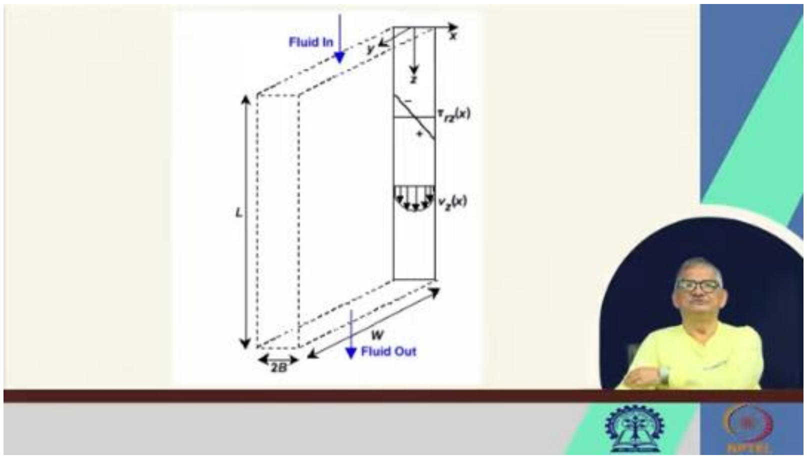
length L and width W, formed by two flat parallel walls which are at a distance of 2B apart. Now, the flow is laminar, and end effects are not considered; they are neglected because B is much less than W, which is much less than L. The fluid flows under the influence of both a pressure difference of delta and also under gravity. Now, we are asked to use a differential shell momentum balance to determine expressions for the steady-state shear stress distribution and the velocity profile for a Newtonian

Fluid having viscosity μ and also obtain expressions for the maximum velocity, average velocity, and the mass flow rate of the fluid through the slit. So, as usual, when we deal with a problem, first we read the problem and then we reread the problem so that the intricacies are before us. We know everything: what to do, what is asked for. So, in this case, We have considered an incompressible fluid having density ρ flowing through a narrow slit of length l and width w formed by two flat parallel plates or walls which are at a distance of 2B apart.

The flow is laminar; there is no end effect, the reason being B is very, very low compared to W, which is also very, very low compared to L. The fluid flows under the influence of pressure obviously, differential pressure, pressure difference Δp, and also under gravity. So, we are asked that you use differential momentum balance to determine expressions for the steady-state stress distribution and the velocity profile for a Newtonian fluid having viscosity μ and obtain expressions for the maximum velocity, average velocity, and the mass flow rate through the slit of the fluid flow. A similar problem we have done earlier also; shell momentum balance we have started with pipe flow and many others.

So, we can do it here also. The differential equation from shell momentum balance we can say that for a plane narrow slit, the obvious choice is rectangular. and Cartesian coordinates. Since the fluid is flowing in the z-direction, as we are showing it in the next diagram, then v_x is equal to v_y is equal to 0. Only vz exists.

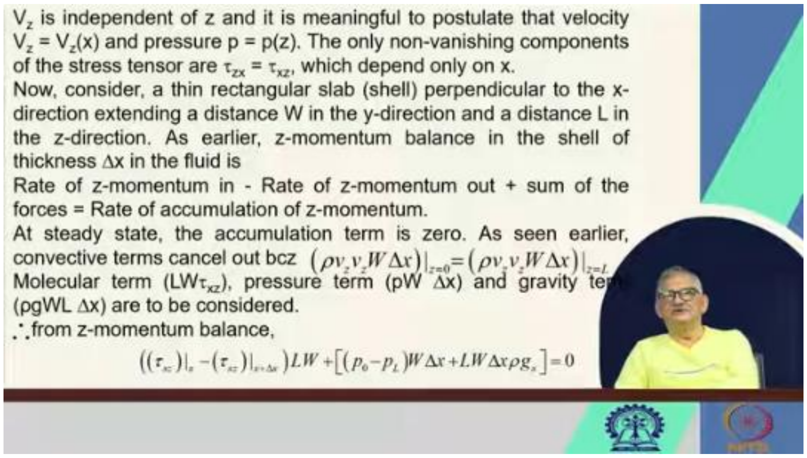
Let us see. So, this is that fluid and the slit. As you see, the fluid is flowing through the slit from top to bottom. The gap between the parallel plates, or whatever you call it, is to be. The width of the slit is W, and the height



