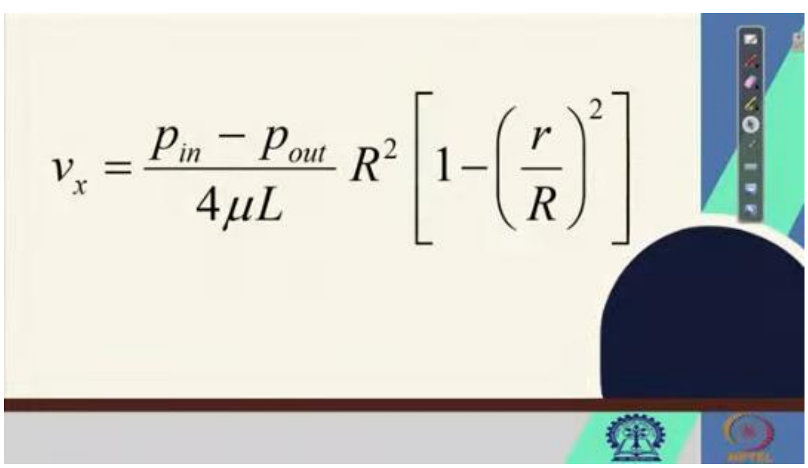
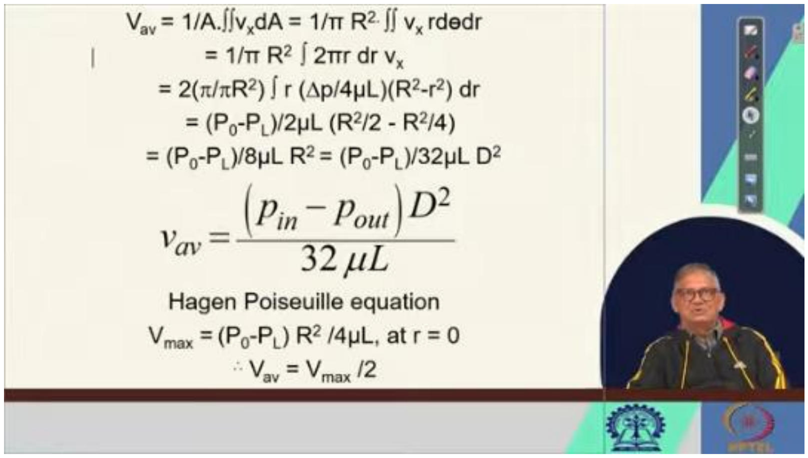
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture19

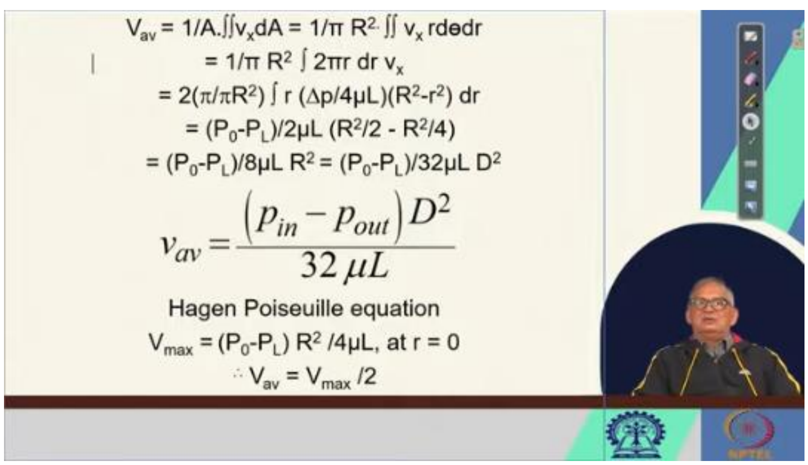
LECTURE 19 : NAVIER STOKES EQUATION USING BOUNDARY CONDITIONS

Good morning, my dear friends and students. We are dealing with flow through pipes, right? We have found out the instantaneous velocity in the pipe, v_x. We have found out v_x we have found out; it is (P_{in} minus P_{out}) over 4 μ L into R square into (1 minus r by R square), which is parabolic in nature. We have also found out the average velocity.

