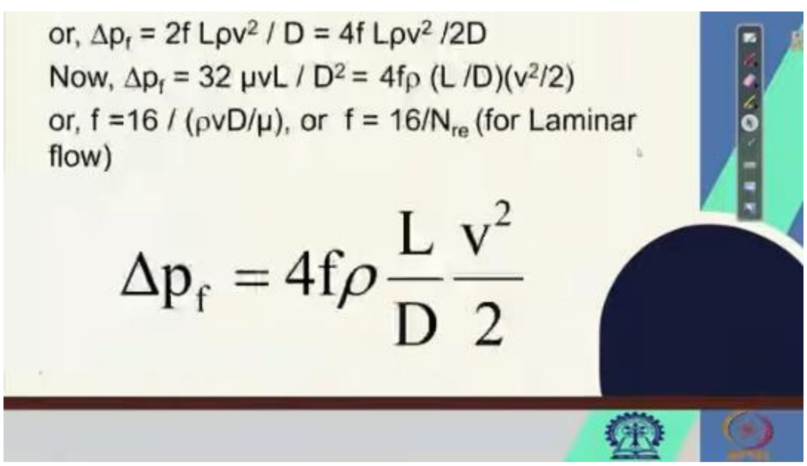
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture21

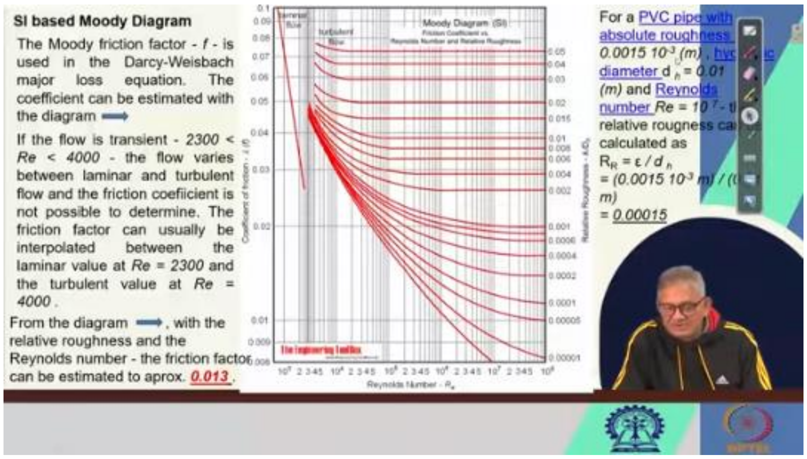
LECTURE 21 : LAMINAR FLOW THROUGH PARALLEL PLATES

Good evening, my dear students and my dear friends. We are in the class of fluid flow, that too in a flow through pipes, and yes, it is happening in a continuous class. The reason being that we have in between shown some new things like the Fanning friction factor, which we have shown. Another new thing we have shown is the pipe which we are taking that is not actually the length of the pipe.

It is somewhat more, which we have to take, and this is because of the development of a scientist called Moody, and he made Moody's diagram. And before that, we have done some problems, and we have shown how the calculations are being made, okay. Now, we go to that Moody's chart. Moody's chart is like this: this is one, right? It is a log-log graph where You see, for PVC pipe, absolute roughness, that is mu or rather epsilon, is 0.0015 × 10 to the power of minus 5 or minus 3 meters, right.



Similarly, some others with Reynolds number 10 to the power of 7, this thing all are given. And this Moody's chart, from there, we can find out the Fanning friction factor. You see, the curve is like that, where this is for turbulent flow, oh my goodness, this is for turbulent flow, this is for laminar flow, right, and it is plotted on log paper, that is, Reynolds number is also in the log, and friction factor is also in the log, and corresponding relative roughness, that is here, it is given k by d h, but we have taken that to be alpha by d or rather epsilon by d, right, and epsilon is the absolute roughness, okay. So, if we know that, then if the flow is transient, that is between 2300 to 4000, then



The flow varies between laminar and turbulent flow, and the frictional coefficient is not possible to determine. The friction factor can or usually be interpreted between the laminar value that is somewhere at less than 2300, and the turbulent value, which is somewhere at more than 4000 Reynolds number, right. Then, from the diagram with the relative roughness and the Reynolds number, the frictional factor can be estimated, right.

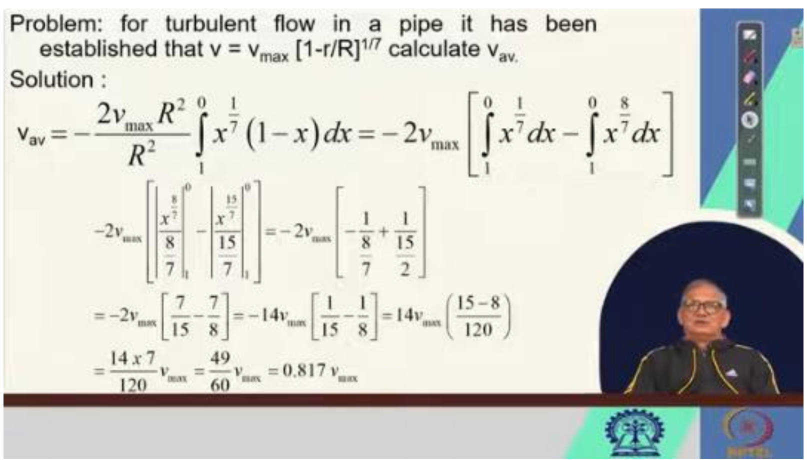
And it may vary like 0.013, 0.022, or whatever value it comes to, depending on the material you are using, like here we are seeing that for PVC, it is given something for some other also, right. This is one graph just to make you understand, just to show you how it looks like, right. And obviously, that relative roughness, which is epsilon by D, according to our notation and here it is obviously given a new notation, that is epsilon by d also d h, ok.