z, which is from 0 to L, right. So, our xy axis, we have already shown that this is the x, and this is the z axis. Here it is, this is the x-axis, and this is the z-axis. Then we are asked to find out this, that is the line or locus of the shear force. And also the locus of the velocity profile like this.

We are asked to find it out, right. So, from this, what we have seen is that it is a laminar flow. Right, and all other things which we have said from there, we can say that v_x equals v_y equals 0, right, which we have said in the previous slide, that v_x is equal to v_y equal to 0, Only v_z exists, ok. So, that v_z exists only, so we have to deal with that.

So, v_z is independent of z, and it is meaningful to postulate that velocity v_z is a function of x, and pressure P equals P_z. The only vanishing components of the stress tensor are that is, tau_{zx} equals tau_{xz}. Again, I am repeating, earlier also I said when tau is coming, it is associated with two subscripts. The first subscript, we said that it is nothing but the direction

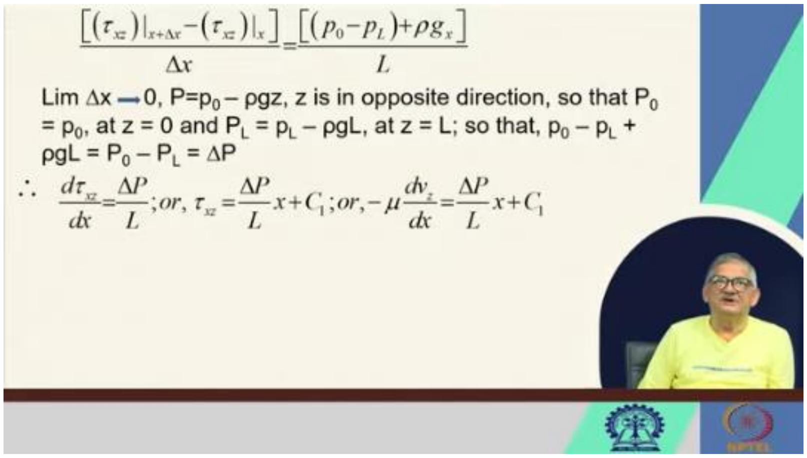


And the second subscript, we said, it is the velocity which is acting. So, tau$_{zx}$ and tau$_{xz}$, these two stress tensors, we can say to be vanished, which depends only on v_x, of course. Now, consider a thin rectangular slab shell perpendicular to the v_x direction extending a distance of W in the Y direction and a distance of L in the Z direction. As earlier, we have said that the z momentum balance in the shell of thickness

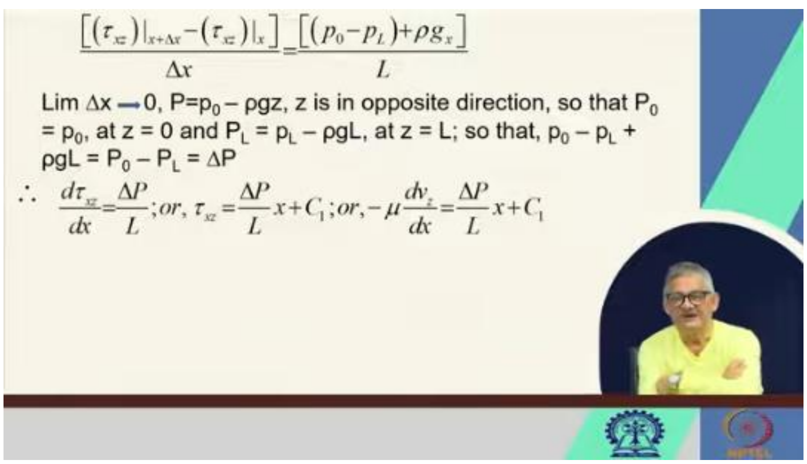
delta x in the fluid can be written as the rate of z momentum in minus the rate of z momentum out plus the sum of the forces acting on it is the rate of accumulation of the z component of the momentum. So, at steady state, the accumulation term again vanishes, is zero. As seen earlier, convective terms also cancel out because it is at steady state, right? So, rho v_z v_z w times delta x at the face z equals 0 is equal to rho v_z v_z w delta x at the face x plus delta x, that is, x equals L. And the molecular term L w tau$_{xz}$

