IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

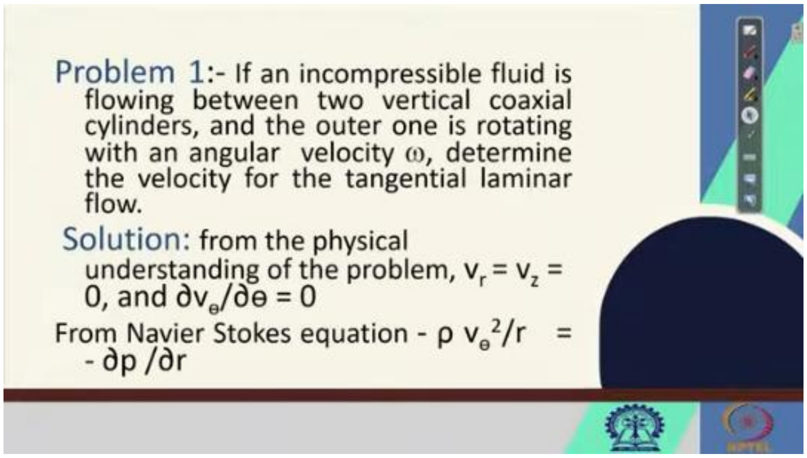
Lecture16

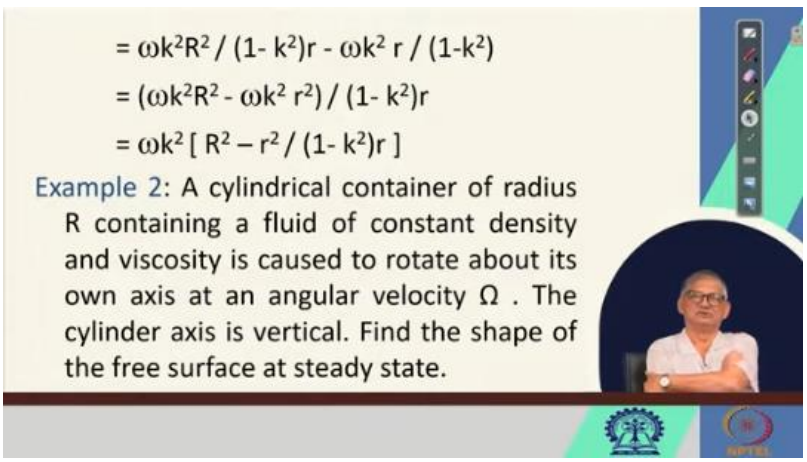
LECTURE 16 : PROBLEMS AND SOLUTIONS WITH THE HELP OF NAVIER STOKES EQUATION

Good afternoon, my dear boys and girls, students, and friends, right? I had given you one assignment. I don't know whether you have done it or not. We are continuing with the problems and solutions of the equation of motion, right?

So, it is the second class, second problem we will try to do, and if you have done the solution, nothing like that; we will also try. It may not be today because the time constraints are, as you see, that in one class, one problem is good enough to cover up. And the problems are, unless we understand, not so easy. So, in the previous class, we have done the problem of one, rather two coaxial cylinders: the inner one is fixed, and the outer one is moving with an angular velocity. And we said, as your assignment, that if you do

reverse one, that is, if the angular velocity, what is the angular velocity which you will find out when the inner one is rotating with an angular velocity of omega and the outer one is moving with, rather, the outer one is fixed, right? This I have given you as a problem, okay. So, I do not know whether you have already done it or not. If you have done it, I will be the happiest person because that also tells that yes, you have understood. Otherwise, now let us go with another problem, okay.





This problem says, this is example 2, that a cylindrical container of radius r. Containing a fluid of constant density and viscosity, is caused to rotate about its own axis at an angular velocity of omega. The cylinder axis is vertical, and the shape of the surface is steady state. I repeat, a cylindrical container of radius r, containing a fluid of constant density and viscosity, is caused to rotate about its own axis at an angular velocity of omega. The cylinder axis is vertical.

Find the shape of the free surface at steady state, okay. This is another problem. There are innumerable problems, and you can utilize this solution, right, using the Navier-Stokes equation. To understand this problem, let me ask you, I do not know whether during your childhood or even now, you have played with such a system that a bucket full of water may not be full, half-filled, or three-fourths filled like that.

