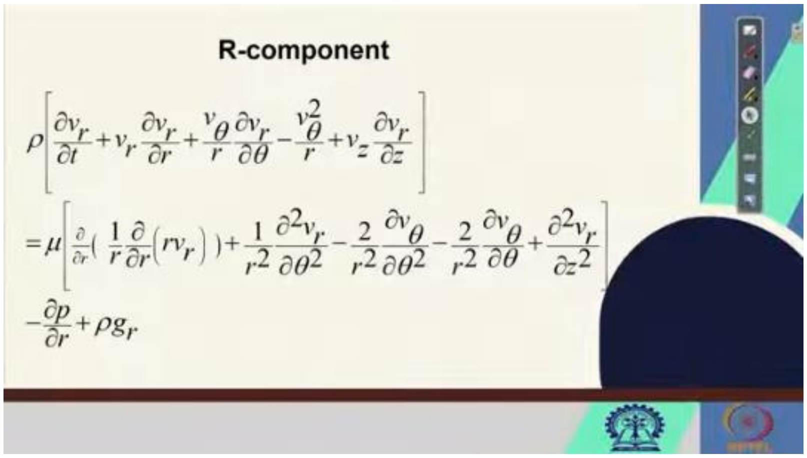
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture15

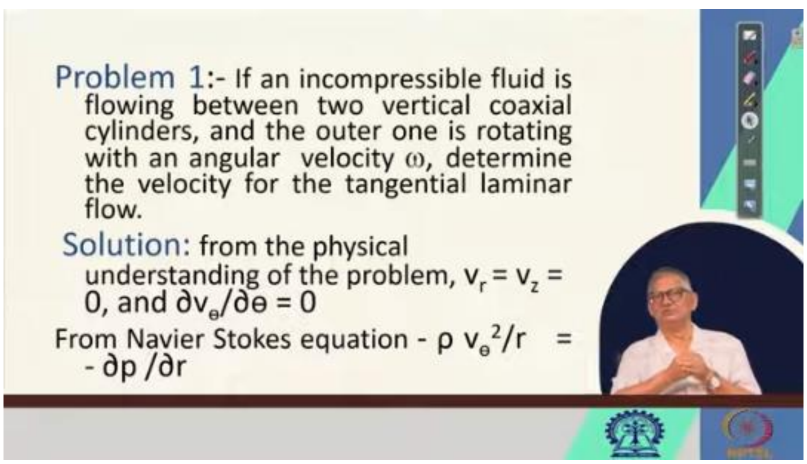
LECTURE 15 : PROBLEMS AND SOLUTIONS WITH THE HELP OF NAVIER STOKES EQUATION

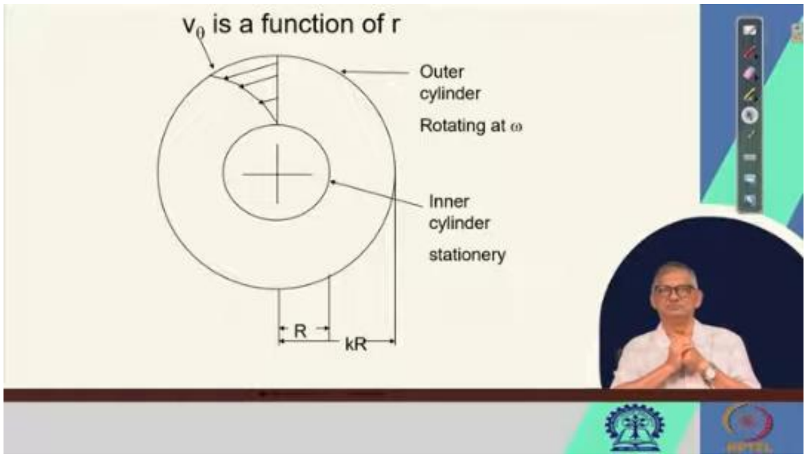
Good afternoon, my dear boys, students, and friends. We are handling the equation of motion, which is the Navier-Stokes equation, that we have derived. We have derived it for Cartesian coordinates, but we have also provided the equations for cylindrical coordinates, which we have also explained, right? Now, we have taken a problem where

we are doing the solution, right? So, let us go directly to the problem first. So, these are the Navier-Stokes equations. So, we had given this problem that two coaxial cylinders, the inner one is fixed, and the outer one is rotating with an angular velocity omega. Then, what will be the tangential laminar flow, right?