When the pressure term, that is P w delta x, and the gravity term rho g rho g w l delta x, are to be considered. Then, from the z component of momentum balance, we can write tau$_{xz}$ at the face x minus tau$_{xz}$ at the face x plus delta x into the area Lw plus the pressure term, that is Poutlet rather, p_0 minus p_L, not poutlet, p_0 minus p_L times w into delta x plus Lw delta x rho g_x, this is equal to 0, right. So, we can rewrite and bring it to the form so that if we put the limit, then it becomes a derivative like tau$_{xz}$ at the face x plus delta x minus tau$_{xz}$ at the face x over delta x, we have also divided.

All with delta x delta y or w and L, right. So, here L w, w delta x, here L w. So, you have divided both sides by L w delta x, and that is why we are getting tau$_{xz}$ at the face x plus delta x minus tau$_{xz}$ at the face x over delta x is equal to P_0 minus P_L plus rho g over L, right. Now, if we put the limit delta x tends to 0 p becomes equal to p_0 minus rho g_z, z is in the opposite direction of gravity.

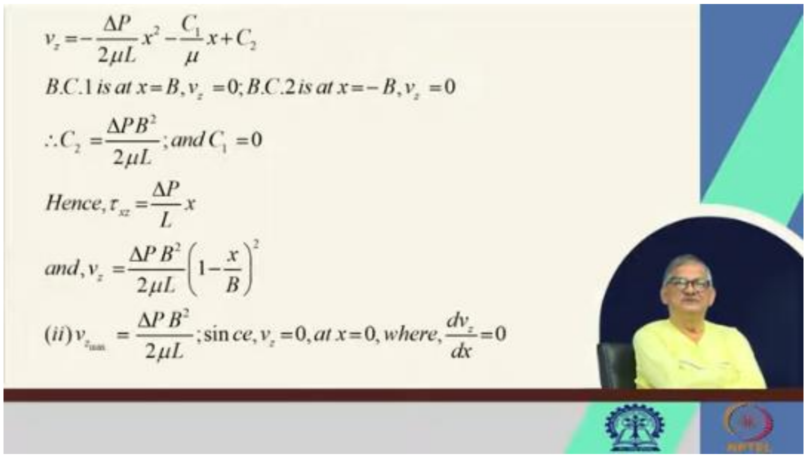


So, that p_0 becomes equal to Capital P_0 becomes equal to small p_0 at z is equal to 0, and capital P_L equals to minus small p_L minus rather small p_L minus rho g_L at z is equal to L. So, that p_0 minus p_L plus rho g_L is equal to p_0 minus p_L is equal to delta p. Mind it, how we proceeded? We proceeded, we have said, delta x tends to 0, then it becomes a derivative, and on the other side, we said P is P_0 minus rho g_z, z is obviously in the opposite direction of gravity.