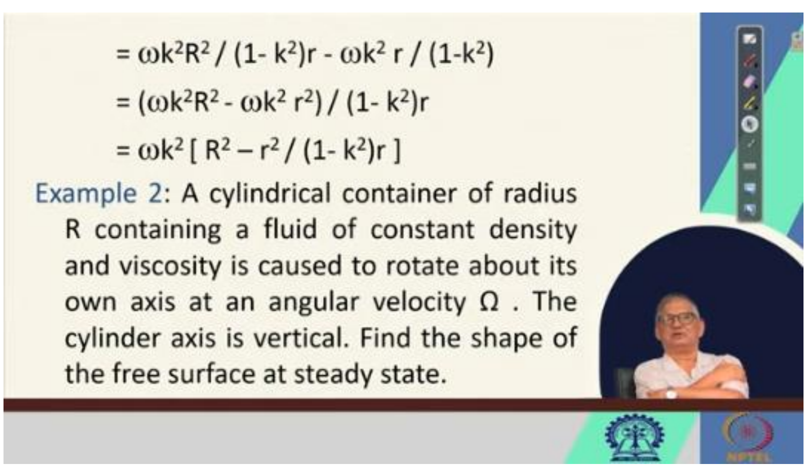
And you have taken it with your hand and started rotating it around you; you are rotating around with that, right? So, you take this like this and rotate, right? If you have done it with a moderately high speed, then you might have seen that in that bucket, the layer of water has become curvilinear like this, if you have ever tested it or not. If you have not, do it today or tomorrow when you are taking a bath.

And maybe this is in a small bathroom also. If you have a bucket, you can try. In earlier days when things were not so complicated, it was simple. And yeah, you had the tube oils or the big those. pots from where you were drawing water through buckets.

People used to take both and many others. So, you might have tried or you might have played with this, right? Yeah, we have also done it in our childhood. I'm not saying that. So, if you do that, if you have done it, you have seen that yes, there is a curvilinear surface of the water like this.

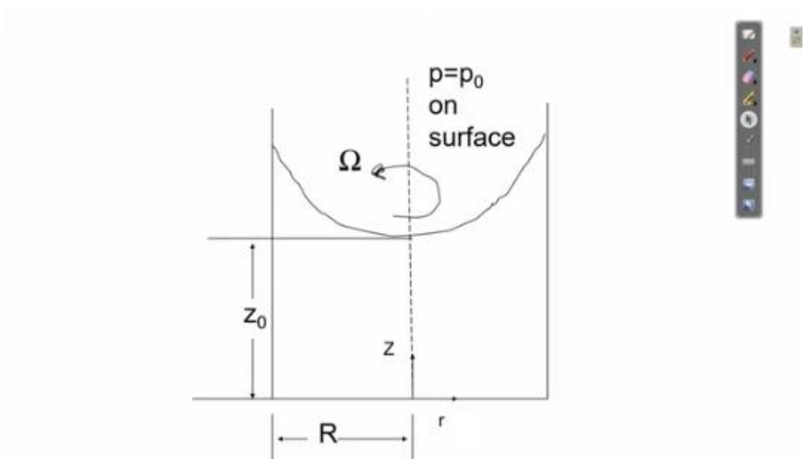
And if you have not done it, you see today or tomorrow and see that you will get one curvilinear shape of the water in the bucket. So, this is another problem which we need to solve. Very good. Now, here we have to first understand the problem. So, a cylindrical container of radius R. So, you have a container of radius r.

Containing a fluid of constant density and viscosity. So, viscosity μ (mu) is constant and density ρ (rho) is also constant is caused to rotate about its own axis with an angular velocity. It is moving with an angular velocity omega. Obviously, the axis of the cylinder is vertical, right, as you yourself are rotating. So, you are the axis, and the tub or bucket is getting moved right in a circular motion like this, you are getting that motion with an angular velocity of omega.