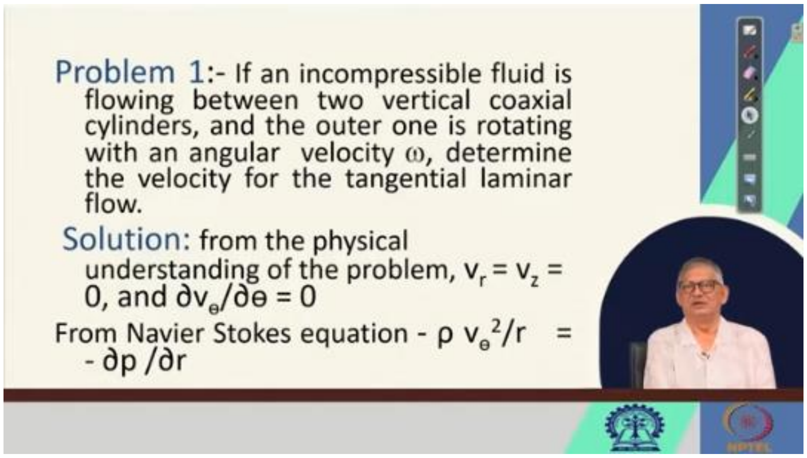
So, from this, we have to understand the individual terms. We have understood, right? We have said that v_Θ is a function of r, right, and that what is the value of v_Θ, you have to find out, right? We have explained in the previous class what the individual terms mean, and we have taken the r component, theta component, and z component of momentum individually, and from there, we have we have deleted the terms which are not required for this problem or which are cancelling out because of the problem defined, right.



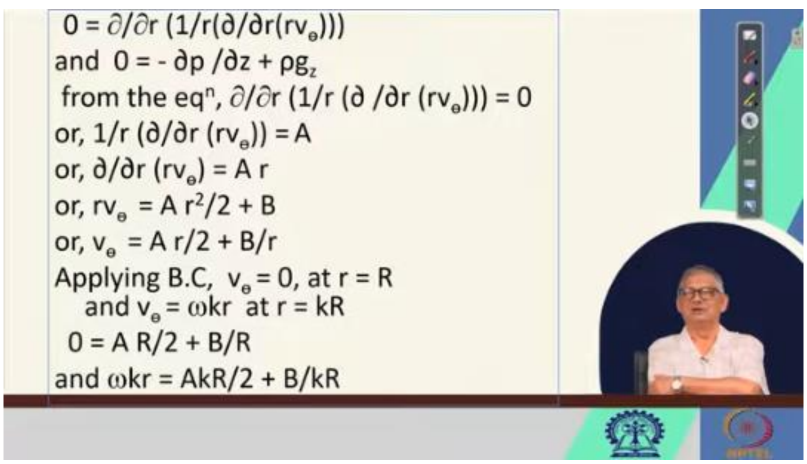


And we have taken this figure as the inner one is fixed with a radius capital R, and the outer one is rotating with an angular velocity omega, having a radius of K R, where K is a multiple of R, definitely greater than 1, because otherwise it would not have been more than R. Okay. So, This is arbitrarily taken; you can take R_1, R_2, any damn thing. It is not necessary that you have to take R and K R. This R and K R we have taken, why?

Because there is a similarity, only K is a constant, right. Here, r is a variable because it is varying between 0 to K R, right? 0 is for 1, and the other one is K R, the outer one, OK, and the inner one is 0 to R. However, that is not required; our fluid is between R and K R, and it is moving as shown through the arrow. So, it is a function of r, right? So, we have arrived at the three equations if you remember, with the r component, the theta component, and the z component, right? So, if we look at them, you see, we arrived at this: 0 is del del r of 1 by r del del r of r v_Θ. This was for the theta component, and for the r component, here we had done. That from the understanding of the problem, v_r, v_z is 0, and del v_Θ del theta is 0, because it is laminar, and since again it is laminar, we can take it to be a pseudo-steady state. That is why, at one point, the rate of change of some velocity component with time we have taken to be 0, right.