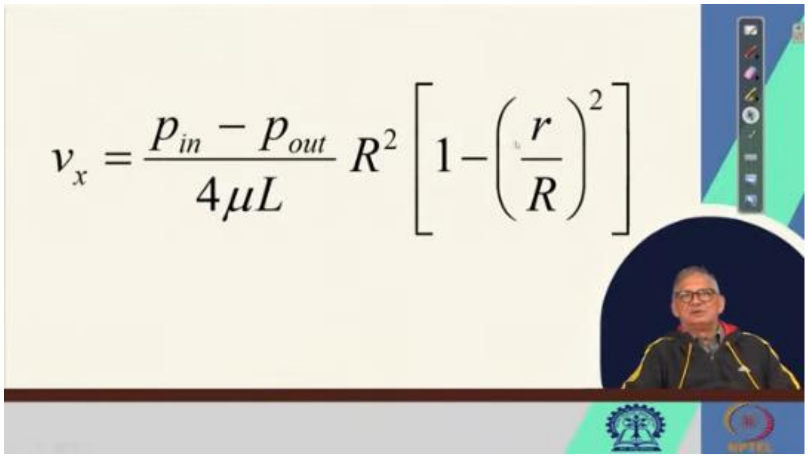




The average velocity is (P_{in} minus P_{out}) into capital D square, that is diameter over 32 μ L, and this equation is known as the Hagen-Poiseuille equation, which is valid for pipe flow, in laminar flow, where the flow is fully developed, there is no end effect, and it is under steady state as well. The fluid is incompressible and the fluid is also having constant density, right? Incompressible and, yeah, incompressible means constant density. So, under this situation, the second Poiseuille equation is valid, right? And we have found out the maximum velocity, which is equal to P_{in} minus P_{out}. It is not P_0 minus P_L, okay? P_0 minus P_L means

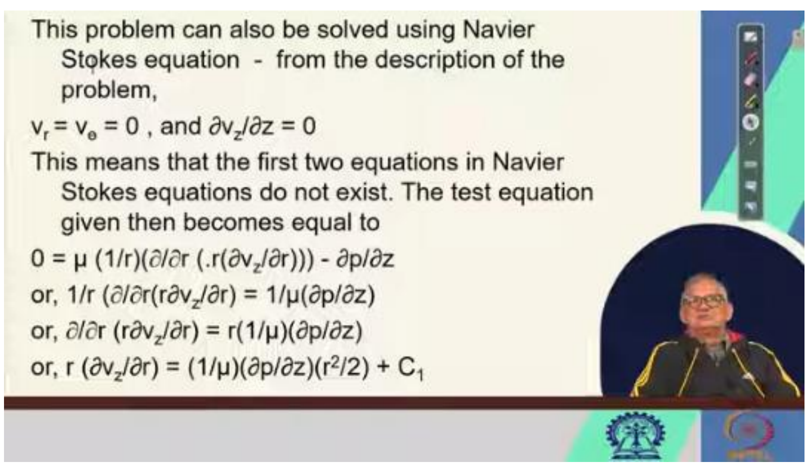


inlet to outlet at x is equal to 0, P_0; x is equal to L, P_L. So, inlet minus outlet into R square by 4 μ L is the v_{max}, and which is valid at r is equal to 0, right. Which is valid at r is equal to 0. At r is equal to 0, this becomes 0, this is 1, and this is P_{in} minus P_{out}. So, if you make r as D, then it becomes D square by 4.



So, it becomes 16 μ L; otherwise, r square by 4 μ L, right? That is the v_{max}. R square by 4 μ L, at r is equal to 0, and $v_{average}$, what we have seen is 32 μ L D square (P_{in} minus P_{out}). So, the relation between average velocity and maximum velocity is like that: average velocity is half of the maximum velocity, v_{max} by 2. Okay. This is what we have till now established.

This can also be solved using the Navier-Stokes equation. I told you, Navier-Stokes has immense, it has immense applications. So, we can say that from the description of the problem, we said v_r is equal to vtheta is equal to 0. So, there is no velocity in the r direction and in the theta direction.