Whatever, d h could be the hydraulic diameter in many cases depending on. However, so that epsilon by d, if we know, we can find out the fanning friction factor, right from the Moody's diagram. One great thing out of this invention was that for calculation, yes, I don't say that whatever pipe you get, everything is very smooth and fine.

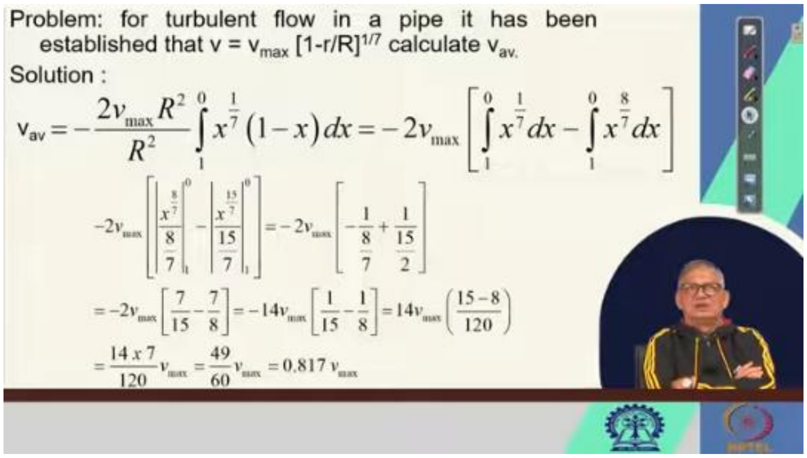
But for calculations, you have, for flow, city map, then state map, where you have to supply water all over. So, unless you are more or less correct about your pressure drop, you will not be able to fix the pump and other things. So, there, this relative roughness is very, very important, where the the actual length, if it is 100 meters, may be more than that, maybe 110, or if it is 1 kilometer, more than 1 kilometer is the length, right.

So, it all depends on the values, that is what is important, that the length calculation, pressure drop calculations, everywhere this epsilon by d is very important, and Moody's chart is so much important, right. So, from there, we go to a problem again, solving a problem. This is nothing but simple arithmetic, not arithmetic, simple mathematics rather. Right, it is said that for a turbulent flow in a pipe, it has been established that v is equal to vmax into 1 minus r smaller by capital R to the power of 1 by 7, then calculate the average velocity vaverage. Right, I repeat, for a turbulent flow,

in a pipe, it has been established that v is equal to v_{max} into 1 minus small r by capital R to the power of 1 by 7, then calculate the average velocity, right. From the definition of average velocity, we know that $v_{average}$ is 2 v_{ma}x into capital R square by r square and say x to the power of 1 by 7 into 1 minus x dx right. That is equal to between 1 to 0 x to the power of 1 by 7 dx minus 1 by 1 to 0 x to the power of 8 by 7 dx.



Now, what we have made x to the power of 1 by 7 or x rather, that we have made by substituting the value of 1 minus r by R, right? So, that is 1 minus r by R to the power of 1 by 7. And if you take that 1 minus r by R is equal to x and you see your limit that is 0 to r. That is changed to 1 to 0, right?



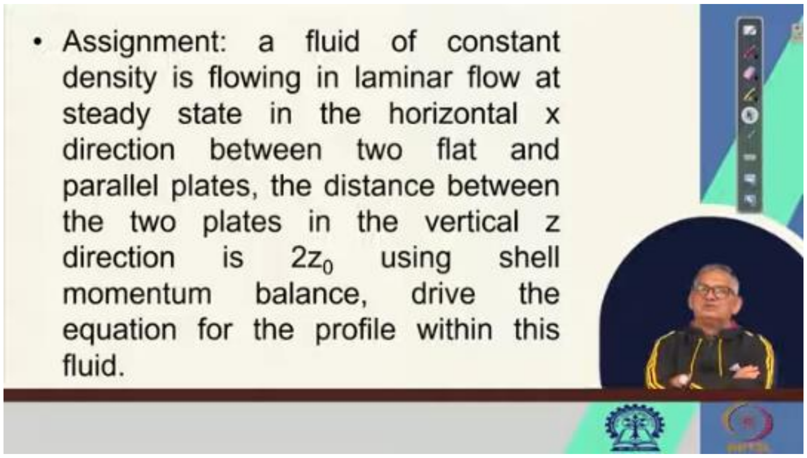
So, that you can find out very easily by substituting 1 minus r by R as dx or x rather, right. So, you can find out what the value of dx is and what the value of dr is accordingly. So, on integration, you get that equals to minus 2 v_{max} into rather it equals minus 2 v_{max} between 0 1 to 0 x to the power of 1 by 7 dx minus 1 to 0 x to the power of 8 by 7 that is 1 minus 1 by 7.

