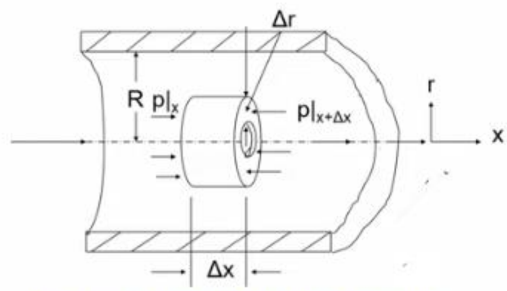
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture18

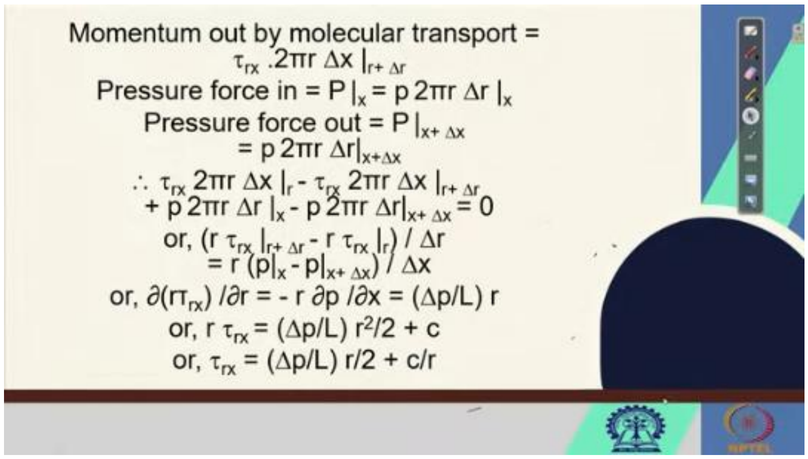
LECTURE 18 : HAGEN POISEUILLE EQUATION

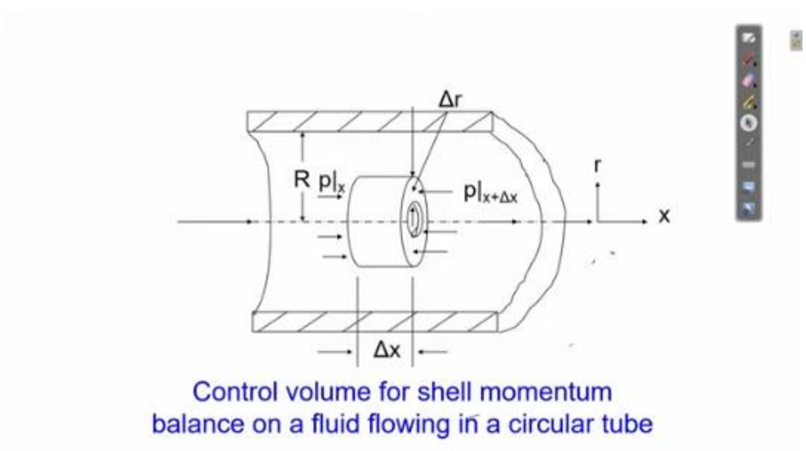
Good afternoon, my dear students and friends. Today, we are continuing with the flow through pipes. This is a real experience because anywhere you are handling flow with pipes. So, how do we know the flow behavior and characteristics of the flow? So, to do that, we have taken a shell momentum balance, and by doing that shell momentum balance, we have come up to this point.



Control volume for shell momentum balance on a fluid flowing in a circular tube

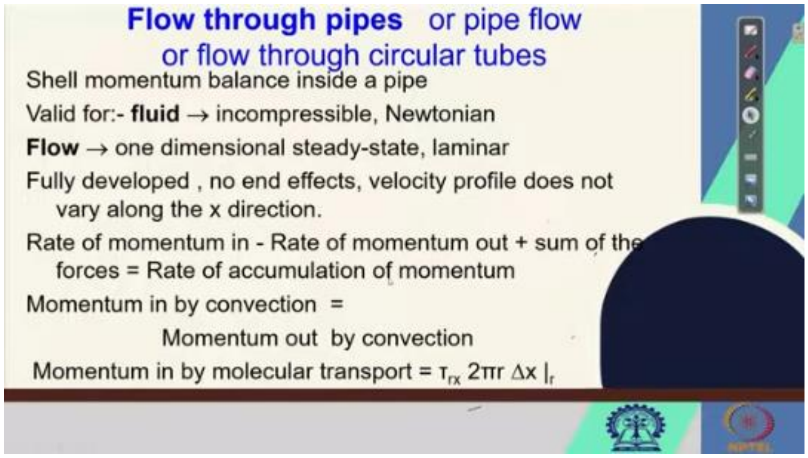
The shear stress τ_{rx} is equal to delta P over L multiplied by r by 2 plus C by r. Now, the question is, how do we find out the constant C, right? So, to determine the constant C, we need to know at least one boundary condition. Now, I go back to the diagram. Why? Because I would like to let you know what could be the boundary by which we can solve this, right?