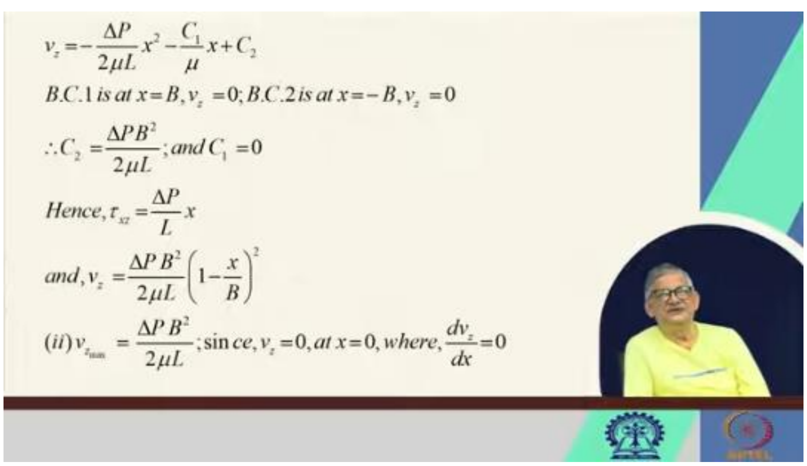
So, P_0 equals to capital P_0 is equal to small p_0 at z equals 0, and P_L is equal to rho g_L at z equals L, such that. P_0 minus P_L plus rho g_L becomes equal to P_0 minus P_L equals delta P, or capital P_0 minus capital P_L equals delta P. Therefore, we can write the equation as d tau_{xz} dx. Or that could have also been written as del tau_{xz} del x equals delta P over L, right. So, on integration of this, we get tau_{xz} equals delta P over L into x plus C_1. So, here we substitute tau_{xz} with minus mu d v_z / dx, minus mu dv_z / dx equals delta P by L into C_1, or rather plus C_1, right.

So, now, we have to find out the value of C_1. The value of C_1 to find out, or if we do the full integration, like if we do the second integration, we get vz equals delta p by 2 mu L into x square minus C_1 by mu into x plus C_2, right. Obviously, boundary conditions will be at plus B and at minus B. So, at x equals B, v_z is 0, because it is clinging to the surface. So, the velocity component is 0.



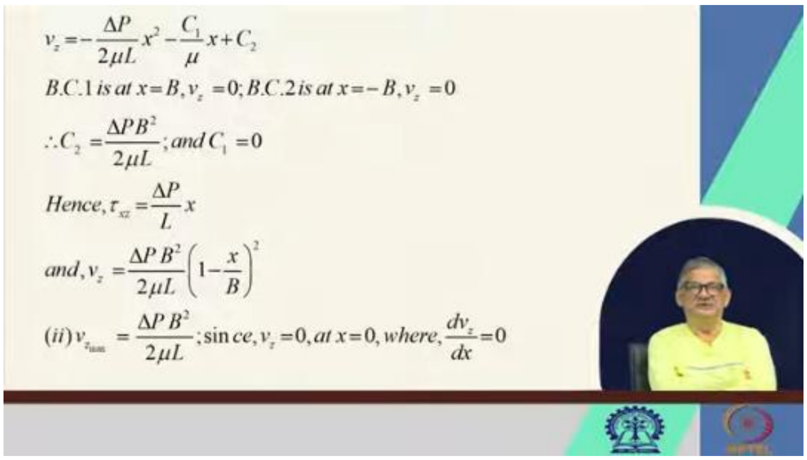
And the other boundary condition 2 is at x equals minus B, v_z again equals 0. Because in both cases, the fluid is clinging to the surface of the container, and depending on the other property values of the container and the fluid, this is easily found out. This is an approximation we have given. That v_z, OK, is 0 at x equals minus B. Right.

Therefore, C_2 can be written as delta P B square by 2 mu L, and C_1 can be written as 0. In both cases, it was a definite boundary, that is, at x equals to B, v_z is 0, and at x equals to minus B, v_z is also 0. Hence, C_1 or C_2 rather is delta P B square by 2 mu L, and C_1 equals to 0. Therefore, tau_{xz} equals delta P by L into x. Rather tau_{xz}, not t, tau_{xz} equals delta P by L into x.