So, it is 8 by 7 x to the power of dx, right. So, that minus 2 v_{max} and this can be written equal to 2 v_{max} minus of course, x to the power of 8 by 7 by 8 by 7 between 1 to 0 minus x to the power of 15 by 7, right. Because, 8 by 7 plus 1. So, it will be 15 by 7 by 15 by 7 between 1 to 0. This equals minus 2 v_{max}.

If we simplify, it is minus 1 by 8 by 7 plus 1 by 15 by 2, right? So, that equals minus 2 v_{max} 7 by 15 minus 7 by 8, which equals 14 v_{max} into 1 by 15 minus 1 by 8. So, 1 by 15 minus 1 by 8 is 14 V_{max} 15 minus 8 by 120. That is 14 into 7 over 120 v_{max}. So, that is 49 by 60 v_{max}, which is 0.817 v_{max}, right.

We have noticed earlier, in earlier classes, somewhere we have said for a pipe flow that $v_{average}$ is v_{max} by 2, right. So, that is 0.5 v_{max}, but here, it is 0.817 v_{max} with the relation as it is given that v is v_{max} into 1 minus r by capital R to the power 1 by 7, right. So, this is nothing but just one problem to understand how the solutions are being made, okay. Then we come to this assignment.

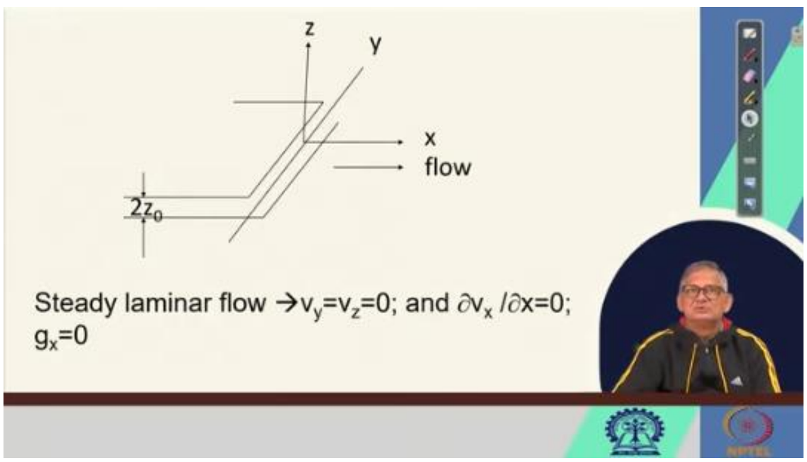
Again, this assignment, I do not expect that you will be sending me back. That is why I try to solve it. So, where it is a liquid or rather a fluid of constant density flowing in laminar flow at a steady state. In the horizontal x-direction between two flat and parallel plates, the distance between the two plates in the vertical z-direction is 2 z_0. Using shell momentum balance, not derived.



So, one E is missing. Derived the equation for the profile within this fluid where the fluid is of constant density and is flowing in laminar flow. At a steady state in the horizontal x direction between two flat and parallel plates. The distance between the two plates in the vertical z direction is 2 z_0. Using shell momentum balance, derive the equation for the profile within this fluid. The first thing is that we should draw what is said.

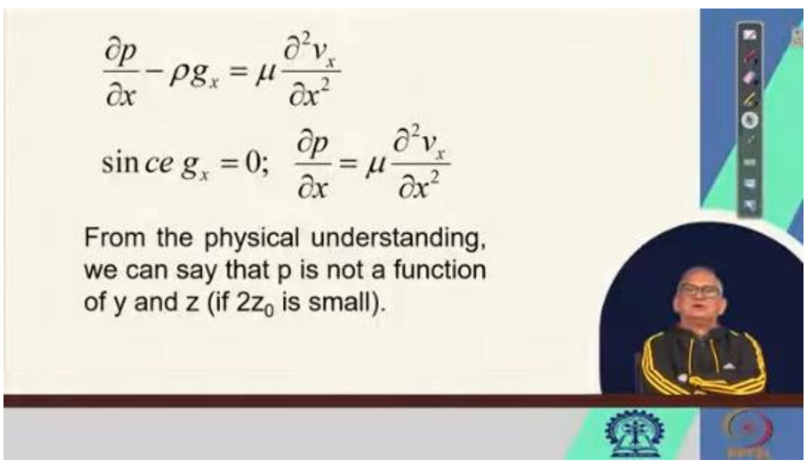
So, we are told that the density is constant, the flow is laminar, and the flow is steady and horizontal in the x direction, right between two flat and parallel plates. So, this is a flat and this is a flat parallel plate, right. This type of flow is very helpful for finding out the flow when you have two very, very narrow, narrow plates, like for homogenization, you use similar kinds of things.

