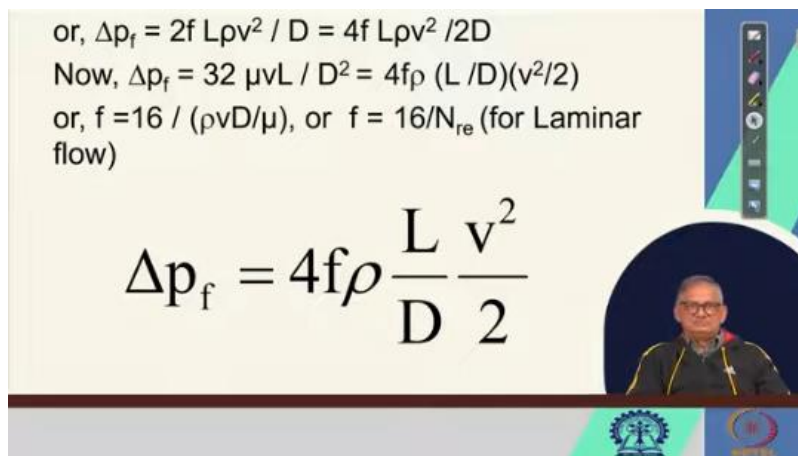


# IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture20

### LECTURE 20 : FRICTION FACTOR

Good evening, my dear students and friends. We are handling flow through pipes, and from there, we have come to the point that the flow through pipes also deals with a frictional term called the Fanning friction factor. Right. So, if we go to that Fanning friction factor, we have done in the previous class that  $\Delta P_f$  is equal to  $4 f \rho L v^2 / D$ . Right, and also we have said that the Fanning friction factor  $F$  is equal to  $16 / N_{Re}$ , where  $N_{Re}$  is  $\rho D v / \mu$  and is valid for laminar flow, right? Up to this, we have already said.



or,  $\Delta p_f = 2f L \rho v^2 / D = 4f L \rho v^2 / 2D$   
Now,  $\Delta p_f = 32 \mu v L / D^2 = 4f \rho (L / D)(v^2/2)$   
or,  $f = 16 / (\rho v D / \mu)$ , or  $f = 16 / N_{Re}$  (for Laminar flow)

$$\Delta p_f = 4f \rho \frac{L}{D} \frac{v^2}{2}$$

Now, next comes If we do some problems, then the solution of the problem becomes a very confidence-determining session, because if you can solve the problem, you can be assumed that you have the subject or the text which is being covered. Now, a problem is like this: a small capillary with an inside diameter of  $2.54 \times 10^{-3}$  meters and a length of 0.4 meters. is being used to measure the flow rate of a liquid having a density of 870 kg per meter cubed and viscosity  $\mu$  equal to  $1.15 \times 10^{-3}$  Pascal seconds, right.

Example : A small capillary with an inside diameter of  $2.54 \times 10^{-3}$  m and a length of 0.4 m is being used to measure the flow rate of a liquid having density of  $870 \text{ kg/m}^3$  and  $\mu = 1.15 \times 10^{-3} \text{ Pa.s}$ .


(1) Calculate the flow rate if pressure drop across the capillary is 0.07 m of water ( $990.24 \text{ kg/m}^3$  density).

(2) Calculate the pressure drop using fanning friction coefficient factor.

Solution: 1)  $\Delta p_f = h \rho g = 0.07 \times 990.24 \times 9.81$   
 $= 679.99 \text{ N/m}^2 \approx 680 \text{ N/m}^2$ .

also,  $\Delta p_f = 32 \mu v L / D^2 = 32 \times 1.15 \times 10^{-3} \times v \times 0.4 / (2.54 \times 10^{-3})^2$

or,  $v = (2.54 \times 10^{-3})^2 \times 680 / 0.01472$   
 $= 0.298 \text{ m/s}$



Then you are asked to calculate The flow rate if the pressure drop across the capillary is 0.07 meters of water, that is 990.24 kg per meter cubed density, and calculate the pressure drop using the Fanning friction coefficient or friction factor. So, these two we have to find out. So, for doing the problem, it is always easy to see that we are understanding the problem, for which we need to read the problem carefully. So, we read again that a small capillary with an inside diameter of 2.54 times 10 to the power of minus 3 meters and a length of 0.4 meters is being used to measure the flow rate of a liquid having a density of 870 kg per meter cubed and viscosity 1.15 times 10 to the power of minus 3 Pascal seconds.