So, from the Navier-Stokes equation, the first r component, we get this: ρ (rho) v_Θ square by r is equal to minus del P del r. This is one equation. The second equation is this: 0 equal to del del r of r 1 by r del del r of r v_Θ, and the third equation is 0 is equal to minus del P del z plus rho g_z, right. Now, we have here only one unknown, right? We have only one unknown. What is that? That is v_Θ.



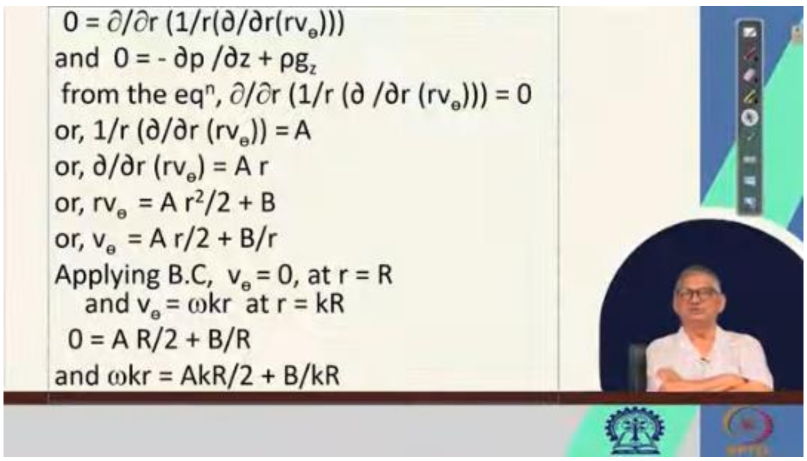
So, to solve one unknown, one equation is good enough. So, any one of these three we can take. But obviously, we would like to take which one? Like del v del z rho g_z. In this, you have no v_Θ term. But in the earlier one, we have one v_Θ term. But that is also not leading to a tangible solution.

Now, the second one we see that. 0 is equal to del del r of 1 by r del del r of r v_Θ. So, this v_Θ we can solve, right. So, if we take this equation as del del r of 1 by r del del r of r v_Θ, this is equal to 0. Then on first integration, we get

1 by r del del r of r v_Θ, that is equal to a constant of integration, that is, A, right. And on simplification, we can write it to be 1 del del r of r v_Θ is equal to A r, right. So,

on second integration of this equation, we get r v_Θ is equal to A r square by 2 plus B, right, where B is the integration constant. So, del del r of r v_Θ is equal to A r on integration it gives, r v_Θ is equal to A r square by 2 plus B.

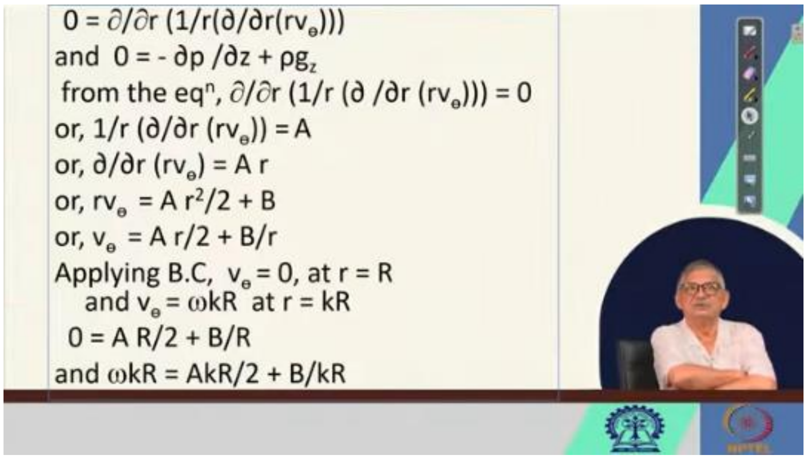
On simplification, we get v_Θ, because r, if we divide both sides, then v_Θ is equal to A r by 2 plus B by r, right. So, we have this integration solution, but we do not know the constants A and B. So, we have to find out the constants A and B, for that, there are two constants. So, we need two boundaries. So, two boundaries are, one at the boundary at r is equal to capital R,