So, there it may be very, very helpful, right. So, two plates horizontal in the vertical z direction at a distance of 2 z_0. So, you find out through shell momentum balance the equation for the profile, ok. So, we draw

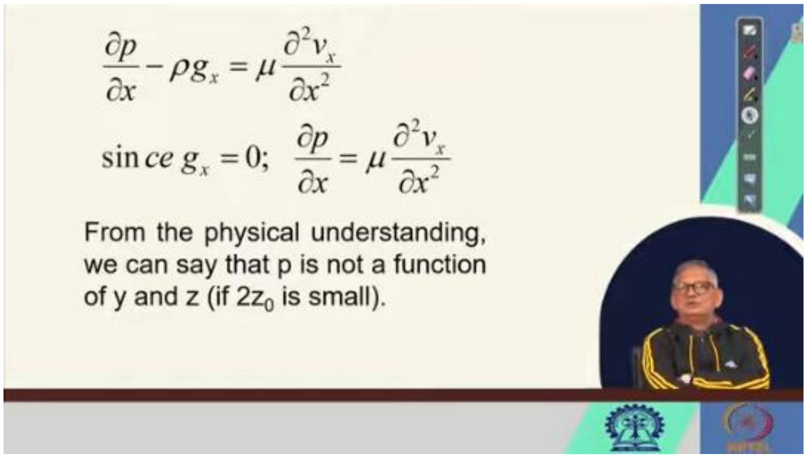


There are two directions, which are z and x. The flow is happening through the x direction, right? The gap between the two plates is 2 z_0, as it is said, right? The third dimension is y. Okay, then, at steady laminar flow, obviously, v_y and v_z are 0. v_x cannot be 0 because, if v_x is 0, then there will be no flow. So, v_y and v_z are 0, and del v_x / del x is also 0. Since it is horizontal, we can say that the value of g or g at x is also equal to 0, right?

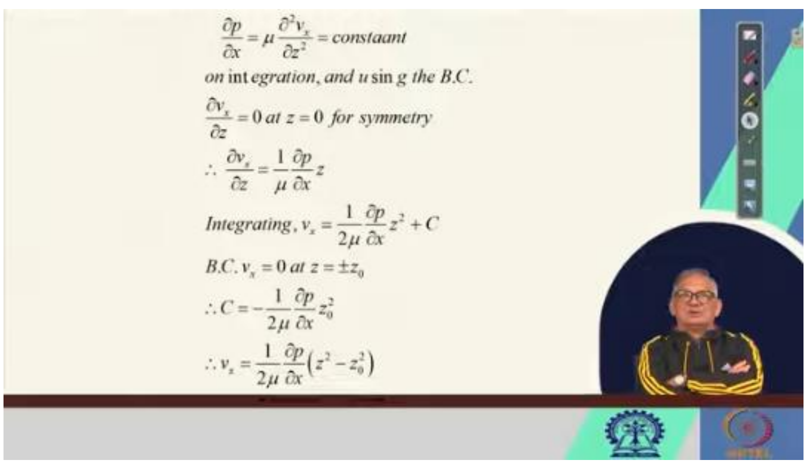
So, I repeat, from the given condition, we have drawn the two plates parallel, which are at a distance of 2 z_0. The flow is happening through the x direction, and since it is laminar steady flow, as it is said, then we can say that v_y and v_z are equal to 0, and also there is no flow, change of flow with respect to the direction, that is del v_x / del x, is also 0. And since, again, it is horizontal, we assume that g_x is equal to 0. Therefore, we can say that del P / del x minus rho g_x is equal to mu del 2 v_x / del x square, right? This we have also done earlier.