Now, calculate the flow rate if the pressure drop across the capillary is 0.07 meters of water having a density of 990.24 kg per meter cubed. So, that is the density of water, and also calculate the pressure drop using the Fanning friction factor or Fanning friction coefficient factor. So, I hope you can easily determine it, right? But as a part of the course material, we need to solve it again here also. So, that after solving, you can have a check with your calculation. So, the first one is to calculate the flow rate if the pressure drop across the capillary is 0.07 meters of water having a density of 990 kg per meter cubed density.

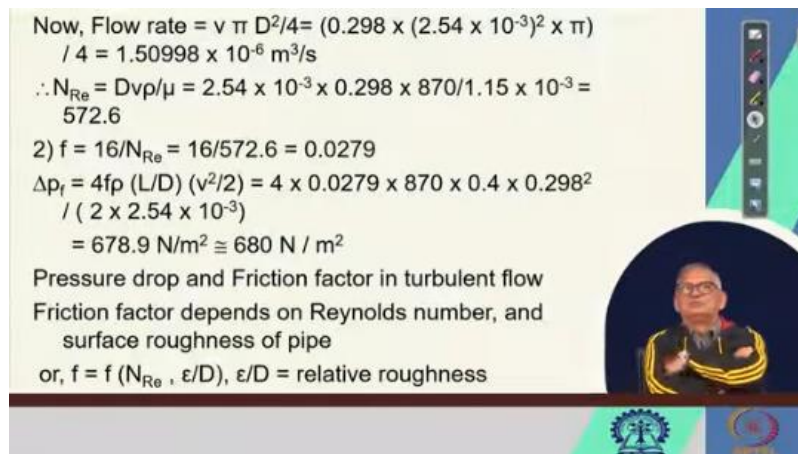
Therefore,  $\Delta P_f$  can be written as  $h \rho g$ , right?  $h$  is 0.07, capillary height 0.07 meters,  $\rho$  ( $\rho$ ) is the density of water, which is 990.24 kg per meter cubed, and  $g$  is obviously 9.81. So, if you multiply, you will see that it becomes 679.99 Newton per meter squared. So, this we can roughly make equal to 680 Newton per meter squared, right? So, this we calculated from the given value. And the given condition, because we have said to calculate the flow rate if the pressure drop

across the capillary is 0.7 meters of water having a density of 990.24 kg per meter cubed.

Density. So, we have found  $\Delta P_f$  to be 680 Newton per meter squared. Also, you can find  $\Delta P_f$  from the relation  $32 \mu v L$  by  $D$  squared, 32 is known,  $\mu$  is given as  $1.15 \times 10^{-3}$ .  $v$  is not known,  $1.15 \times 10^{-3}$  into  $v$  into  $L$ , given as 0.4, divided by  $D$ , which is also given as the inside diameter of  $2.54 \times 10^{-3}$ . So, into  $2.54 \times 10^{-3}$  squared, right?

So, from this, we know everything.  $\Delta P_f$  has already been found to be 680 Newton per meter squared. So, equating this with that,  $32 \mu v L$  by  $D$  squared, where  $v$  is unknown. So, you can write that  $v$  is equal to  $2.54 \times 10^{-3}$  whole squared, into 680 over 0.01472. So, that becomes equal to 0.298 meters per second.

You can also use your calculator and find out whether the values are correct or not. So, it is 0.298 meters per second as the velocity. Now, if the velocity is known, what we have been asked is to calculate the flow rate, right? So, the flow rate is then  $v$  into  $\pi D$  squared by 4,  $\pi D$  squared over 4 is the cross-sectional area. So, that means,



Now, Flow rate =  $v \pi D^2/4 = (0.298 \times (2.54 \times 10^{-3})^2 \times \pi) / 4 = 1.50998 \times 10^{-6} \text{ m}^3/\text{s}$

$\therefore N_{Re} = Dvp/\mu = 2.54 \times 10^{-3} \times 0.298 \times 870/1.15 \times 10^{-3} = 572.6$

2)  $f = 16/N_{Re} = 16/572.6 = 0.0279$

$\Delta p_f = 4fp (L/D) (v^2/2) = 4 \times 0.0279 \times 870 \times 0.4 \times 0.298^2 / (2 \times 2.54 \times 10^{-3})$

$= 678.9 \text{ N/m}^2 \cong 680 \text{ N/m}^2$

Pressure drop and Friction factor in turbulent flow  
Friction factor depends on Reynolds number, and surface roughness of pipe  
or,  $f = f(N_{Re}, \epsilon/D)$ ,  $\epsilon/D$  = relative roughness

$v$  is 0.298 into  $2.54 \times 10^{-3}$  whole square, that is so much  $D$  square into  $\pi$  by 4, this is equal to  $1.50998 \times 10^{-6}$  meter cube per second. You can do a cross check again with your calculator and see whether it is coming, the flow rate to be  $1.50998 \times 10^{-6}$  meter cube per second or not, hopefully it should come. Therefore, we can write that the Reynolds

number to be  $D v \rho$  by  $\mu$ ,  $r$  is equal to  $2.54 \times 10$  to the power minus 3 into  $0.298 \times 870$  over  $1.15 \times 10$  to the power minus 3 is equal to  $572.6 N_{Re}$ . So, the value of  $N_{Re}$  is  $576.2$ , then it comes what is the value of  $f$ ? So,  $f$ , we have seen earlier for laminar flow, we can write it is  $16$  by  $N_{Re}$ .