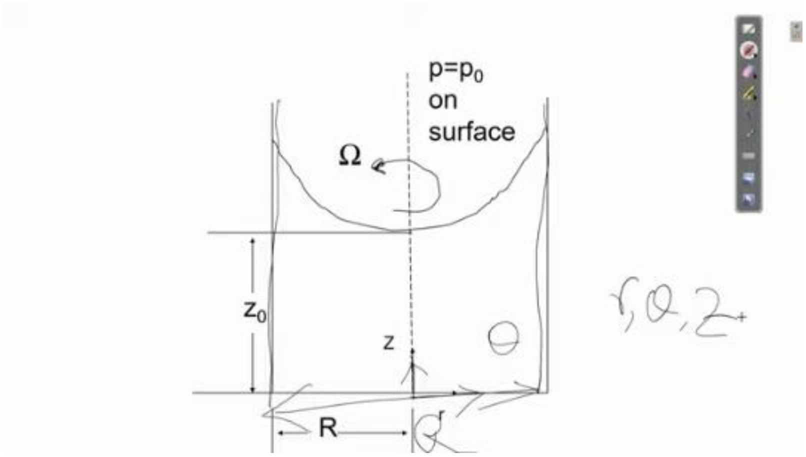


Your problem is that what is the shape of this shape of the free surface, and the free surface is at steady state, right? So that means when you are moving After some time, it has become a steady state, and the surface is like that. Before that, it was maybe horizontal, and then after a little more, then after even a little more like that. When it becomes a steady state, then the surface is also horizontal.

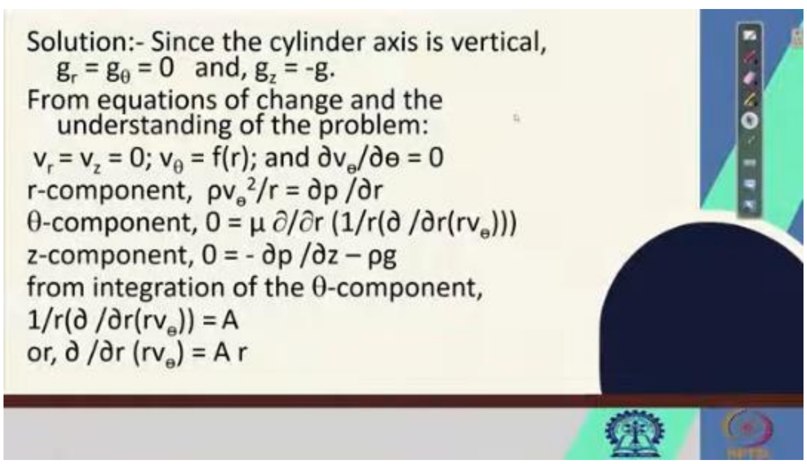
The locus of the surface can be said to be steady, ok. Now, if we want to solve it, we have to first find out the problem and understand the problem, right. So, if you want to understand the problem, this is the drawing of the problem. That you have the bucket, right, you have the bucket. This bucket is this and this, and we said the radius of the bucket is R, right, and r,z.



These are the three coordinates of r, Θ (theta), and z. This is also a cylindrical coordinate, right? So, r, theta, and z are what we are asked to find out, and this bucket is moving with an angular velocity of omega. So, we are asked to find out that at steady state, you get this kind of curvature. So, what is the locus of the curvature? At steady state.

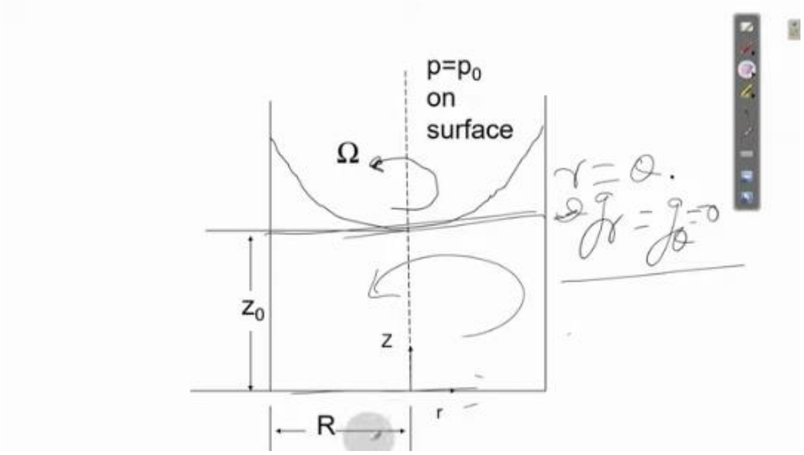


So, we need to understand the problem and then solve it accordingly, okay? This is a very good problem, and obviously, we do come across it in the everyday program of our life, right? We go to the understanding of the problem. And it is said that since the cylinder axis is vertical, we said the axis of the cylinder is vertical because if I am the axis and around me it is rotating, then I am the axis, and it is vertical, right?