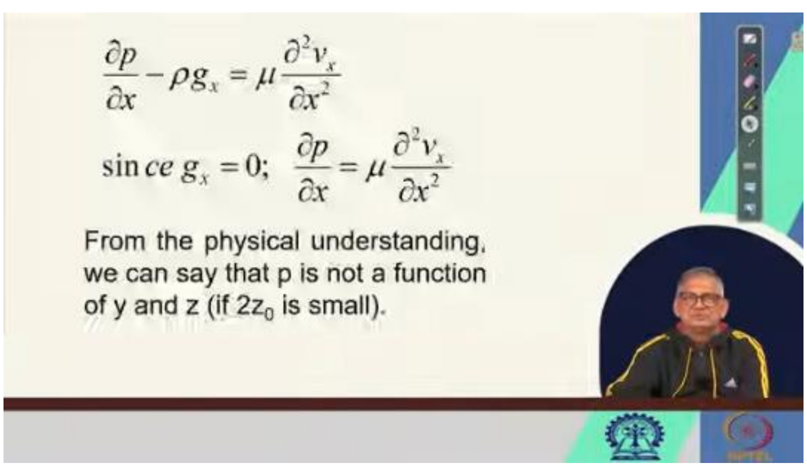


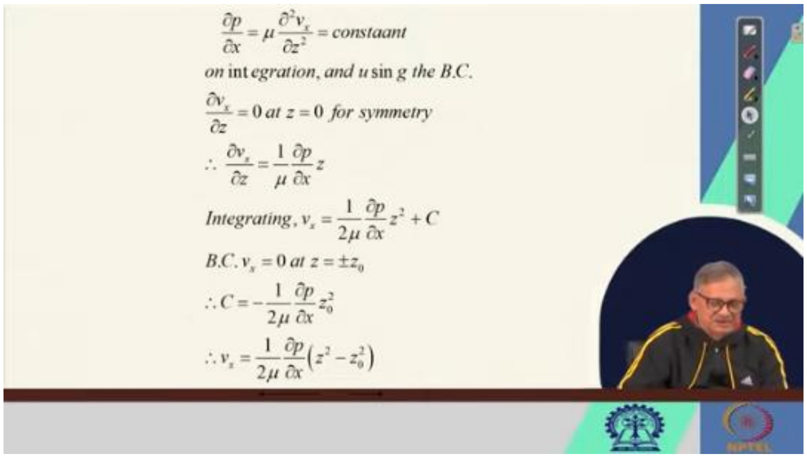
So, del P / del x minus rho g_x is equal to mu del 2 v_x / del x square, and since g_x is 0, del P / del x is mu del 2 v_x del x square, right? So, already, we have said from the physical understanding that we can say P is not a function of y or z. If 2 z is small, if 2 z is big, then it may not be, but if 2 z or 2 z_0 is small, then p is not a function of both y and z.



Now, del p / del x being equal to mu del 2 v_x / del x square and equals to constant on integration, we get On integration and using the boundary condition, then we get del v_x del x, del v_x / del z is equal to 0. You look at here, it was del v_x / del x is equal to 0, and now we are saying that del v_x / del z. This is the boundary condition. del v_x / del z is equal to 0 again at z is equal to 0, right? Because, it is symmetrical.

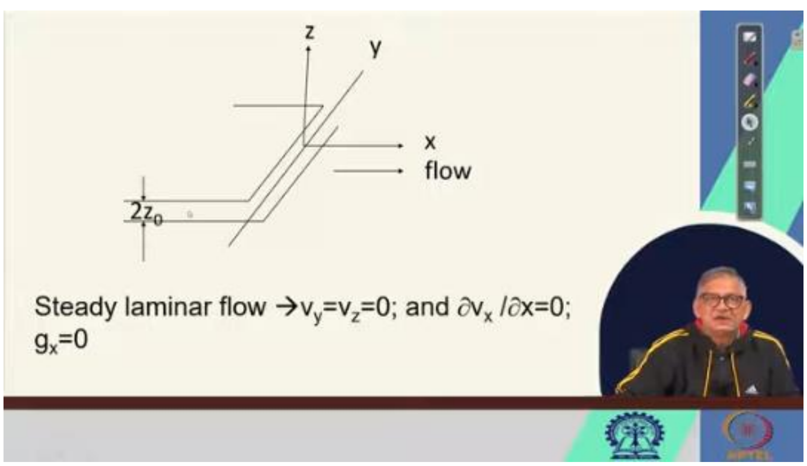




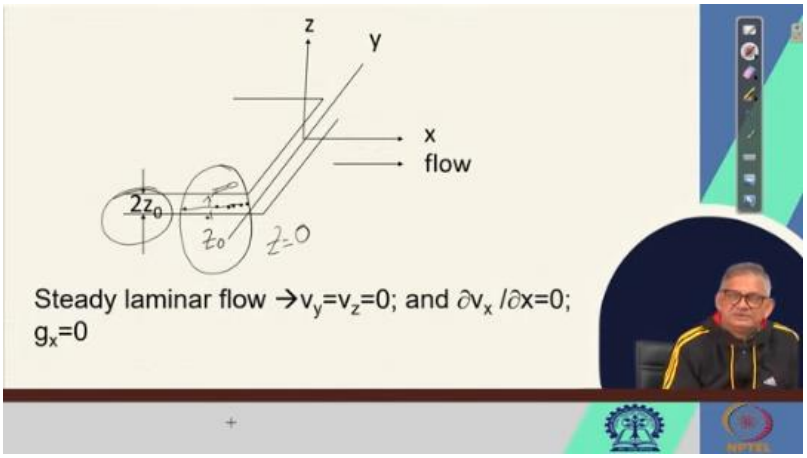


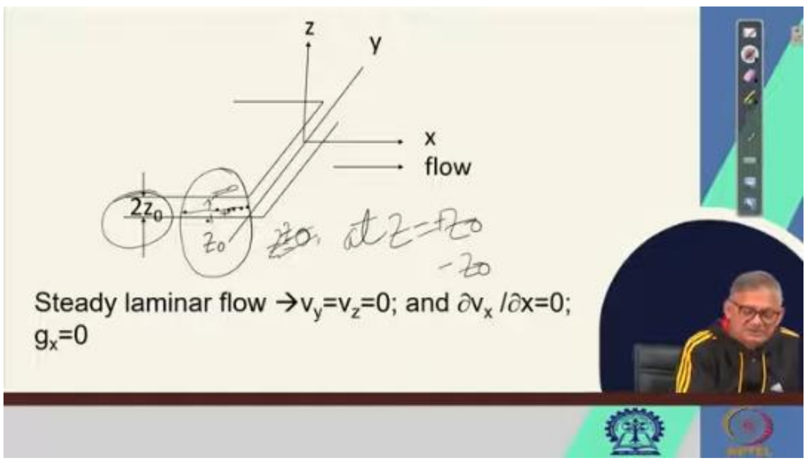
2 z_0 we said. So, one is z_0, the other is also z_0. So, being symmetrical, We can say that at z is equal to 0, we can say that del v_x / del z is equal to 0. So, we can write del v_x / del z is equal to 1 by mu del P / del x into z, right. So, on integration, we can say that v_x is equal to 1 by 2 mu del P / del x z square plus C, that is the integration constant, right. v_x is 1 by 2 mu del p / del x z square plus C, right. Now, what will be the boundary condition?