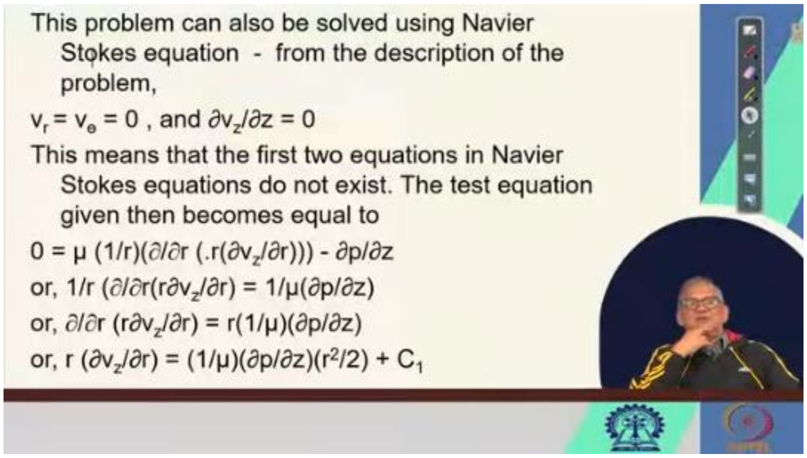


So, these two are 0, and del v_z del z, that is the third dimension, is also 0 because there is no velocity profile along the depth. So, this means that the first two equations in the Navier-Stokes equations do not exist because v_r, v_{theta} both being 0 and del v_z del z being 0, the first two equations in the Navier-Stokes equations do not exist. The test equation given then becomes equal to 0 is equal to mu into

1 by r del del r of r del v_z del r minus del p del z, right. Here, the only thing which we have taken is del v_z del z is equivalent to del v_x del x,

right, del v_z del x is equal to del v_x del x, that is, whatever velocity was at the inlet, the same velocity is at the exit, right. There is no velocity profile along the x axis, right, there is no velocity profile along the x axis, or no, I should not say velocity profile, I should say that there is no change of velocity with the distance in the direction of flow. So, this means that the first two, ok, and then we come to 0 is equal to mu 1 by r del del r of r del v_z del r minus del p del z, this is the third one, ok. One was for r, one was for theta, and the third one is for z, or x, whatever we call it.

So, we can rewrite 1 by r del del r of r del v_z del r that is equal to 1 by mu del P del z or del del r of r del v_z del r is equal to r 1 by mu del P del z. On rearrangement, we can write r into del v_z del r is equal to 1 by mu del p del z into r square by 2 plus C_1, that is the first integration constant, right. Again, we can rewrite as del v_z del r equal to 1 by mu del p del z into r by 2 plus C_1 by r, this is our rearrangement. Now, again, at r is equal to 0, del v_z del r is also 0, right. If we consider this instead of x, as v_z as z, then at r is equal to 0.

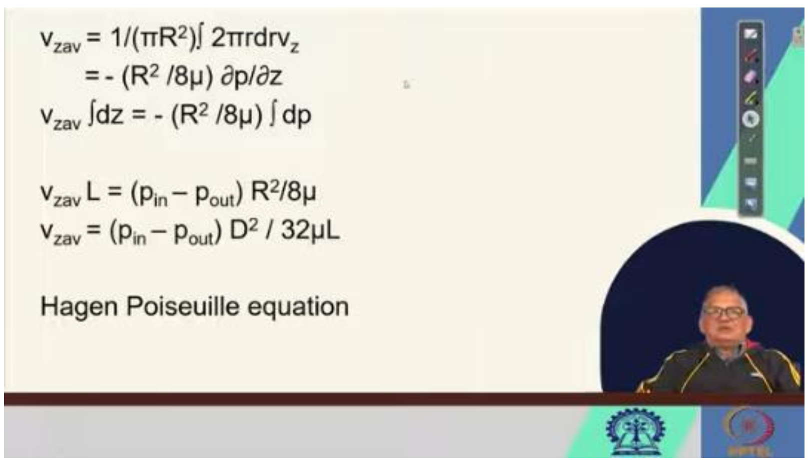


del vz del r or del v_x del r, whatever we call it, is also 0. Therefore, C_1 is equal to 0. Therefore, we can say that del v_z del r is equal to 1 by mu del p del z into r by 2. So, on integration, it gives v_z is equal to 1 by mu del p del z into r square by 4 plus C_2.