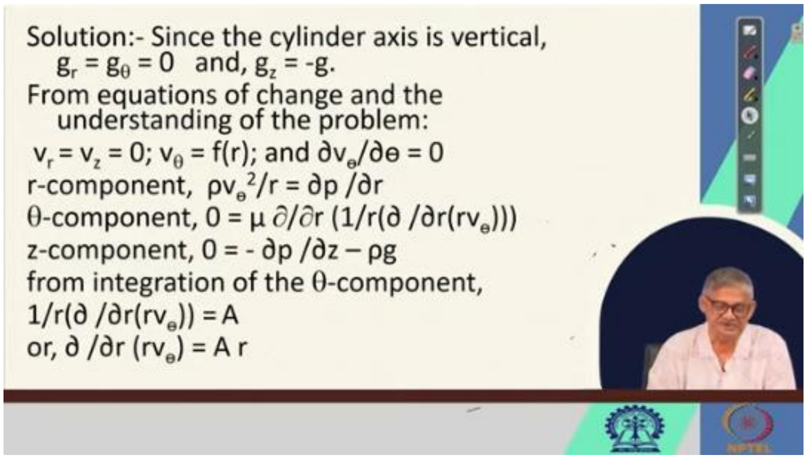


Since the cylinder axis is vertical, g_r is equal to g_{theta}, which is equal to 0, right? I go back to the problem that g_r and g_{theta} are equal to 0, meaning g_r is equal to g_{theta} is equal to 0, meaning that r is in this direction, right, and theta is in this direction, right? So, g_r and g_{theta} are 0 if it is the surface. Here, r and theta at this point have g equal to 0, right? That is true, okay.

If that be true, we try to solve the problem. This is the first understanding, and the first thing which you have said is that this is the condition. Next, if we go into So, if we go next, g is in the z direction because z is the vertical.



So, g_z is equal to minus g; it is acting in reverse of g, right? g is like this. So, it is acting because you are rotating. So, it is acting as the negative of that. So, from the equation of changes and the understanding of the problem, if we use the basic equation of Navier-Stokes, then we need to identify from the solution, from the equations, the different

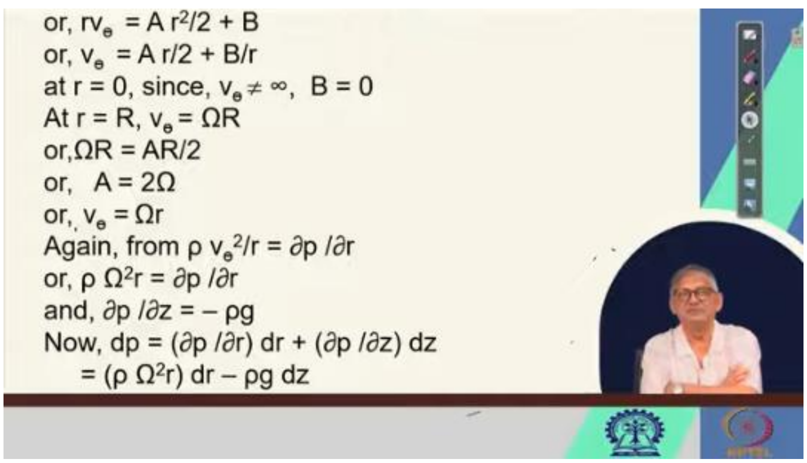


parameters or different terms in that equation. One is that we have already said v_r is equal to v_z is equal to 0, and also vtheta is a function of r and del vtheta del theta is 0 because it is at steady state, so it is at laminar condition, right. So, from the Navier-Stokes equation we can say that the r component is like rho vtheta squared by r is del P del r, theta component is 0 is equal to mu del del r of 1 by r del del r of r_{vtheta}. This was also the same in the last whatever we said.