So, to find out the boundary condition, let us go to the drawing diagram. Here, we said it to be 0, 2 z_0, right? That means at the center, we have this side. Sorry, we have this side. Let us take this help. We have, if it is here, right, in the z_0, then this side is z0, and this side is also z_0, right. So, this makes this 2 z_0, ok, and since at z is equal to 0.



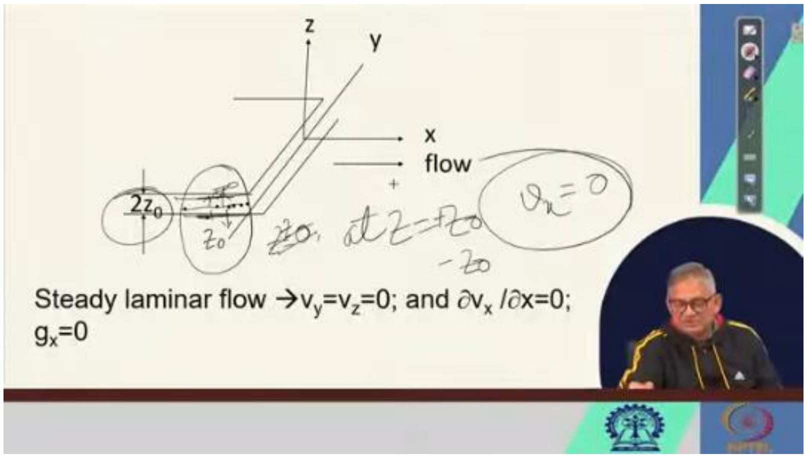
What is the boundary? At z is equal to 0? No, at z is equal to z_0, right? At z is equal to z_0, it could be plus z_0, or it could be minus z_0. If this is plus z_0, then this is minus z_0, right. So, at both z is equal to plus minus z_0 at the center, we can say v_x is equal to 0, because this is the inner surface. This is also the inner surface at z is equal to z_0 or z is equal to minus z_0.

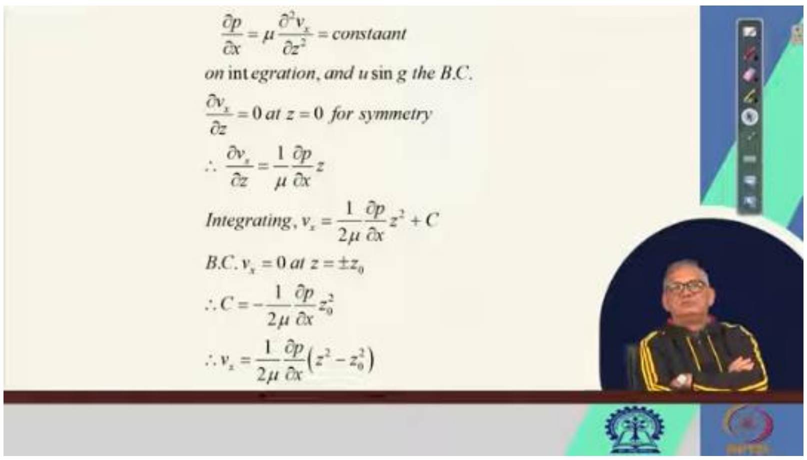




So, that liquid which is on that surface is clinging onto it. So, we cannot take it that the liquid is flowing at the surface. So, the surface velocity must be 0. So, v_x is equal to 0.

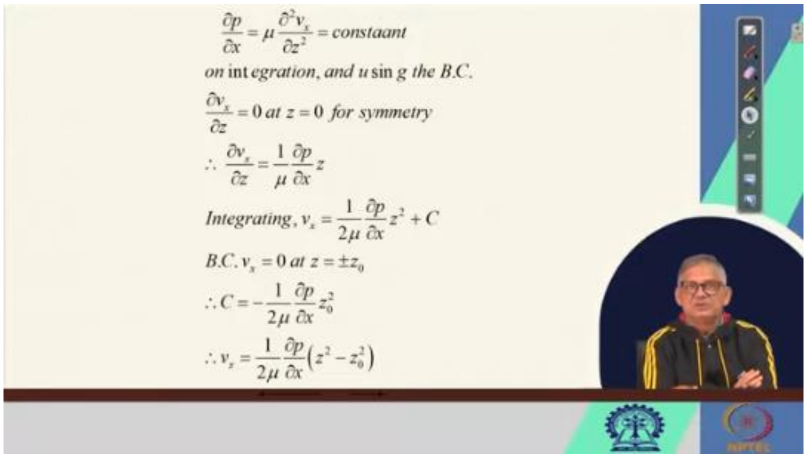
So, if that is true, let us remove this; otherwise, it will remain. If that is true, then we can definitely say that. We can definitely say that at plus minus z_0 is equal to plus minus z_0; that is the boundary. v_x is equal to 0, right? Therefore, by putting this value of v_x, we can find out the value of C. The value of C is minus 1 by 2 mu del P / del x z_0, right?