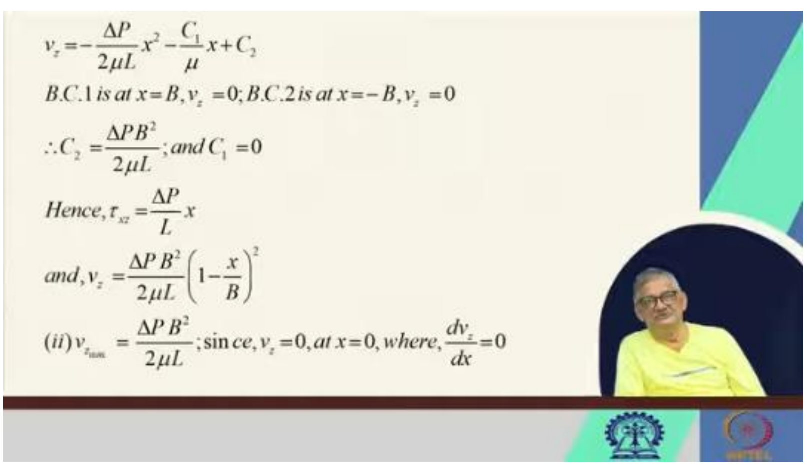


And v_z equals delta P B square by 2 mu L into 1 minus x by B whole square, right? This arithmetic or this mathematics we have done is simple. So, there is nothing hard-rocking or rocket-launching technique here. So, we got both tau_{xz}, that is the shear stress profile,

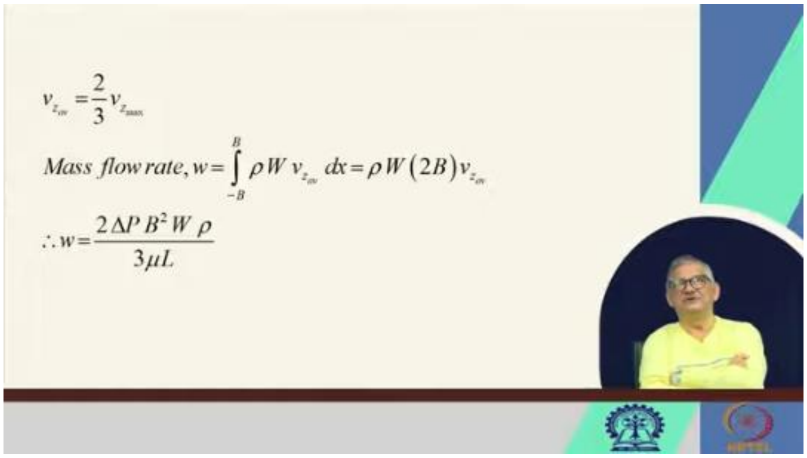
v_z equals 2 delta P into B square by 2 mu L into 1 minus x by B whole square. So, also similar parabolic, right? So, from here you see, if we put x equals to 0, then v_z becomes delta P by 2 mu L into B square, right, because after negative x by B that becomes 0, and this is 1 minus. So, this is the maximum.



So, v_{zmax} should be delta p B square by 2 mu L into L. Then the other one, that is the average velocity, that we can find out. The delta $P_{average}$ is delta P by 2 v square divided by mu L. Since v_z is 0, then d v_z / dx is also 0. Since v_z is 0 at x equals to 0, then d v_z / d x is also equal to 0. Then we can say that $v_{zaverage}$ is nothing but two-third of v_{zmax}.