And the z component is 0 equal to minus del P del z minus rho g, ok? Because g is negative, that is why it was plus rho g, but it has now become minus rho g. So, these are the three r, theta, z components. Which we have identified by eliminating the, I mean, appropriate terms, right. Then from the integration of this equation, we get the theta component as 1 by r del del r of r_{vtheta} that is equal to A, right. So, on rearrangement, we can write del del r of r_{vtheta} is equal to A r small.

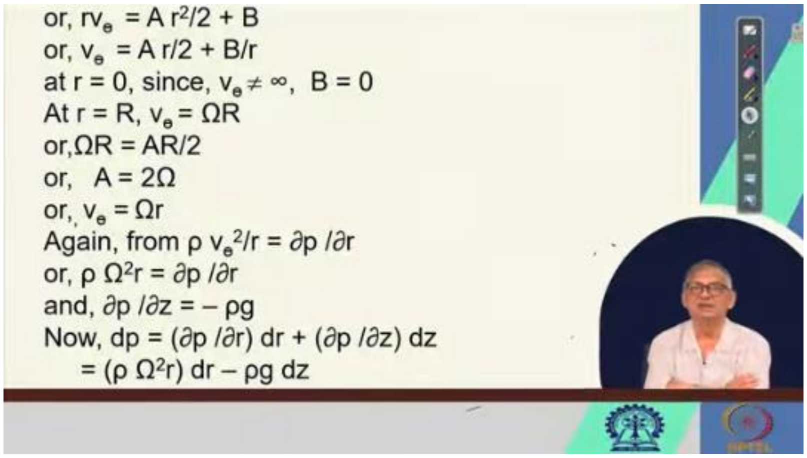
And on the second integration, we can write r_{vtheta} is A r squared by 2 plus B and v_{theta} we write same as earlier, A is equal to r by 2, rather V theta is equal to A r by 2 plus B by r, right. So, we have to put now the boundary condition; at least two boundaries are required. So, the first boundary is obviously at r is equal to 0. At r is equal to 0, since vtheta is not negative or rather infinite, since vtheta is not infinite, right?

Because you see the solution v_θ is A r by 2 plus B by r, right? If v_θ r is 0, then vθ has to be infinite; otherwise, B cannot be said to be 0. So, since it is not infinite v_θ, we can say that B is equal to 0, right? Therefore, at r is equal to capital R, v_θ is equal to ω R.



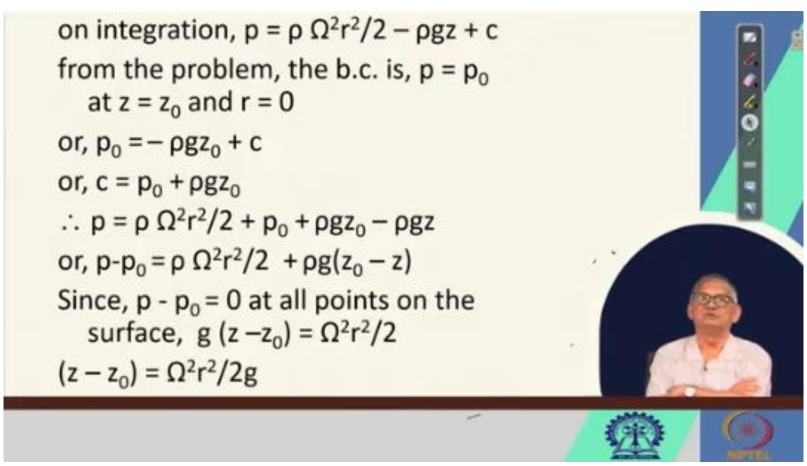
So, we write by supplying there B has become 0. So, ω R is equal to A R by 2, right. So, ω r is A R by 2, or A is equal to 2ω; r, r goes out, 2ω, right. Therefore, substituting the value of A, we get vθ is equal to 2ω, right. And from the previous one, we can say that ρv_θ^2 by R,,

that is equal to ∂p/∂r, the third equation. No, this is the first equation, that is ρv_θ^2 by r is equal to ∂p/∂r. So, by substituting, we get $\rho \omega^2 r$ is equal to ∂p/∂r that $\rho \omega^2 r$ is ∂P/∂r, and ∂P/∂z, this is equal to -ρg, which we have also shown. Now, P is a function of both r and z; therefore, we can write dP is equal to ∂P/∂r into dr plus ∂P/∂z into dz, right.