We have found out  $N_{Re}$  is  $572.6$ . So,  $16$  by  $572.6$  is  $0.0279$ . So, the value of the friction factor,  $f$  is  $0.0279$ . You can cross check that  $\Delta P_f$  which was  $4 f \rho L$  by  $D v$  square by  $2$ , that is,  $4$  into  $0.0279$  into  $\rho$  is  $870$ ,  $L$  by  $D$  is  $0.4$

and we know what is the value of  $v$ , that is,  $v$  is  $0.298$  square by  $D$  is  $2$  into  $2.54 \times 10$  to the power minus 3, right  $D$  is  $2 R$ , which comes equal to  $678.9$  Newton per meter square again. We can say it is the same as  $680$  Newton per meter square. So, the pressure drop and friction factor in turbulent flow. If we want to find out, we must have done the pressure drop and the friction factor for the laminar flow, which we have just now finished with a problem, but this was for laminar flow. Then comes if it is available for laminar flow, why cannot it be used for

turbulent flow? Right. So, for pressure drop and friction factor in turbulent flow, we say that the friction factor depends on the Reynolds number and the surface roughness of the pipe, which I said also earlier, if you remember that, if it is a pipe, if it is a pipe, this is made of some good or bad, I do not know, plastic, right. So, the pipe roughness may not be so, it is more or less smooth. Now, you think that this is made of mild steel.

So, its roughness could be much more than this. Here, if it is made of some other material, say copper, stainless steel, galvanized iron, or whatever different materials. This surface roughness will be different. I hope at home you have also seen that. There are certain thermostable or something like that, it is called some container, right, where down below there is a bulb that is nothing but a vacuum bulb.

And it is double-walled, right? And outside is so polished that you can see your face also. There, the surface roughness is also very, very poor, right? Very, very low, whereas. You have seen that people are making what we call pond-like, not pond, some circular things where the rings are joined, one over the other, and it becomes, say, a pipe.

Now, Flow rate =  $v \pi D^2/4 = (0.298 \times (2.54 \times 10^{-3})^2 \times \pi) / 4 = 1.50998 \times 10^{-6} \text{ m}^3/\text{s}$


$\therefore N_{Re} = Dvp/\mu = 2.54 \times 10^{-3} \times 0.298 \times 870 / 1.15 \times 10^{-3} = 572.6$

2)  $f = 16/N_{Re} = 16/572.6 = 0.0279$

$\Delta p_f = 4fp (L/D) (v^2/2) = 4 \times 0.0279 \times 870 \times 0.4 \times 0.298^2 / (2 \times 2.54 \times 10^{-3})$

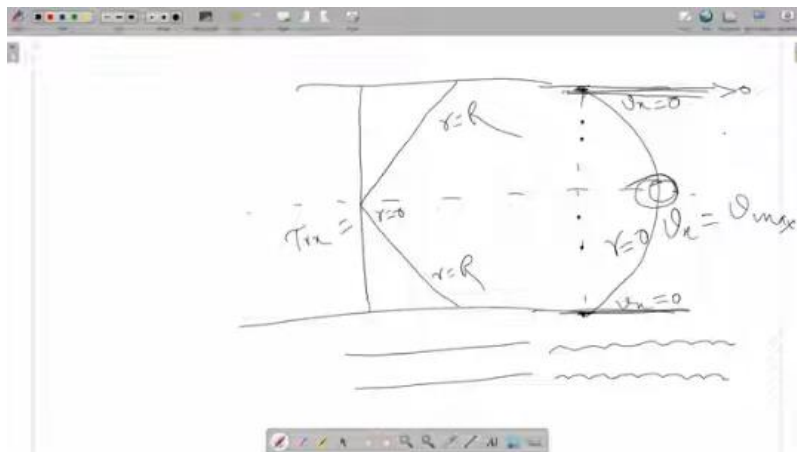
$= 678.9 \text{ N/m}^2 \cong 680 \text{ N/m}^2$

Pressure drop and Friction factor in turbulent flow  
Friction factor depends on Reynolds number, and surface roughness of pipe  
or,  $f = f(N_{Re}, \epsilon/D)$ ,  $\epsilon/D$  = relative roughness



So, there the flow or roughness is very, very high. So, like that, depending on the roughness of the pipe, the friction factor, when we consider it to be turbulent, then the drop of the friction factor is related to Reynolds number as well as the roughness of the pipe. Or surface roughness of the pipe. This was again highlighted by some very renowned scientist, right. So, we call him definitely a pathfinder.