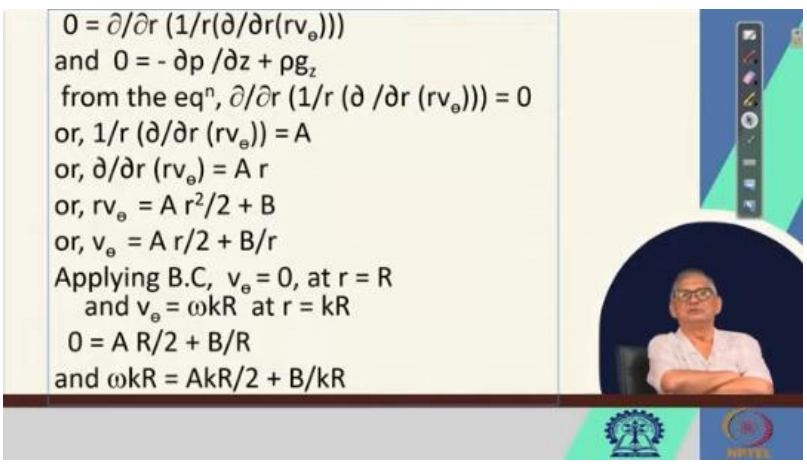


right, and, at r is equal to capital R the boundary is v_Θ is 0, because at r the cylinder is constant or fixed, not rotating, it is not moving, it is not rotating, it is fixed as it is said in the problem that it is fixed. So, boundary condition becomes v_Θ is 0 at r is equal to capital R. And the second boundary, which is given that the outer cylinder is rotating with an angular velocity of omega. So, that we can write as v_Θ is equal to omega K R, this is this is capital R right omega K R, at r is equal to K capital R, right.

This is capital R; I should make this change here itself, otherwise, you will be in confusion, right. So, this is r is equal to capital K R, ok. Now, omega K R at r is equal to K capital R. So, we have two boundaries and two unknowns, A and B. Solving it, we get both A and B. To solve it, we first take that v_Θ is equal to 0. At r is equal to capital R, right. So, if we take that equation as a solution, then v_Θ is 0 at r is equal to capital R. So, A r by 2 plus B by capital R is one solution.

And the second boundary is v_Θ is omega K R. At r is equal to K R. So, by substituting the value of v_Θ, we get omega K R, again it is capital R. Actually, the problem arises when we do this cut and paste of similar equations; that becomes a problem, ok. So, omega K R equals, by substituting the values of v_θ, we get omega KR is equal to A KR by 2 plus B by K R, right. So, we have two equations and two unknowns. A and B in terms of r, right.





So, if that is true, then we solve it like this: 0 equals to A K R by 2 plus B by K R, right, or we can take A K R by 2 is equal to minus B K by R. This, on simplification of the first boundary, and the second boundary is again omega K R. Yeah, omega K R equals to A K R by 2 plus B by K R. So, from there, omega K r equals to B by K R minus B K by R, from where, by taking common B, it is becoming 1 by K R minus K by R, right. So, we can write again here; there is a mistake that has to be corrected. So, we have seen B equals to omega K R by 1 by K R minus K by R, that is equal to