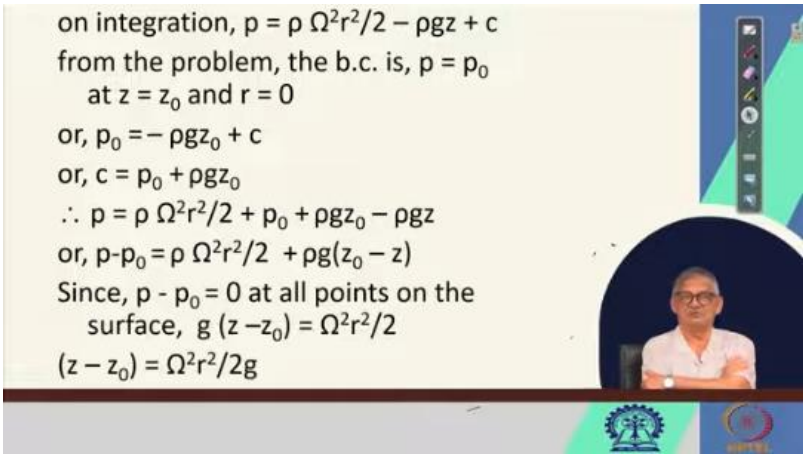
That is a variable P, how it is functioning with other variables like r and z. We are saying that dP is equal to $\partial P/\partial r$ dr plus $\partial P/\partial z$ dz. Therefore, ω^2 ρ r into dr minus ρg dz, so that is equal to dP, right. This we have done very simply. That is purely logic; based on that logic, we have come to this.

Now, integrating we get P is equal to omega, rather density, that is rho omega square r square by 2 minus rho gz plus c. Then, the boundary condition is like that p is equal to p_0 at z is equal to z_0 is equal to 0, right. This is from the definition p is equal to 0, then all becomes equal to 0, p is equal to 0, then p is equal to 0 and p becomes equal to p_0. And at r is equal to 0. Therefore, we can write p_0 equals to minus rho g z_0 plus c, a constant or, c is equal to p_0 plus rho g z_0.



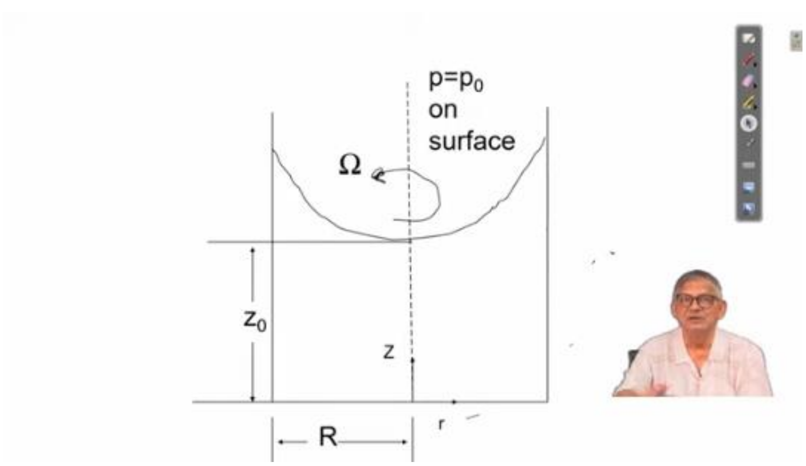
Therefore, dp is equal to dp, which we have made, that we have made, this dp as p minus p_0 is equal to 0. At all points on the surface and g into z minus z_0 is equal to omega square r square by 2. Therefore, we can write z minus z_0 is equal to omega square r square by 2 g. Omega square r square by 2 g that is the z minus

z_0 and this is what we can say that the locus of the line which we have found out is z minus z_0 is equal to omega square r square by 2 g, right.