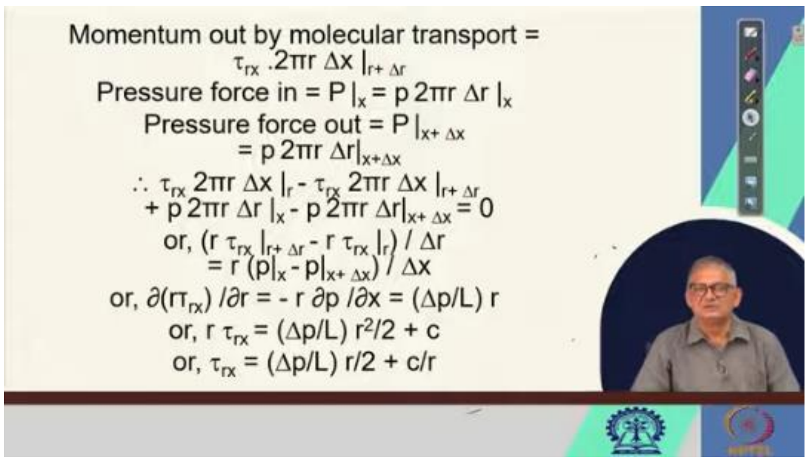




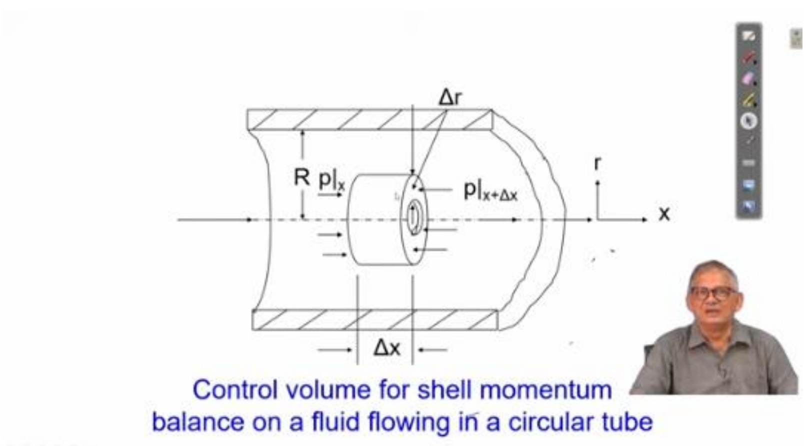
Control volume for shell momentum balance on a fluid flowing in a circular tube

Here, τ_{rx} because we are saying that this is the v_x which is going this is the delta P which is there, right. So, in between the layers, there is the momentum transfer, molecular, right. So, what can make this C a constant? Now, can't we see that at r is equal to 0, where is r equal to 0.



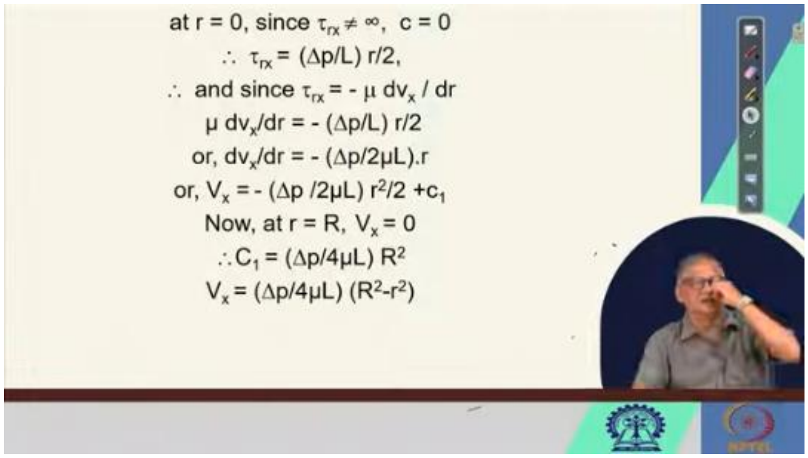


Here, right? At the center, at r is equal to 0, because r is moving this way, right? This is the r, x direction. So, at r is equal to 0, since τ_{rx} is not infinite.