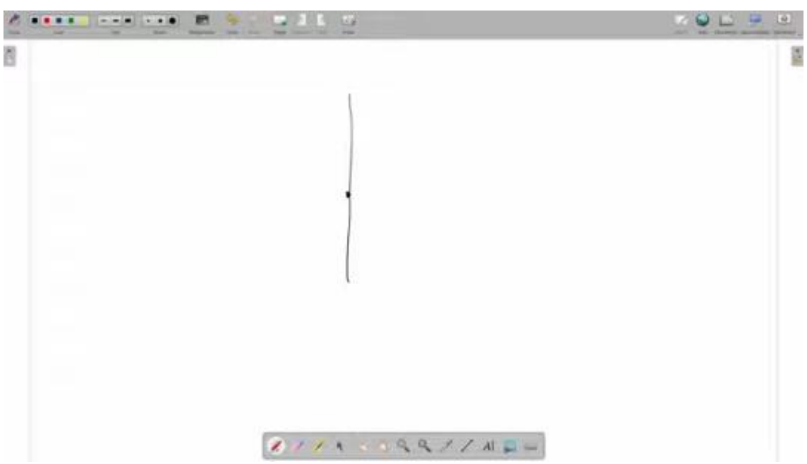


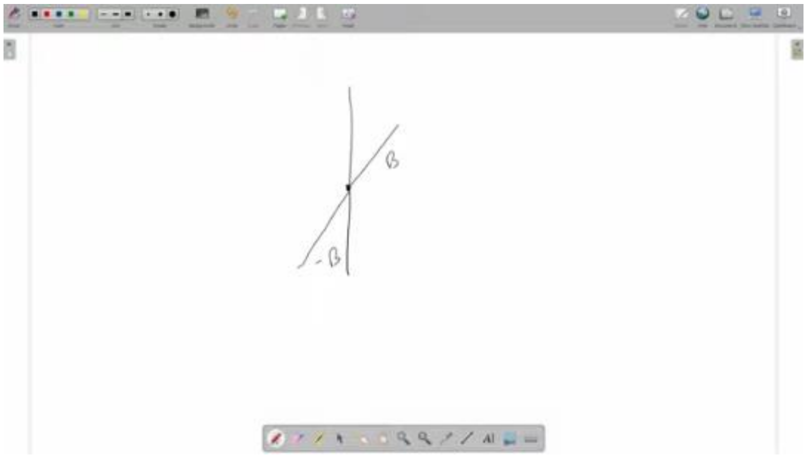
In the earlier cases, we have shown how the average velocity is found out. In the same way, you are finding out the average velocity. We have not done here, and then we can show that $v_{zaverage}$ is nothing but two-third of $v_{zmaximum}$. Similarly, we can find out the mass flow rate, that is say a w

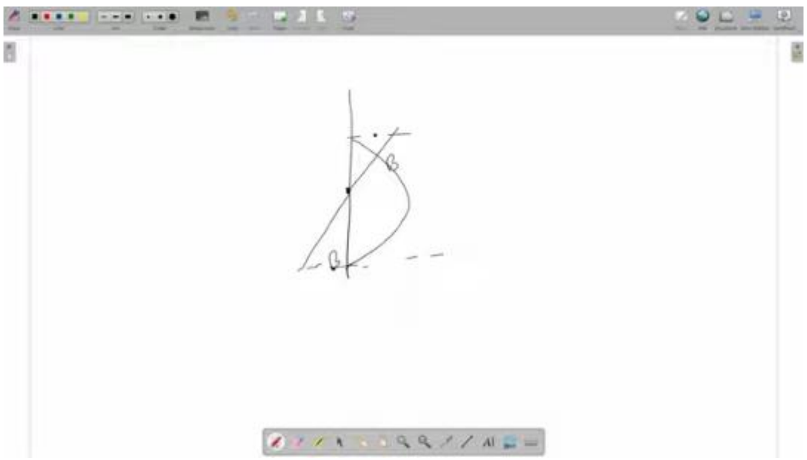


This is equal to integration within minus B to plus B, rho capital W $v_{zaverage}$ dx, is equal to rho W into 2 B into $v_{zaverage}$. Which on further simplification, we can write that the mass flow rate w is 2 delta P B square W rho divided by 3 mu L. Hence, mass flow rate w equals to 2 delta P B square W rho divided by 3 mu L, is the mass flow rate. So, we have found out mass flow rate, average velocity, velocity profile, and also stress profile, right. So, before saying thank you, I would like to show you one more thing here.

If you look at the profile right, if this is the center, then the profile of stress right, it went like this plus B minus B right. And if this is the outer layer that is the shell that is the plate, then we see that the velocity profile looks like this parabolic in nature right. So, with this, we come to the end of the class.







of this particular and we thank you for attending the class. Thank you.