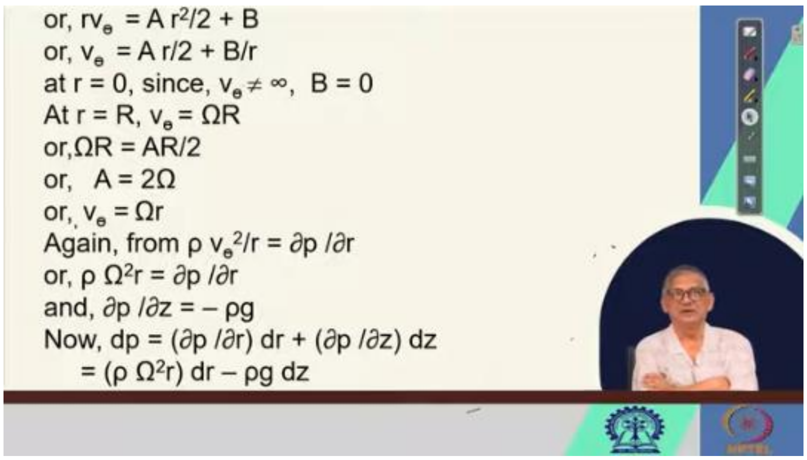
So, what we can know that P is constant or P is equal to rho into omega square into r square by 2 plus, of course, P_0 rho z_0 minus rho gz, right. So, from there we found out P minus P_0 in terms of omega rho and r as rho omega square r square by 2 plus gz, z_0 minus z, right. And since p minus p_0 is 0, why p minus p_0 is 0?

Because, we go back to that pictorial view. Here you see at time 0, p is p_0, z is z_0, right, and the surface is steady and horizontal, right. Now, with the velocity vtheta, we can find out what is the velocity with respect to z_0, we will try, right. However, so that P is P_0 on the surface of the liquid.

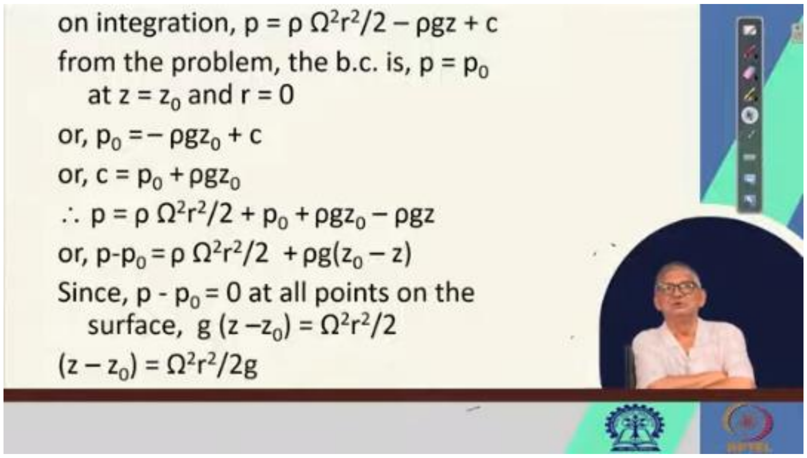


Steady state, right? So, it never changes, even at steady state, right? And we are saying, So, omega has to be found out, right. So, we are saying that at steady state, p is equal to 0 on the surface, and this is true for all because this omega is under steady state, ok.

So, we found out this 1 by r del del r of r vtheta that is A, and then we found out A r, that is del del r of r vtheta is equal to A r. And we can find out what is the vtheta by substituting the values of A and B. Right, and we got omega r is A r by 2 or A is 2 omega, and vtheta is omega r. We substituted the values of vtheta and we got rho omega square r is equal to del p del. And del P del z is equal to minus rho g. Therefore, we could find out dP because dP is a function of P, which is a function of r and z, right. So, dP is del P del r d r and del P is del z d z. p omega square r dr minus p g dz is the dp.



So, again, on integrating this P, we get rho omega square r square by 2 minus rho gz plus c. So, from this problem's boundary, we can get that z is equal to z_0 and r is equal to 0. So, therefore, P_0 is rho g z_0 plus C and C is nothing but P_0 rho g z_0. So, p is omega square r square rho over 2 plus omega plus over 2 plus p0 plus rho g z_0 minus rho g z. Therefore, by separating the variables, we get at all points on the surface, g is equal to g into z minus z_0 equals omega square r square by 2. Or z minus z_0 is omega square r square by 2 g. So, this is the locus of the curvature which you have created while rotating with the



angular velocity omega, ok. These are the two problems we have solved, and we have given you one assignment in the class. So, if you can do it, do it; else, whenever time permits us with you that day, we can try.

Thank you very much.