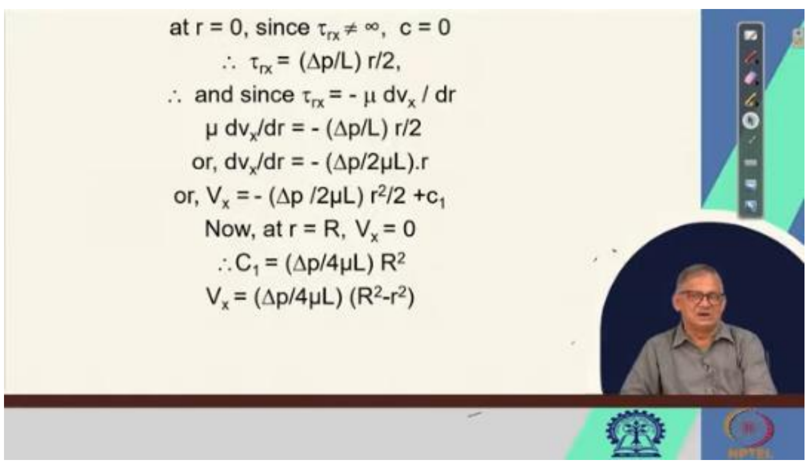


Control volume for shell momentum balance on a fluid flowing in a circular tube

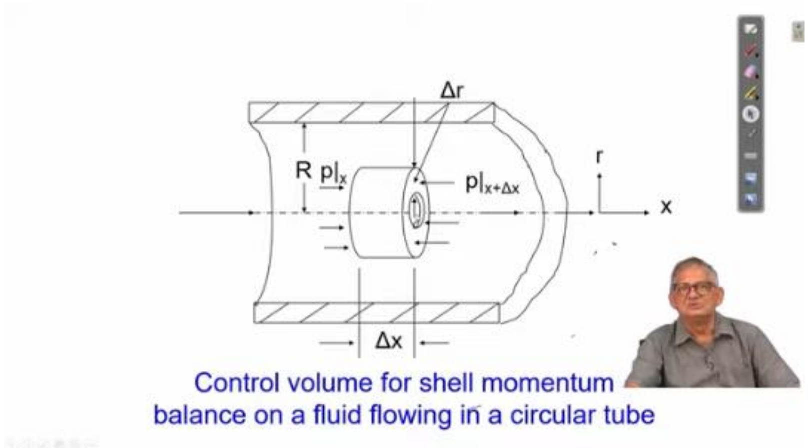
So, C becomes equal to 0, right? This can be one boundary. At r is equal to 0, since τ_{rx} is not infinite, C has to become 0. So, we can write now, the expression for τ_{rx}, that is the shear force, equals delta P by L into r by 2, right? Because, plus C by 2, that C has become 0, C by r has become 0.



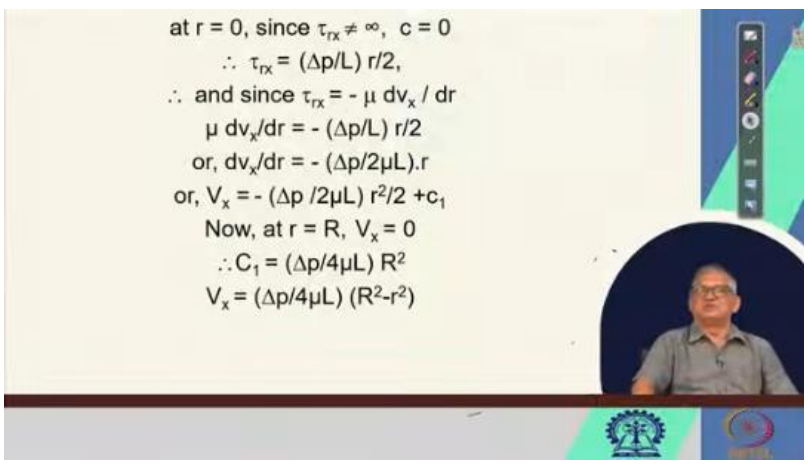
So, it has condensed to delta P by L into r by 2, that is the shear force. τ_{rx} or the expression for the shear force to find out. Here, you see at r is equal to 0, τ_{rx} becomes equal to 0, and at r is equal to capital R, τ_{rx} becomes some number, right. And r can be, as we have seen, capital R, small r can be, so that is a number.



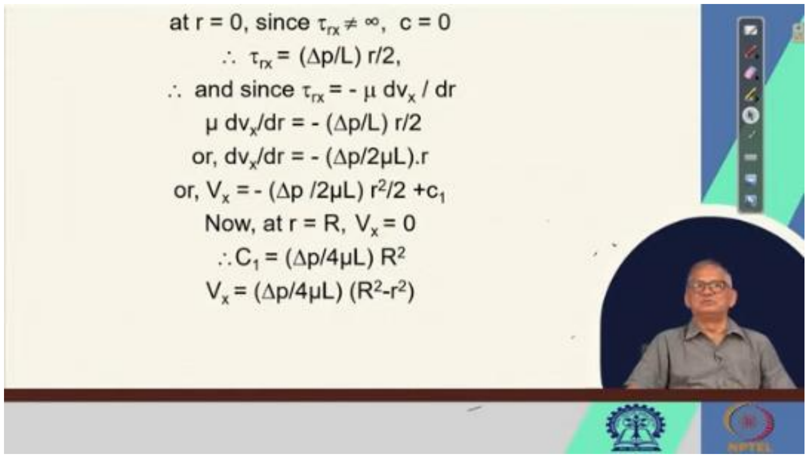
So, we will have the τ_{rx} locus of the shear force that is passing through the origin, right, that is what we explicitly show here, that here it is τ_{rx} is 0. So, τ_{rx} is some finite number here, and similarly, τ_{rx} is a finite number here. So, it will be like a 'k', right? This is one part, this is another part, and this is the vertical, right. We have shown that τ_{rx}, what is the profile of τ_{rx}? It can be and that is having a maximum number at r is equal to capital R and a minimum of 0.



