IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture17

LECTURE 17 : FLOW THROUGH PIPES

Good afternoon, my dear students and friends. We have done the Navier-Stokes equation, its derivation. We have done it in Cartesian coordinate derivation. We have seen how not complex or complicated, how big it is to derive; it takes time. We have also given the Navier-Stokes equations for cylindrical coordinates, that is, r, theta, z, and you have done some problems and their solutions using the Navier-Stokes equation. I hope you will find many other problems like that and solve them. This is for your knowledge only. Right, because it is not possible to give either in, we can, whatever questions we provide. Or we can call it to be whatever we give, as an assignment, or in the final question also, because they are very short; this kind of derivation is just not possible in such a short period of time.

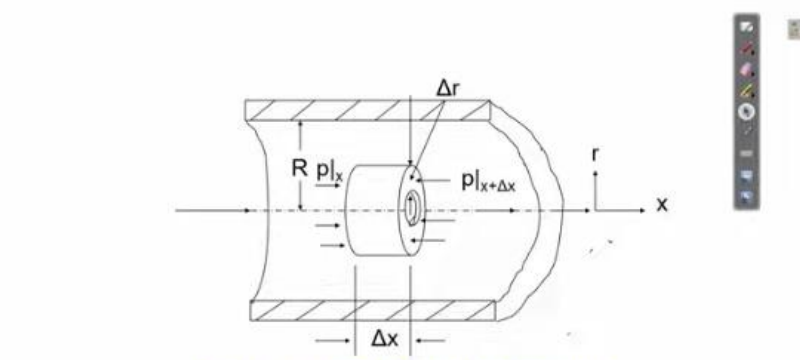
It does not mean again that you should not practice, because unless you do practice, you will not be able to capture the subject, right. Now, we shall go to another topic, that is, flow through pipes. Now, flow through pipes, you know. At home, you have seen that from different sources, different waters are coming, and from different sources, different outlets are going. So, that means there is a flow through pipes.

Another very important associated thing, I am telling you, I do not know how many of you are from any hostel, or if you have any experience in staying at a hostel, wherever it be, it does not matter. The topmost hostel, bottommost hostel, hostel life is more or less similar. So, you might have experienced that whenever you went to the washroom for taking a bath. Sometimes, generally it happens during summer time, where there is a drought of water supply problem. I am not saying in India, all over the country, the supply of water is uniform or universal.

So, the supply of water could be different. So, you went to the washroom to take a bath. So, you have opened the tap, maybe the shower is coming. So, you have put a lot of soap and shampoo, etc. Suddenly, you saw that the water started going push, push, push, push, and then no more water.

It must have happened during your lifetime or to anyone you know in a hostel, right? It may not be so common at home, may not be so common, but in hostels, it is really not uncommon. That is when it is stopping. So, by chance, if you shout, 'No water, pump,' etc., if people are there. So, what will they do?

They will turn on the pump, and some water will come, and that will again start falling on you. And there also, when falling, you might have noticed that it is making some push, push... then push, push, push, then push, push, push, push like that, and after that, maybe a steady flow. These are said to be the end effects. These two are said to be the end effects, right? That is the effect before it is getting stopped, and the end effect before it is getting a regular flow. So, unless a steady flow is attained, we are not considering that as a pipe flow.