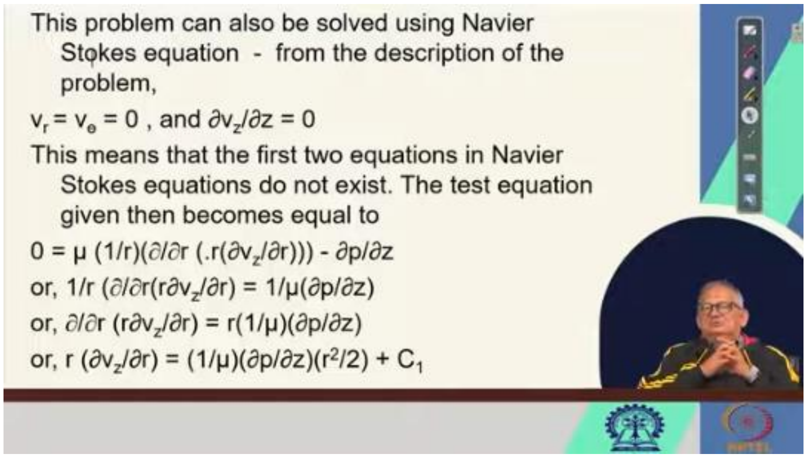
Again, we need another constant, another boundary to find out another constant C_2. Since r is capital R, at r is equal to capital R, v_z is also 0. Therefore, C_2 is equal to minus 1 by mu del p del z into R square by 4. Therefore, v_z, on rearrangement, we can write, 1 by 4 mu del p del z r square minus R square, right?

So, at r is equal to 0, now v_z is minus 1 by 4 mu del P del z into R square. Therefore, v_z is P_{in} minus P_{out} into R square by 4 mu L is equal to v_{max}, right? So, v_z is vmax into 1 minus r by R whole square, right. This is what we find out from the basic equation that is our

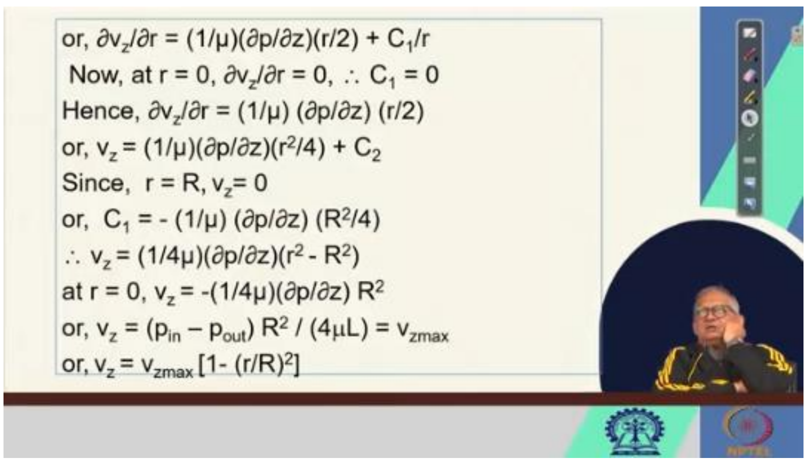
Navier-Stokes equation, right. So, we can write that vz average is 1 by pi R square into integration of 2 pi r dr v_z. So, that is equal to minus R square by 8 mu into del p del z. So, v_z average times integration of dz. is minus R square by 8 mu into delta P. Now, vzaverage into L is P_{in} minus P_{out} into R square by 8 mu. Therefore, $v_{zaverage}$ is delta P or P_{in} minus P_{out} into D square by 32 mu L, which is nothing but the Hagen-Poiseuille equation.



So, we have also found out the Hagen-Poiseuille equation from the Navier-Stokes equation. Mind it? What we needed is that from the Navier-Stokes equation, the boundaries, whatever we know, we have put that: vr and vtheta are 0. And either del v_x del x or del v_z del z, whatever you write, is equal to 0, right? So, from the Navier-Stokes first two equations, that is r and theta, there is nothing which we can take.