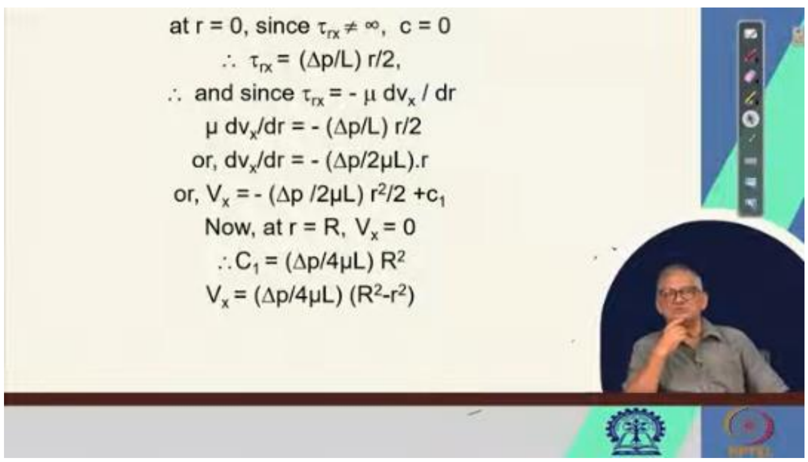
So, passing through the origin, that is the locus of the expression for the shear force, ok. Again, since we know τ_{rx} is nothing, but it can be defined in terms of viscosity, minus mu d v_x / dr. So, I again remind you that the first one is direction, the second one is velocity component. So, the velocity component here will be v_x, and the direction will be r, or dr. So, τ_{rx} can be defined as minus mu d v_x dr.



So, if you put minus mu d v_x / dr in place of τ_{rx}, then it becomes mu d v_x / dr, is equal to that minus you have taken to this side, is equal to minus delta P by L into we had r by 2. If we take mu to the other side, then we can write d v_x / dr is equal to delta P by 2 into mu L into r. Because, we have taken 2 inside, that is, delta P by 2 mu L, right, and these are, we have kept it there outside. Now, if we integrate it again, then we get v_x is equal to minus delta P by 2 mu L, this r becomes r square by 2, plus one integration constant C_1, right.

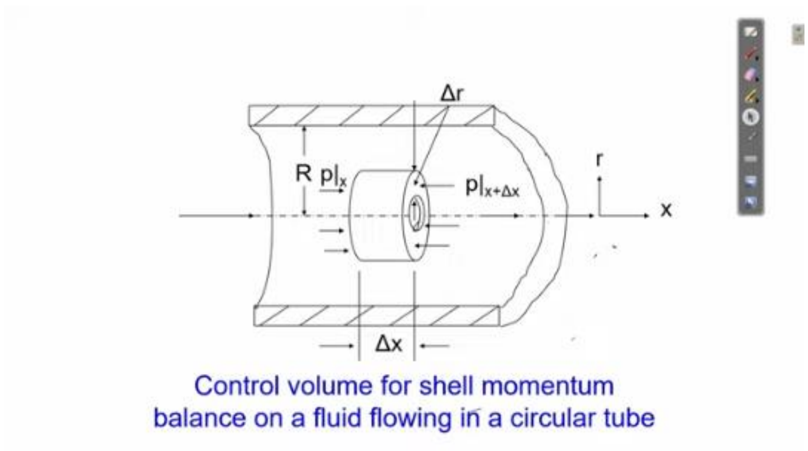


So, we got one expression velocity component v_x in the pipe, and that is minus delta P by 2 mu L into r square by 2 plus integration constant C_1. Again, one unknown C_1. To know that we need to know one boundary condition. Boundary condition, what can it be? At r is equal to R, v_x is equal to 0.