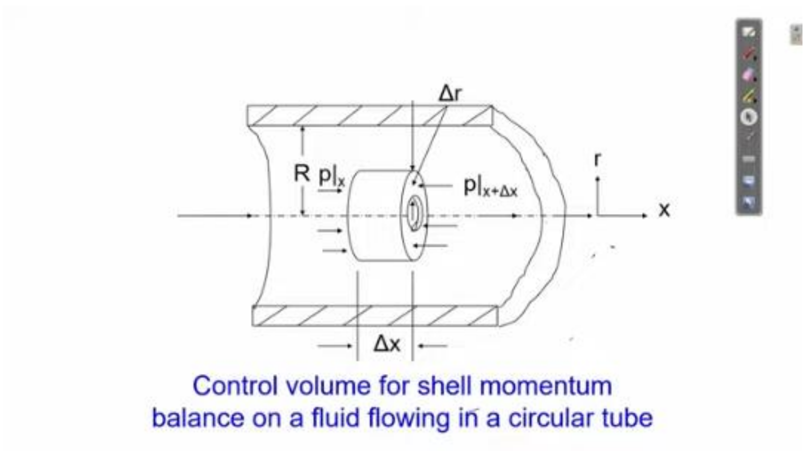


Control volume for shell momentum balance on a fluid flowing in a circular tube

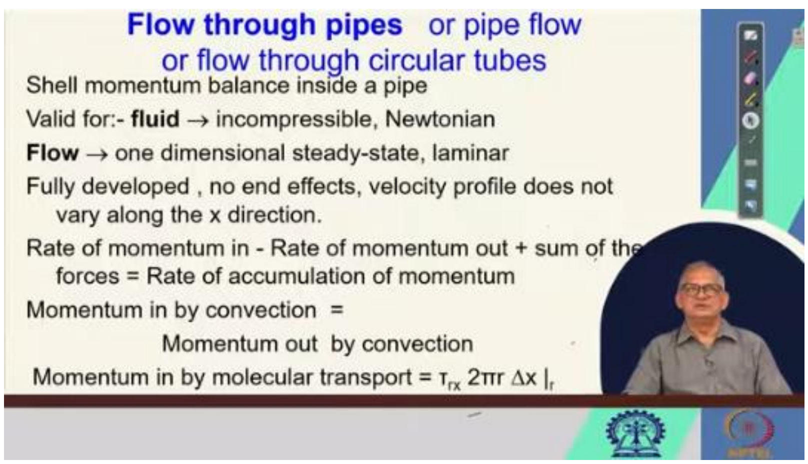
This is my intention to let you know, right? For a pipe flow, it is mandatory that the flow is steady, right? So, you have taken a control volume; we have taken a section of the pipe as shown in the diagram, right? A pipe of diameter or radius capital R, and we have taken a small volume where one side is delta r and another side is delta x, and the third dimension is unit, right? So, it is delta r delta x into 1 $\Delta r\ \Delta x\ 1$.

That is the volume of this volume element. And pressure force is acting on the inlet, that is at P at x, and pressure force is also occurring at the face P plus P at x plus delta x, right. The control volume having delta x may be whatever you call thickness, breadth, or width, or whatever. The other one is delta r again; we call it breadth, width, or whatever, and the third dimension is unity. So, the volume is delta r delta x into 1, right.

So, this spectral view tells us the control volume for shell momentum balance. So, we will do shell momentum on a fluid flowing in a circular conduit or tube. Now, when we are saying flow, right, we are saying fluid, so flow through pipes, or pipe flow, or flow through circular tubes, or circular conduits, whatever we call it, right. We are doing shell momentum balance inside a pipe, right. So, what is that?



It is valid for fluid which is incompressible and also Newtonian. So, incompressible and Newtonian fluid, that is the prerequisite, and then flow is one-dimensional, steady-state, laminar flow. Right, this I said in the beginning. So, these two are the prerequisites: fluid has to be incompressible, Newtonian, and flow is one-dimensional, steady-state, laminar.



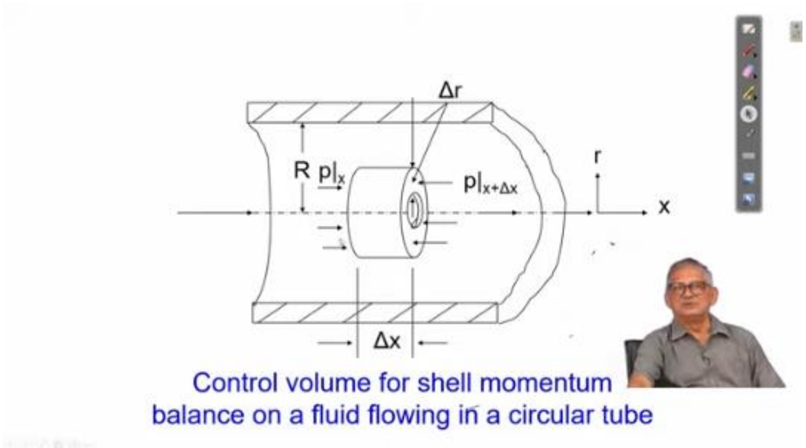
Apart from this, as just I said, it has to be fully developed; that end effect which I explained to you is not there. So, there are no end effects. The velocity profile does not vary along the flow of the fluid, that is, in this case, along the x-direction because x is the direction of flow, as we have seen from the pictorial view, right. So, now, we come to the governing equation.

So, the governing equation is like this: rate of momentum in minus rate of momentum out. Plus, the sum of the forces acting on the volume element must be equal to the rate of accumulation of momentum in the volume element, right? We have not written 'volume element' everywhere because then it becomes very, very big. So, rate of momentum in minus rate of momentum out plus the sum of the forces should be equal to the rate of accumulation of momentum. This is all with respect to the volume element, which we have taken as delta x, delta r, and 1, right?