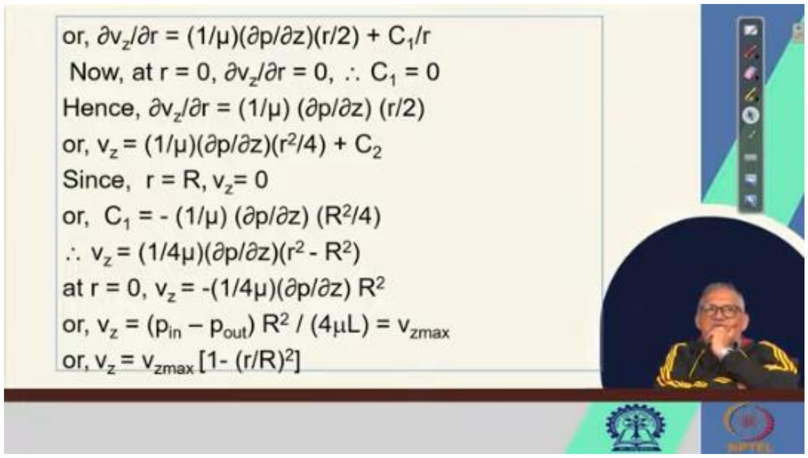


And the third equation, that is there, we have given in terms of z. So, that is why it is also in terms of z, but it can also be in terms of x because all other things are the same, only instead of z it is x. So, 0 is mu times 1 by r del del r of r del vz del r. Into r del v_z del r minus del p del z, that is from the Navier-Stokes equation we get. And on simplification, we write 1 by r del del r of r del v_z del r is equal to 1 by mu del p del z, right? Therefore, we can say that del del r of r del v_z del r is equal to r into 1 by mu del p del z or r del v_z del r is equal to 1 by mu del p del z r square by 2 plus C_1 and by putting a boundary del v_z del r, we can write 1 by mu del p del z r by 2 plus C_1 by r, rearranging. And putting the boundary, we can write that at r is equal to 0, del v_z del r is 0, therefore, C_1 is 0.

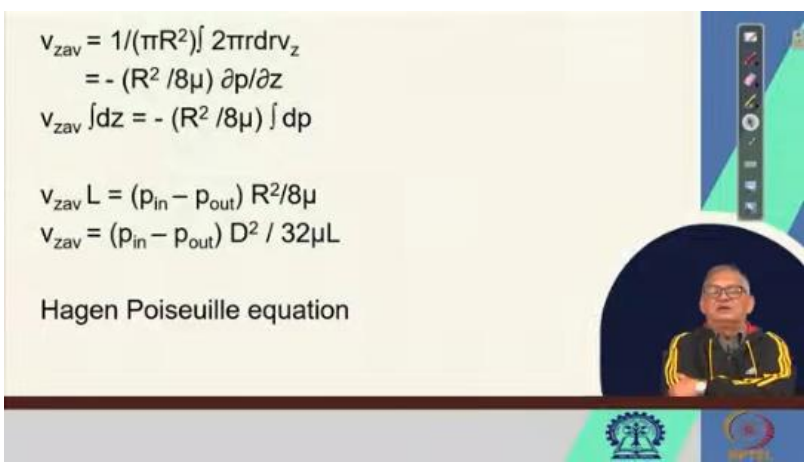


Hence, del vz del r is 1 by mu del p del z into r by 2. So, we can integrate, vz is 1 by mu del p del z r square by 4 plus C_2, here we write v_z is 0 at r is equal to capital R. Therefore, C1 becomes minus 1 by mu del p del z r square by 4 and vz becomes 1 by 4 mu del p del z small r square minus capital R square. At r is equal to 0, v_z is equal to minus 1 by 4 mu del P del z into capital R square or v_z is equal to P_{in}

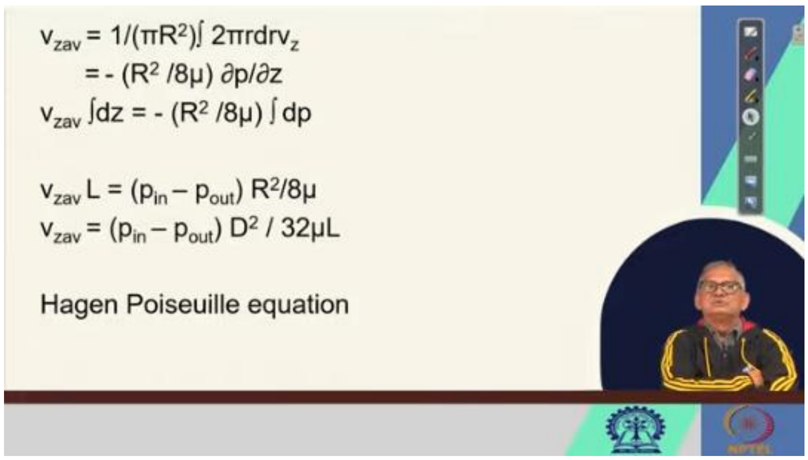
minus P $_{out}$ into R square by 4 mu L, that is equal to v_{max} or v_z is equal to v_{max} into 1 minus small r by capital R square. Therefore, we can write $v_{zaverage}$ is 1 by pi r square.