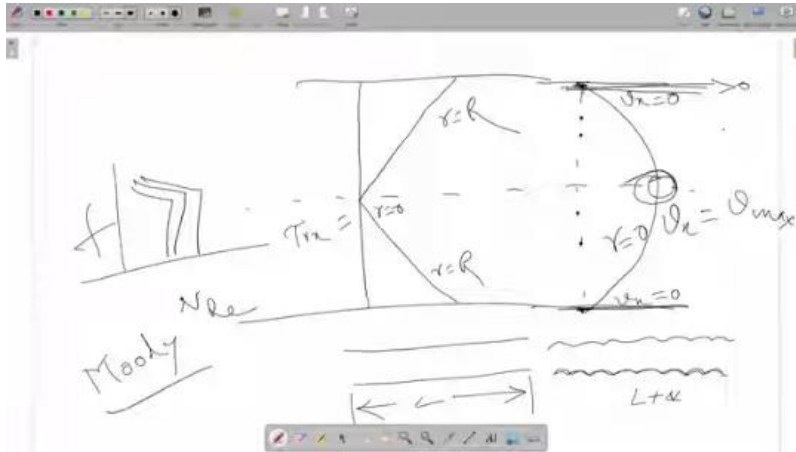
For the pipe flow because he said that if we are assuming that for a flow through pipe, if the length of pipe we consider this. The actual flow of the pipe may not be this much only. I show you here that the actual flow may not be this much only, because if this is a pipe, right, depending on the roughness of the pipe. Right? So, if this becomes L.



Then it cannot be that the movement of the fluid which has to move through this, through this, this, this, this, this, this, this same as L. This is something L plus delta

L. Right? This was first shown by the scientist called Moody, right? M-O-O-D-Y, and he made a chart. I will show you afterwards.

He made a chart of  $f$  versus  $N_{Re}$ . For different other parameters, right. So, that we will find out. So, what is said is, if we elongate it, then the length of the pipe could be this, which is actually  $L$  plus  $\Delta L$ , not  $L$ . Which depends on what is the value of  $N_{Re}$  and this is called relative roughness or epsilon ( $\epsilon$ ), right.



So, that will dictate what is the value of the frictional factor. So, depending on Reynolds number and surface roughness of the pipe, the frictional factor has been related by this Moody with the Reynolds number and relative roughness, that is epsilon by  $D$ , right. So, epsilon by  $D$  versus, rather,  $N_{Re}$  versus  $f$  for different relative roughness epsilon by  $D$ , that scientist Moody plotted, right. From there, if we know the epsilon by  $D$ , and  $N_{Re}$ , you can find out the value of the  $f$  for a given construction of the pipe, right, pipe material.

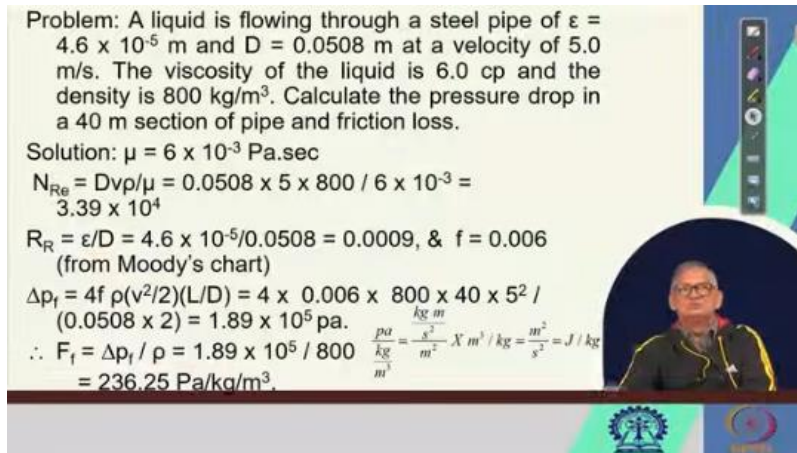
So, if we do again a problem quickly, then we may come to a solution. The problem is like this. A liquid is flowing through a steel pipe of epsilon equals to 4.6 into the power minus 5 meter, and  $D$  is 0.50508 meter at a velocity of 5 meter per second. The viscosity of the liquid is 6 centipoise, and the density of the liquid is 800 kg per meter cube.