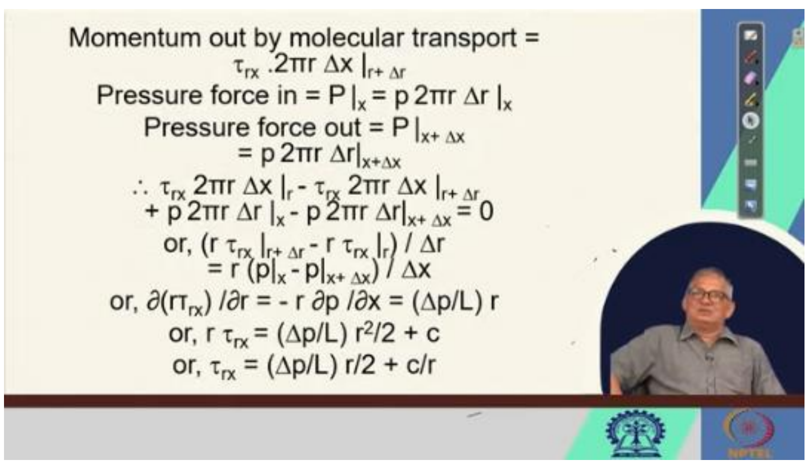
Now, momentum in by convection—we have already done this earlier, while developing the unique equations, right? There, we have said that momentum transfer takes place, one by convection or bulk movement, and another by molecular transport. So, this we have done in developing the Navier-Stokes equation, right? So, momentum in by convection that we can write is equal to—obviously, at steady state, it must be momentum out also by convection because otherwise, it will not be at steady state.

If it is steady state, whatever is coming in must be equal to whatever is going out, right? So, momentum in by convection must be equal to momentum out by convection. So, momentum by convection is canceling out. Then, it remains the other one, which is molecular transport. So, momentum in by molecular transport—that is (tau) τ_{rx} again, that tangential or this, what we call this tangential or shear force, that is (tau rx) τ_{rx} again—I hope you could have understood what I am asking you, that what stands for r and x, right? I said earlier repeatedly that keep in mind the first letter stands for the direction, and the second letter stands for the velocity component, right? So, here, with respect to (tau rx) τ_{rx}, we can say that r is for the direction, and x is for the velocity component.

So, τ_{rx} into its area, area is how much? 2 pi r into delta x, right? 2 pi r into delta x, that is the shell, that is the area at the face r, right? We go back to the that, it is at the face r. So, where is it? This is at the face r, right. So, or here, if you take that small volume element, right, and at the face r plus delta r, another one, right. So, we get that momentum in by molecular transport is τ_{rx}

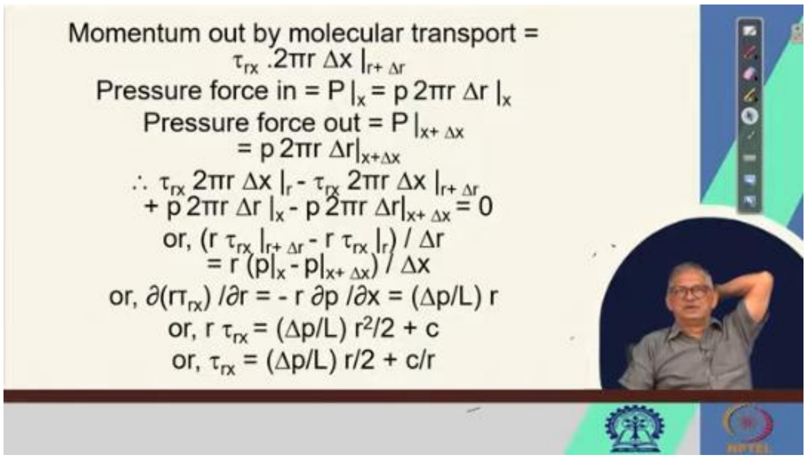


times 2 pi r delta x at the face r, and momentum out by molecular transport is equal to τ_{rx} times 2 pi r into delta x at the face r plus delta r. This is for molecular transport momentum. Other forces are also there, like pressure force, because if there is no pressure, there will be no flow of the fluid. I do not know, whether you have ever played or not, but that by taking in a small tube, say this big tube, right. Okay, let me show you.