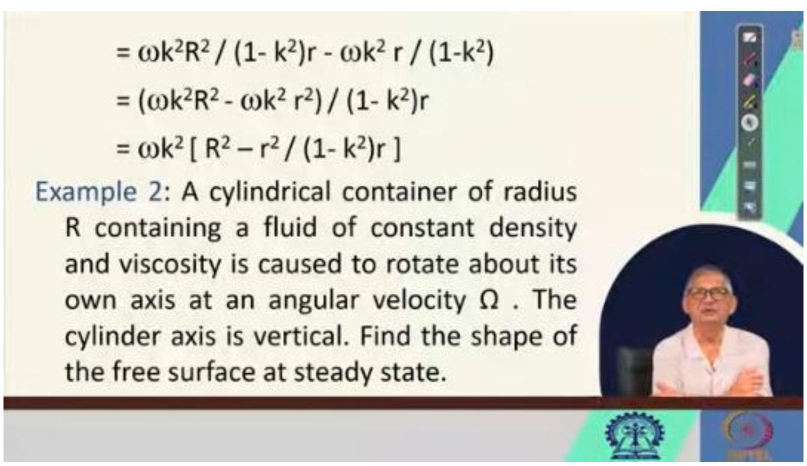
omega K R over 1 minus K square by K R, right. So, that is equal to omega K square R square by 1 minus K square. So, B becomes equal to omega K square R square divided by 1 minus K square. Similarly, we can write from the other equation that omega K R is equal to A K R by 2 plus B by K R. So, we can write 0 equals to AR by 2 K plus B by K R; that was our solution.

So, now, substituting omega, A K R by 2 minus A R by 2 k. So, you have substituted the value of B, right. So, it becomes A R common divided by 2 into K minus 1 by K. Therefore, A is 2 omega K over K minus 1 by K. So, you have found out both B in terms of omega and R and K and also A in terms of omega K, right. So, substituting the values of A and B, we can write v_Θ is equal to

Omega Kr, v_Θ is equal to omega Kr divided by K minus 1 by K plus omega K square R square, right, over 1 minus K square into small r. We go back to what was the previous one. So, v_Θ is equal to A r by 2 plus B by r. So, ultimately it was v_Θ is equal to, by substituting the boundaries, we get this, and now we have got A and B. So, A is 2 omega K divided by K minus 1 into K.

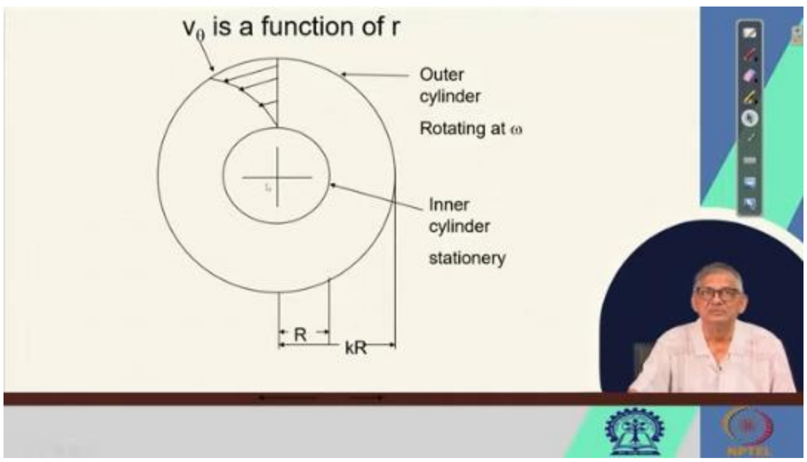
And, B is equal to omega K square R square divided by 1 minus K square, right. So, v_Θ is equal to omega K square r over K square minus 1 plus omega K square R square divided by 1 minus K square into r. So, we get omega K square into r into K square minus 1 plus omega K square capital R square divided by 1 minus K square into r, right; this r is small. So, the second part is plus omega K square R square divided by 1 minus K square into r. So, on rearrangement,

we get omega K square into capital R square minus small r square divided by 1 minus K square into r, right. This r is small because r remains. Why r remains? We go back to r v_Θ, v_Θ is A r by 2 plus B by r, right. We have substituted the values of A and B in terms of omega and K, right. This is nothing but a simple



Algebra, right. So, we have v_Θ equals omega K squared multiplied by R squared minus r squared, divided by 1 minus K squared multiplied by r. So, if we know the value of omega, what is the angular velocity? If we know the value of K and if we know the value of capital R, which is the outer cylinder radius, then we can say that yes, we can find out v_Θ, right? So, this was one problem. So, another problem, but again, before going into this problem, because these are the two problems which we will be doing. These are the two problems because here you see we have

Where is that? We have taken this, OK? We have taken this as constant and this as variable, right. We have taken this as R inner, this as K R outer, and we found out what is the value of v_Θ in terms of K, capital R, and small r. Right, because both K is equal to how much we know, capital R is equal to how much we know, and where we want to find out because v_Θ is a function of R, that we have said here. v_Θ is a function of R.