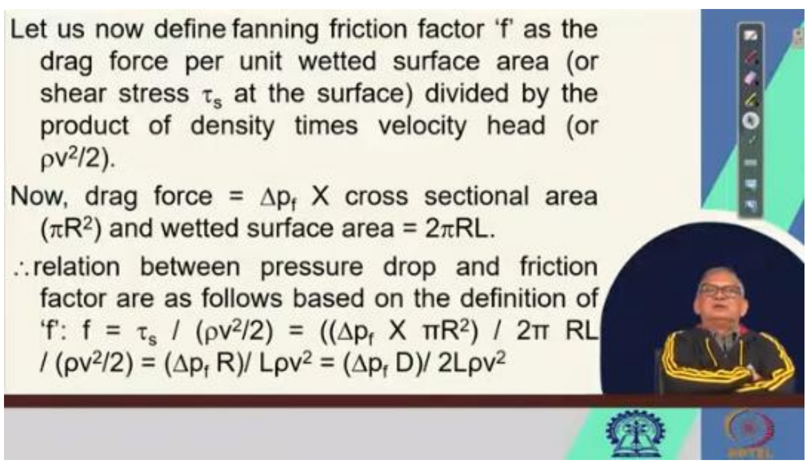


That is, average velocity into integration of 2 pi r dr v_z and that becomes equal to minus r square by 8 mu into del p del z. Therefore, v_z into dz integration is equal to minus R square by 8 mu into dp, integration of dp. So, $v_{zaverage}$ dz on integration between 0 to L is L and dp on integration between pin and pout. It is P_{in} minus P_{out}, because that minus is taken care of into R square by 8 mu. So, vzaverage is P_{in} minus P_{out} into D square by 32 mu L, because. D is equal to 2 r or r is equal to D by 2, right?



So, if we substitute, then it becomes 32 mu L, right, and this curve on the numerator is called the Hagen-Poiseuille equation. Let us now define another parameter, another term called the Fanning friction factor, or small f. The Fanning friction factor is known as small f, very popular in the flow of fluid through pipes and others, right. This is what it is again. I take this help that if it is a pipe, if a liquid is flowing from this side to this side, from a source having some pressure drop. Therefore, depending on the composition or constituents of this container, what is it made of? If it is made of plastic, that will have one type; if it is made of stainless steel, that will have another; if it is made of copper, that will have another; if it is made of some rough solid, then it will have another frictional loss, right.





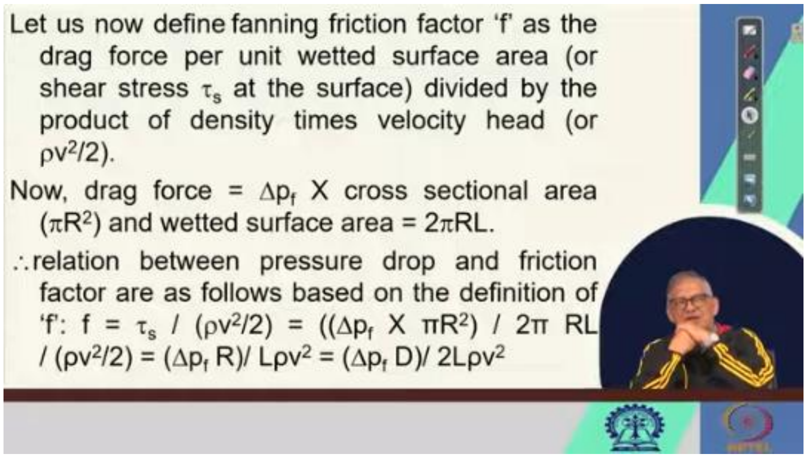
So, this delta P will go on increasing. If that thing happens, and this was established by Fanning, the resistance suffered is known as the Fanning friction factor, right. So, the Fanning friction factor 'f' is denoted as f and can be defined as the drag force per unit wetted surface area, which can also be said equal to shear stress τ s at the surface divided by the product of the density times the velocity head.

So, density times velocity head is known as rho v squared by 2, right. So, I define again the Fanning friction factor. I just explained with the example what it means.