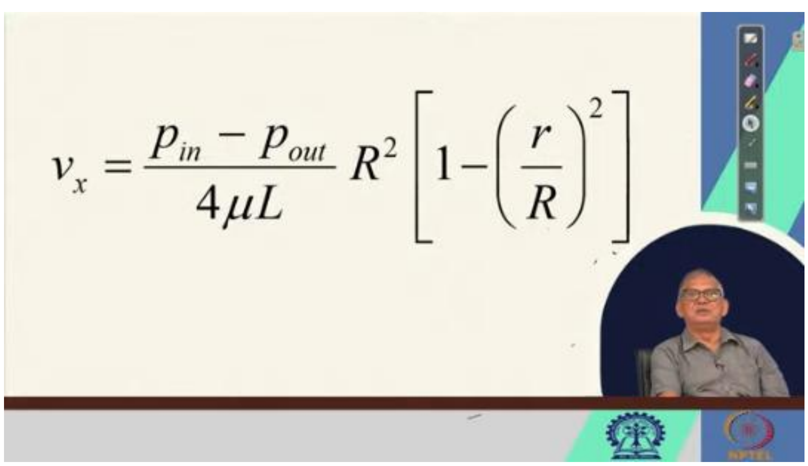


We go back to the pictorial view. At r is equal to R, this is the R, right? At r is equal to R, v_x is equal to 0, because, this surface is not moving. So, the layer clinging to this surface is also not moving, right?

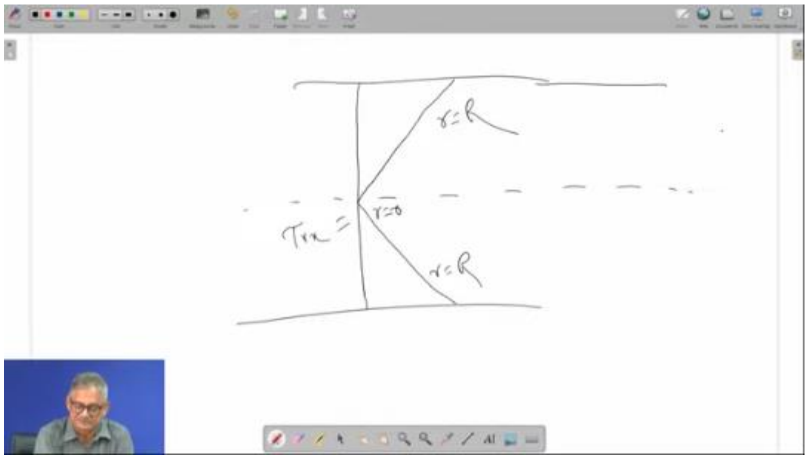
Just like the one, I had shown you that, you have taken a card, a bunch of cards of 52, normally, and you have taken, given, a force at the bottom, then that goes a little inside, and there is a, there is a like a triangle, there is one movement, the top remaining there. So, here also that layer, whatever how many layers you think of. The layer which is clinging to the surface of pipe that is not moving, because, the pipe surface is not moving, right.

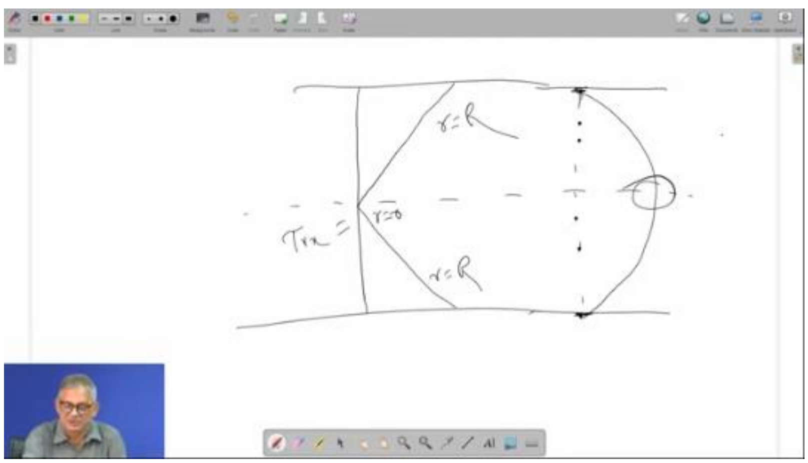


So, at r is equal to capital R, we can say that v_x is equal to 0. Then we can say that. C_1 is nothing, but after putting that, r is equal to capital R, v_x equals to 0, C_1 becomes equal to delta P by 4 mu L into R square. Right, with which on rearrangement, substituting into the expression for v_x, that becomes, v_x equals to delta P by 4 mu L into capital R square minus small r square. So, this we can rearrange, rewrite as the velocity component v_x for the laminar flow of the fluid through the pipe having a radius r with a pressure difference of delta P, P_{in} minus P_{out}, v_x equals P_{in} minus P_{out} divided by 4 mu L into, if we take R square common, then it becomes 1 minus r by R to the power 2. Or 1 minus r by r smaller by capital R to the power 2 or square right. This is nothing, but a. Parabolic curve or in nature. So, we can go back to the flow and say that, yeah, and say that, for the velocity profile, sorry, for the shear stress profile, if this is the. r, right, and if this is the center, then you have some value at r is equal to R capital.