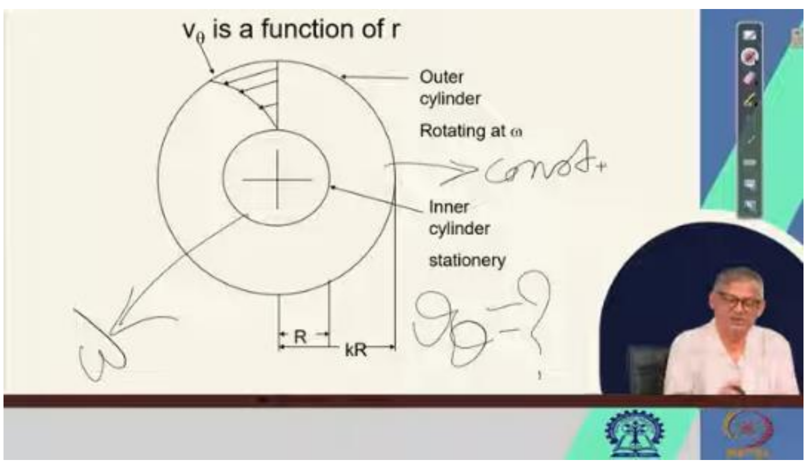
If that be, then we would like to know at what r, meaning this is one r, this is another r. So, at what r we want to know the value of v_Θ? So, we need to know the value of r. So, if we know capital R, K, and small r value, then we find out what is the value of v_Θ. Why did I come back to this? Because I would like to tell you and give you an assignment. Of course, assignments will be given to you by the technical assistant people or students, right?

But for them, it is very difficult to give you such a problem. The way the course is assigned, that the weekend our this thing is not so easy. Weekend our giving assignments are not so easy so big because this will take some time. So, that cannot be given with a number of 2 because your in-term your that weekend assignments, they will be given 10 assignments each of on question number or each of having

The power of the question is 2, right. So, we will not be able to do it. So, that is why I would like to give you a parallel problem which you do on your own. If you are not able to do it, we will try to solve it in the class. That is, instead of saying that this is moving with an angular velocity omega, if

constant, now if we make it that this is moving with the angular velocity omega and this was constant, right. If that be the case, the same this our v_Θ, all other remaining same, we are just changing the situation. One is this inner one was said to be constant, outer one was said to be varying with an angular velocity of omega, right. We got a solution of v_Θ, whatever value we got a value of v_Θ in terms of capital R, small r & K, right.

And obviously, it is a function of r. So, with r also this we got. Now, we change the problem, we change for you as the assignment that you please find out the velocity profile that is v_Θ what is the value of v_Θ when the inner one is moving with an angular velocity omega and the outer one is constant, right, just the reverse. Inner one is moving with an angular velocity omega and outer one is constant, then what is the velocity? or angular velocity v_Θ, keeping you may keep this as r and k r, you may keep as r 1 and r 2, whatever you like, that does not matter, but the condition is there.



The inner one is fixed with an angular inner one is fixed and the outer one was moving with an angular velocity of omega, this time it is not. So, this time it is inner one is moving with the angular velocity omega and the outer one is fixed, outer one is fixed with fixed, rather not with outer one is not moving, right. So, this problem you please solve and come back to us if you can, if you cannot, obviously, we will try to do it here, right. So, with

We can say I have given you this assignment where we have solved it with the inner one fixed and the outer one moving. However, I have given you an assignment where the inner one is moving with an angular velocity omega, but the outer one is not moving; it is fixed. What is the velocity or angular velocity, or what is the tangential velocity, which is v_Θ? You find out, okay. Thank you.