Problem: A liquid is flowing through a steel pipe of  $\epsilon = 4.6 \times 10^{-5} \text{ m}$  and  $D = 0.0508 \text{ m}$  at a velocity of  $5.0 \text{ m/s}$ . The viscosity of the liquid is  $6.0 \text{ cp}$  and the density is  $800 \text{ kg/m}^3$ . Calculate the pressure drop in a  $40 \text{ m}$  section of pipe and friction loss.

Solution:  $\mu = 6 \times 10^{-3} \text{ Pa}\cdot\text{sec}$   
 $N_{Re} = Dv\rho/\mu = 0.0508 \times 5 \times 800 / 6 \times 10^{-3} = 3.39 \times 10^4$   
 $R_R = \epsilon/D = 4.6 \times 10^{-5}/0.0508 = 0.0009$ , &  $f = 0.006$   
 (from Moody's chart)  
 $\Delta p_f = 4f \rho (v^2/2)(L/D) = 4 \times 0.006 \times 800 \times 40 \times 5^2 / (0.0508 \times 2) = 1.89 \times 10^5 \text{ pa}$   
 $\therefore F_f = \Delta p_f / \rho = 1.89 \times 10^5 / 800 = 236.25 \text{ Pa/kg/m}^3$

$\frac{\text{Pa}}{\text{kg}} = \frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^3}{\text{kg}} = \frac{\text{m}^2}{\text{s}^2} = \text{J/kg}$



Then, calculate the pressure drop in a 40-meter section of pipe and the frictional loss. I repeat, a liquid is flowing through a steel pipe. Again, here you see the pipe has been said to be made of steel, right. So, it has an epsilon, which is 4.6 into 10 to the power of minus 5 meters, and D is 0.0508 meters. at a velocity of 5 meters per second.

The viscosity of the liquid is 6 centipoise, and the density is 800 kg per meter cubed. So, we are asked to calculate the pressure drop in a 40-meter section of pipe and the frictional loss. So, we are asked to find out the pressure drop and the frictional loss, that is, the friction factor. So, we are given,  $N_{Re}$  is  $D V \rho$  by  $\mu$ , that is, 0.0508 into 5 into 800 into 6 into 10 to the power of minus 3, that is equal to 3.39 into 10 to the power of 4, right. And the relative roughness, epsilon by D, is 4.6 into 10 to the power of minus 5 over 0.0508

that is equal to 0.0009. And from Moody's chart, if you see that Reynolds number versus friction factor, for a given epsilon by D, again for a given container or made of what material, the values are calculated and found out. So, f became equal to 0.006, as we could find out from the chart. Therefore, we can say that  $\Delta P_f$  is  $4 f \rho v$  squared by 2 into L by d. that is equal to 4 into 0.006 into 800 into 40 into 5 squared by 0.0508 into 2, that is 1.89 into 10 to the power of 5 Pascal, 1.89 into 10 to the power of 5 Pascal.

Now, we are asked to find out the frictional loss, that is, the frictional head. So, we can say that  $F$ , capital  $F_f$ , that is the frictional head, is nothing but  $\Delta P_f$  over  $\rho$ . So,  $\Delta P_f$ , we have found out to be 1.89 into 10 to the power of 5 and  $\rho$  we have seen to be 800 kg per meter cubed. So, this becomes equal to 236.25 Pascal per kg per meter cubed, right.

So, normally head is expressed in terms of joules. So, here we obtained a value of 236.25 Pascal per kg per meter cube, then we have to convert it into joules per kg, right? So, Pascal per meter cube per kg is that joule per kg. So, we make this conversion that Pascal per kg per meter cube is nothing, but Pascal is kg meter per second square divided by meter square into meter cube divided by kg.

So, it becomes that kg and this kg goes out. remains meter square per second square because from meter square meter cube that goes out, but on the top there is 1 meter. So, it becomes meter square and the denominator remains second square. So, meter square per second square is nothing, but joules per kg. That means, we have found

the value of frictional head as 236.25 Pascal per kg per meter cube. So, that is nothing, but 236.25 joules per kg. the unit by which the frictional loss or frictional head is found out, right? So, with this, we have come to the end of this class because time is over. And we will continue the flow through the pipes in the next class also because we have shown, we have obviously not shown till now the.

Moody's chart, right? That is very, very helpful as you have seen that the length which you consider may not be the same as what it is, right? So, thank you for listening to the class.

Thank you.