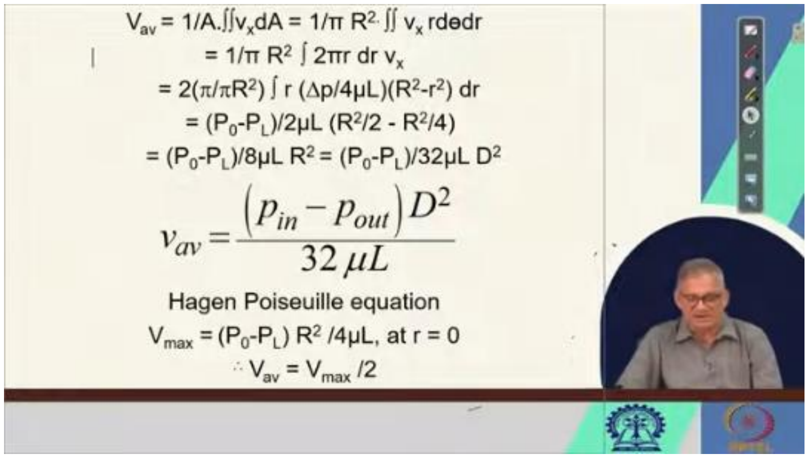
At r is equal to 0, this is τ_{rx}, is equal to so and again we have at r is equal to capital R. This is the shear stress profile right and if we look at velocity profile which it has said that it is to be parabolic. In nature. So, parabolic in nature means, if this is the vertical, then it was 0, at the wall, it is 0 at the wall. So, it can be this, this is parabolic in nature, right, where you see. That v_x is equal to 0, here at the at the interface between the between the surface of the pipe and.

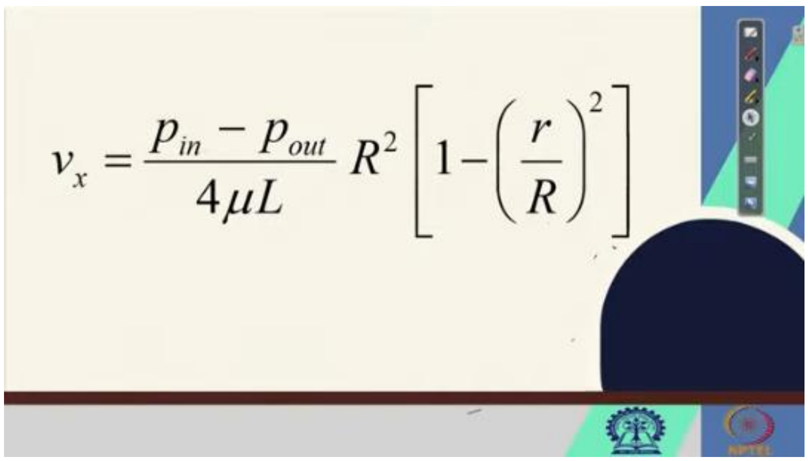


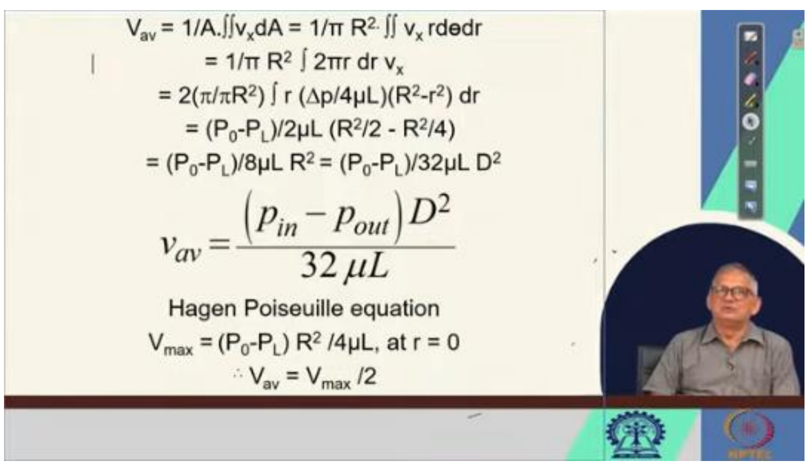


The fluid. So, the layer which is clinging to the surface is not moving and its velocity is equal to 0 right. And this is the another velocity which we will tell, it to be maybe at r is equal to 0. v_x is equal to v_{max}, right. That we will see whether it is really happening or not, right.

Yeah, 1 minus r by R whole square, this is what we have seen. Another term which we call vaverage, average velocity, right, v_{max} will come, it is coming and it is at r equal to 0, v equal to v_{max}, that we have seen, it has to be so. And another one which is called average velocity. Average velocity is defined as $v_{average}$ equal to 1 by area into, 2 integration v_x dA, one integration for v_x, another integration for dA, right.







So, 1 by A, what is the area? Area is pi R square. So, 1 by pi R square, 2 integrations, v_x & dA. So, dA we can replace with r d theta dr, right, r d theta is the arc and dr is the thickness. So, it is v_x r d theta dr.