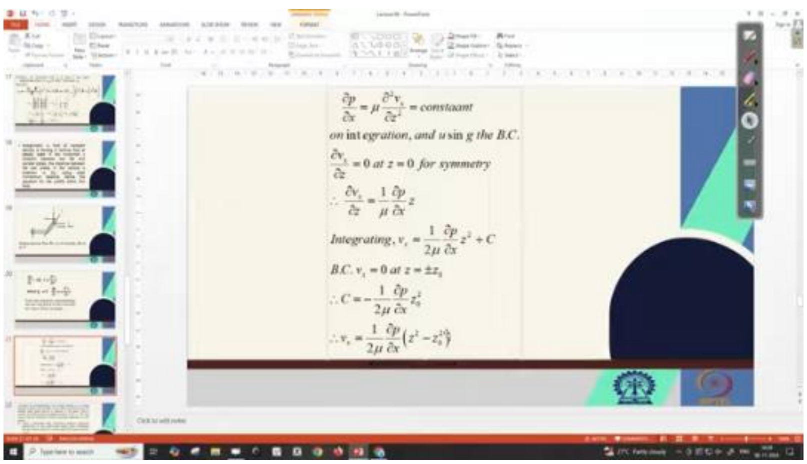


Therefore, v_x can be rewritten as. 1 by 2 mu del P del x into z square minus z_0 square, right? So, this is what we have found out when two flat parallel plates are present, a flow of fluid is happening through that, and we said the flow is laminar. We also said not only is the flow laminar, but we also said that the density is constant, right? And the gap between the two is 2 z_0, right? The gap between the two is 2 z_0.

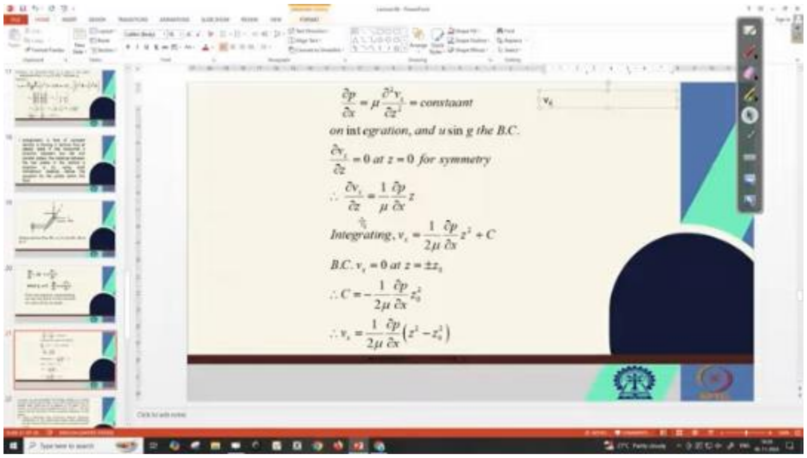
Therefore, we can say that you are asked to find out the velocity profile. So, the velocity profile is v_x is equal to 1 by 2 mu del P del x into z square minus z_0 square, right? Obviously, this z we said plus minus 2 z_0. One half is z_0, and another half is minus z_0, right?



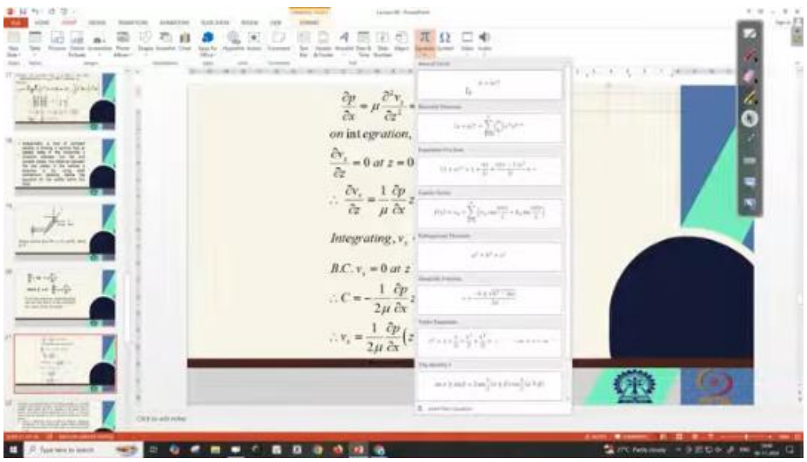
So, between 0 to z_0, that z is functioning, right? And del P is obviously the pressure in minus pressure out, right? P at in and P at out is the del P. And del x is the total length of the x that can be L, or any such, right. So, as you see, this is we can also make it parabolic in nature by simply writing it in a way like this. If we take it here, right.