So, here, right? So, if it is horizontal, absolutely. If I put this cap open, then what will happen? Some liquid will obviously come out. But after some time, when there will be a steady state, no more pressure, no more flow, no more movement, right? The same you might have seen at your home, that you have taken a small tube where you have filled it up with water or any fluid and kept it on a horizontal plate. Then you saw that yes, there is no movement of water. Because, unless there is a pressure force, it will not move. So, in this case, the pressure force will be that from the source, whatever pressure was given, and at the exit, whatever pressure is coming.

So, this is the delta P. So, that delta P is the pressure force. And this is P at the face x that is equal to P times area that is 2 pi r delta r, right, 2 pi r delta r, area of the small control volume which we have taken, right. So, this is at the face x. Similarly, at the face x plus delta x, it will be P into 2 pi r delta r at the face x plus delta x, right. Therefore, we can say that



from the governing equation, which we said that, rate of momentum in minus rate of momentum out plus sum of the forces acting on the volume element equals to rate of momentum accumulation. So, the rate of momentum in by convection or by bulk volume, we have seen that it is not there, because it is at steady state, whatever is coming in that is going out. So, bulk momentum transfer is not there, but molecular momentum transfer is there and that we have found out to be equal to τ_{rx} 2 pi r delta x at the face r minus τ_{rx} times 2 pi r delta x at the face r plus delta r. This plus the forces that is pressure force, p into 2 pi r delta r at the face x minus p into 2 pi r into delta r at the face x plus delta x. And this must be equal to 0, because, the rate of momentum accumulation is also 0, because we have taken it to be a steady state, right. So, then rewriting it, we can write that r into taurx at the face r plus delta r minus r into taurx at the face r and divided by delta x and delta r. So, here we had delta x. So, that goes out. So, in the denominator, delta r remains, right.

And we have taken to the other side that r at p at x minus r p, sorry, r into p at x minus P at x plus delta x over delta x because here delta r was there, that goes out. So, delta x in the denominator. And why did we do this? Because we want to make from the definition of derivative that higher minus lower divided by the difference that is the Derivative. So, here also the term τ_{rx} at r plus delta r and τ_{rx}

at r with the space delta r and p at x and p at x plus delta x with the space of delta x, right. So, this leads to, if we put, limit delta r tends to 0 or limit delta x tends to 0, then we can write the del del r of r τ_{rx} is equal to minus r into del P del x, right.

Again we have taken a horizontal pipe. So, that is why we have not taken the gravitational force into consideration, right. So, del r τ_{rx} over del r is equal to minus r del P del x. is equal to delta P over L into r, right.

So, this delta P is del P del x that del P is delta P and del x is the total length L, right, times r. Now, that negative sign has been taken into account because your delta P is P higher minus P lower, right, and the P higher minus P lower, if it is delta P, then this negative, where does it go? It has gone because in the previous line, if you see, that is equal to r into p at x minus p at x plus delta x over delta x, and we said limit delta x tends to 0.

So, it makes a derivative. So, that derivative would be del p del x, right, del p del x, but this delta p is from the real. So, one negative and there because of the derivative concept higher minus lower, right, higher minus lower. So, that is there and we are making p at x minus p at x plus delta x. Right.

So, this derivative is also giving a negative. So, that negative derivative is in the next line, which is minus r del p del x, that is equal to delta p. This delta p is now, that negative is taken care of because this delta P is not from exit to inlet. The exit P is lower than the inlet because, if the inlet pressure is not higher, it will not flow. So, here, we are taking P at r and P at r plus delta r, or P at x and P at x plus delta x, right. So, this is the delta P we are taking.

So, that takes care of the negative, and we can write delta P as P inlet minus P outlet, and delta x is between 0 to L, which is the length of the pipe, right. So, we can write r τ_{rx} is equal to delta P by L. into r square by 2 because, here, we have taken del del r of r τ_{rx}, and we have done integration. So, it becomes r τ_{rx} equal to delta P by L, which is a constant, and this r becomes r square by 2, and a constant C, right. So, we can write that by dividing both sides with r, we can write τ_{rx} is equal to delta P by L into r by 2 plus C by r, where C is the integration constant.

So, we got an expression for the tangential force, or what is said, this is that force, what we said is, this force, is the shear force, right. So, τ_{rx} is the shear force at r due to v_x, right. So, that is equal to delta P by L into r by 2 plus C by r. Now, we have to find out the value of the constant C. To know the value of the constant C, definitely we need to know, here, one unknown, which is C. So, at least one boundary condition, we must have,

so that we can find out the value of the constant C, okay. Now, today's time is up. So, we will do it in the next class. Thank you.