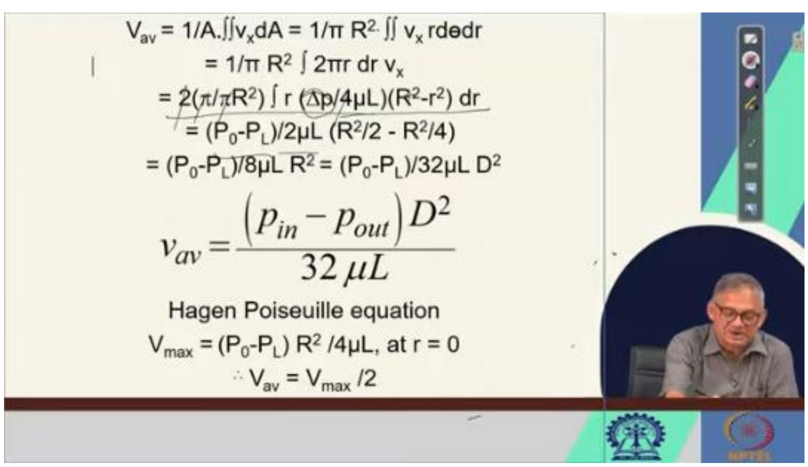
So, that can be written as equal to 1 by pi R square into. So, if we make d theta as 0 to 2 pi, right, then 1 by pi R square, the remaining x that integration, 2 pi r dr v_x.

So, 0 to 2 pi, it is becoming 2 pi, right, and then r dr v_x. So, this on rearrangement, we can rewrite as 2 into pi by pi R square, right, because, in the numerator, we got one pi, in the denominator, we also got pi R square, and 2 also you have taken out. So, integration of r dr.

Now, v_x, we are replacing with delta P by 4 mu L into capital R square minus small r square into dr, right. So, this on integration, and subsequent arrangement, we can write that, delta P is nothing, but P_0 minus P_L. over it was 4 mu L. So, one 2 was on the top and one 4 is on the bottom. So, that 2 and 4 goes out. So, it becomes 2 mu L. Then on integration, we get R square by 2 minus R square by 4, after putting the integration domain value of r equal to 0 to capital R, right. So, P_0 minus P_L by 2 mu L into capital R square by 2 minus R square by 4. So, that on simplification can be written as P_0 minus P_L by. So, this 2 and 4 are taken common. So, 4 is out.

So, 8 we will get, and if we take R square as common, right, then it remains, the remaining one as 1 by 2 minus 1, that means 1 by 2, right. So, this we can rewrite, let us write. delta P, P_0 minus P_L by 8 mu L into R square, before that P_0 minus P_L by 2 mu L R square by 2 minus R square by, oh! this one. So, we can take from here, right.

So, what are you getting, pi, pi goes out, ok, and this we are writing as r delta p by 4 mu L. So, this delta P is P_0 minus P_L, and this was 4 mu L. So, this 2 and this 4 goes out. So, 2 mu L and this is R square right, by 2 minus R square by 4. So, ultimately it is coming to the point that $v_{average}$ is P_{in} minus P_{out} into D square.

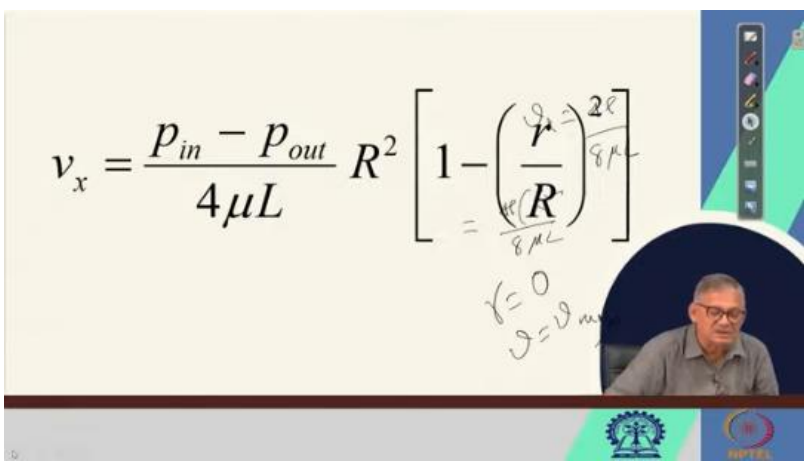


D square means that r is equal to or 2 r is equal to D, right. So, 2 r is equal to D. So, when we took it, it became 32, right. So, from there this 4 and 8, 32 it is becoming, right, 32 mu L P_{in} minus P_{out} D square by 32 mu L, that is the average velocity, right. So, if average velocity is like that, then we can say that vaverage, if it is P_{in} minus P_{out} by D is 32 mu L. And if we say that these times instead of r, that is radius, if we convert it to diameter, then it is P_{in} minus P_{out} into D square by 32 mu L, right.