So, we add a box there, we can write instead of going to some other place. So, we can say that v_x. So, v at x. So, this is equal to. It will not be unless we remove that, ok. So, this is v_x that we can write is equal to, is equal to 1 by 2 mu that we can write by taking, not shape, insert not symbol equation,

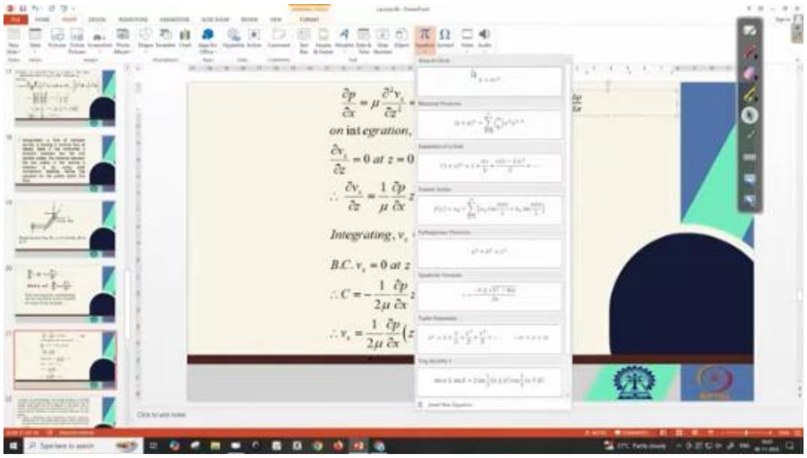


say this is a simple equation. So, v_x is equal to, we can write 1 by 2 mu. So, here we get 1 by 2 mu. So, 2 and mu we can get from here somewhere, 0 Hmm.

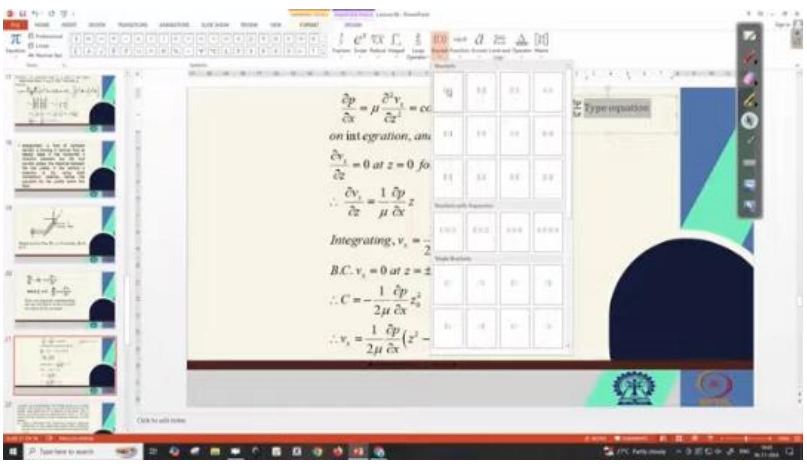


mu ok. So, 1 by 2 mu into del P del x, right into del P del x, that is not here, it should have been there, into del P del x. So, here we write delta p or del p, whatever del is coming here, delta p over delta x. So, delta is that right into we can write that z_0 minus delete ok into we write z_0 minus. So, where is it?

It is drawing tools, no or not format review review insert, yeah. So, here we are. So, into into into view format text file, no no. So, you are writing that. So, we get here delta x. Where did we go?

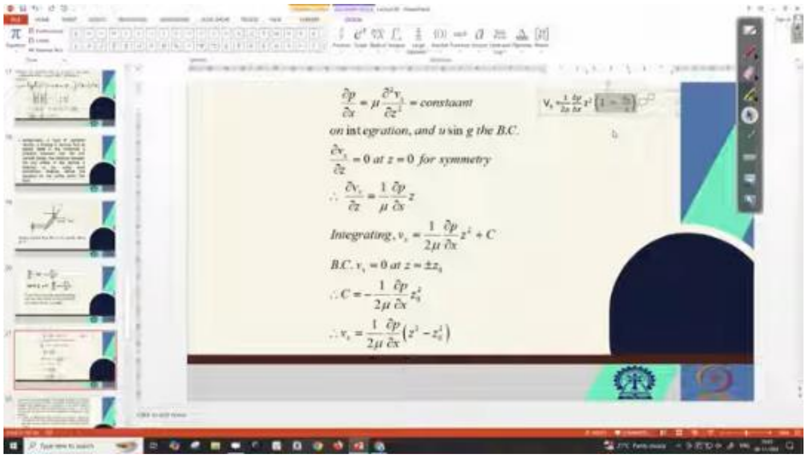


here ok. So, here into of course, here if we take z square, if we take z square, square means you have to do superscript 2 and again come back from the superscript. right, z square and here it is 1 minus. z_0 by z whole square, 1 minus z_0 by z square. So, come back to again insert, come back to again equation right.



So, you can write here z_0 y by this. So, it is z 0 by. Z, where is Z? Z_0, 0, you can do it from here; that is Z, and this is 0, Z_0 by Z, OK, and this is out. Square.

This square, I can do again; insert equation, that square we can do from here. Control X, OK. So, we rewrite here x, and here we make 2, OK. So, like this, we can make it. I am so sorry that I have taken some time, maybe extra, just to show you how we can do it online. Right.



So, we have made some additional points; it can also be written in the form of a parabolic form, right? So, we have done it in the form of a parabolic form, and with this, we have come to the end of the class, and we thank you for attending this class, right? I have also shown how online we can insert something which we are needing at that very moment, OK, because I wanted to show that it is also possible to make it parabolic, OK.

Thank you so much.