It is defined as the drag force per unit wetted surface area or shear stress τ_s at the surface divided by the product of density times velocity, or it should be said product of density and velocity head, or it is rho v squared by 2, that is the product of density and velocity, and rho v squared by 2, right. So, then let us look into what the drag force is.

So, the drag force is nothing but delta P_f, where f is denoted for the friction factor. Delta P_{out} of friction times the cross-sectional area, that is pi capital R squared And the wetted surface area is 2 pi capital R capital L, right. So, whatever the pipe radius and whatever the pipe length, that is the wetted area. So, 2 pi R L is the wetted area.

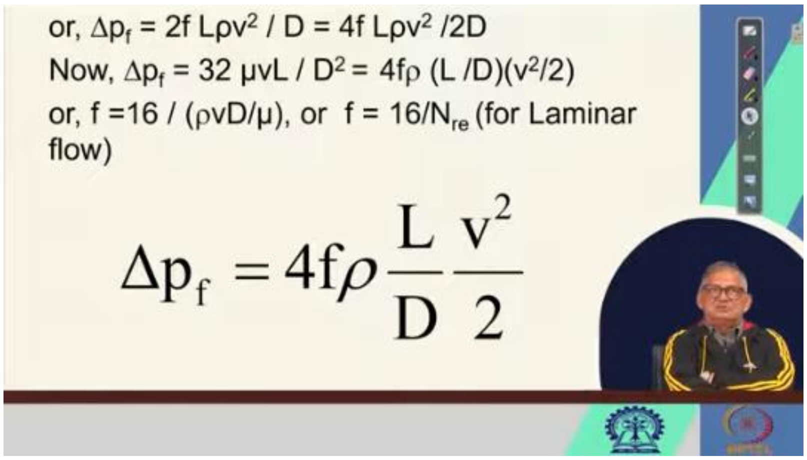
And drag force that is delta P_x or delta P times the sectional area or CS, cross-sectional area that is pi R square and weighted area surface area is 2 pi R L, right. So, from the definition, we can write, we can write the relation between pressure drop and friction factor, relation between pressure drop and friction factor, that is delta Pf and friction factor f, right, that can be related as based on the definition of the Fanning friction factor f. So, f we have defined it to be τ_s divided by rho v square by 2 as it is defined, right, shear stress at the surface.



So, τ_s, that is shear stress at the surface, divided by rho v square by 2. So, this is equal to that, by the definition we have said the shear stress at the surface, that is, τ_s, f is τ_s divided by the product of velocity head and density, that is rho v square by 2. So, this is equal to delta P_f into pi R square over 2 pi R L divided by rho v square by 2, right.

How do you add this? That drag force, we have said, delta P_f into sectional area, cross-sectional area, right, and we have also said that weighted surface area that is 2 pi R L, right. So, f is τ_s over rho v square by 2 and τ_s is delta P_f into pi R square by 2 pi R L, right.

So, this divided by rho v square by 2 is the Fanning friction factor f, which on simplification can be written as delta Pf into R over L into rho v square is equal to delta P_f into capital D over 2 L rho v square, right. So, this on simplification can be written that delta P_f is equal to 2 f L rho v square, right, or it can be rewritten as 4 f L rho v square by 2 D, that in the numerator, we have introduced one 2 and that is why denominator another 2, right. So, delta P_f we can write to be



32 mu v L by D square, right, which is nothing but 4 f rho L by D into v square by 2, 4 f rho L by D into v square by 2, right. From these two relations, we can find out the Fanning friction factor f is equal to rho v D by mu or f can be written as 16 by N_{Re} for laminar flow. So, we can say again, sorry, that f is 16 by N_{Re}, right, which is valid for laminar flow. So, we can write the delta P_f equals to 4 f rho L by D into v square by 2, right?

And f is 16 by N_{Re} for laminar flow. Our time is up. So, thank you for listening. We will add on in the next class.

Thank you.

or, $\Delta p_f = 2f\,L\rho v^2 / D = 4f\,L\rho v^2 / 2D$

Now, $\Delta p_f = 32\,\mu v L / D^2 = 4f\rho\,(L/D)(v^2/2)$

or, $f = 16 / (\rho v D/\mu)$, or $f = 16/N_{re}$ (for Laminar flow)

$$\Delta p_f = 4f\rho \frac{L}{D}\frac{v^2}{2}$$