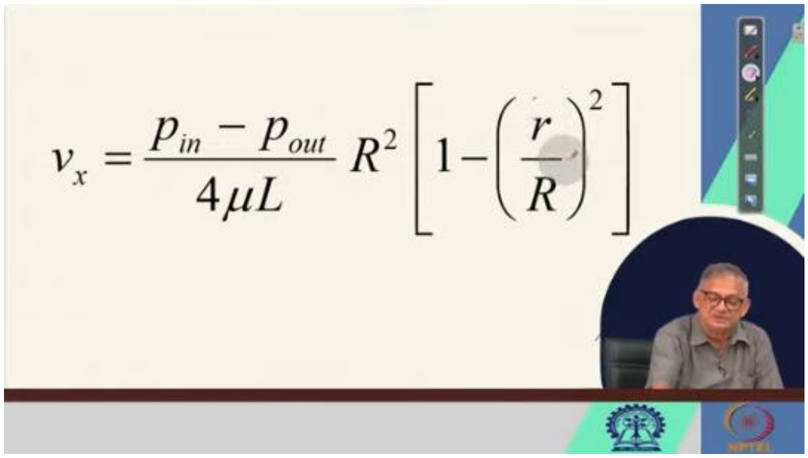
So, this equation is known as Hagen-Poiseuille equation. This equation is known as Hagen-Poiseuille equation if you see the spelling Hagen is H A G E N, but Poiseuille is P O I S E U I L L E. So The average is delta P, that is P_{in} minus P_{out}. P_{in} minus P_{out} is always delta P, because unless there is a pressure difference between the inlet and outlet, it will not flow, right?

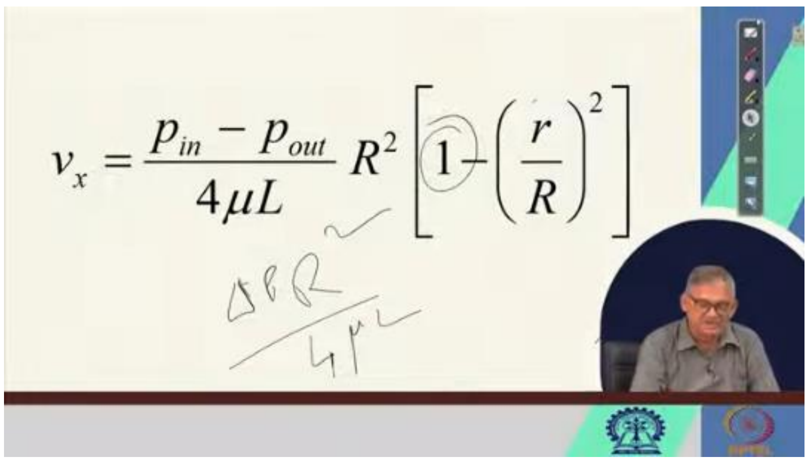
So, that is a must. So, P_{in} minus P_{out} into capital D square by 32 mu L is the Hagen-Poiseuille equation. From there, we can write v_{max}, right? From there, we can write v_{max}. Now, v_{max} is when r is equal to 0, then it is v is equal to v_{max} at r is equal to 0.

is equal to v_{max}, we said earlier, right? Because, this is at r is equal to 0, that means this is D square, that means 2 r square, right. So, 4 so that means, it becomes P_{in} minus P_{out} or delta P into R square by 8 mu L, right. That from here earlier, that v_x equation was delta P by 8 mu L, right? 8 mu L, if we go back a little. Yeah, P_{in} minus P_{out}, delta P by 4 mu L into R square into 1 minus r by R whole square, ok.



So, from there if we write that at r is equal to 0, right? At r is equal to 0, if we write at r is equal to 0. This becomes 0. So, this is 1 and we get that delta P into R square by 4 mu L. This is the maximum velocity or v_{max}. So, this is the maximum velocity or v_{max}. So, that we can get from the Hagen-Poiseuille's equation, right.





So, yeah. So, delta P R square by 4 mu L at r is equal to 0. So, that is $v_{average}$ is v_{max} by 2, $v_{average}$ is v_{max} by 2, okay our time is up. So,

thank you.

$V_{av} = 1/A.\iint v_x dA = 1/\pi R^2 \iint v_x\, r d\theta dr$

$= 1/\pi R^2 \int 2\pi r\, dr\, v_x$

$= 2(\pi/\pi R^2) \int r\, (\Delta p/4\mu L)(R^2 - r^2)\, dr$

$= (P_0 - P_L)/2\mu L\, (R^2/2 - R^2/4)$

$= (P_0 - P_L)/8\mu L\, R^2 = (P_0 - P_L)/32\mu L\, D^2$

$$v_{av} = \frac{\left(p_{in} - p_{out}\right) D^2}{32\, \mu L}$$

Hagen Poiseuille equation

$V_{max} = (P_0 - P_L)\, R^2/4\mu L$, at $r = 0$

$\therefore V_{av} = V_{max}/2$