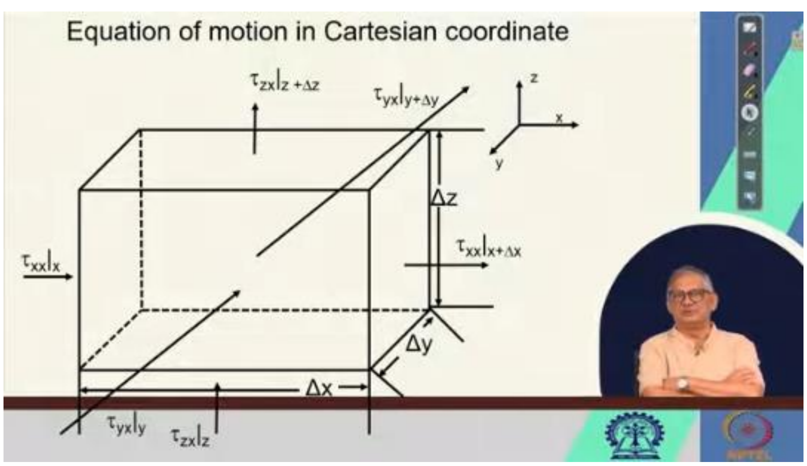
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture14

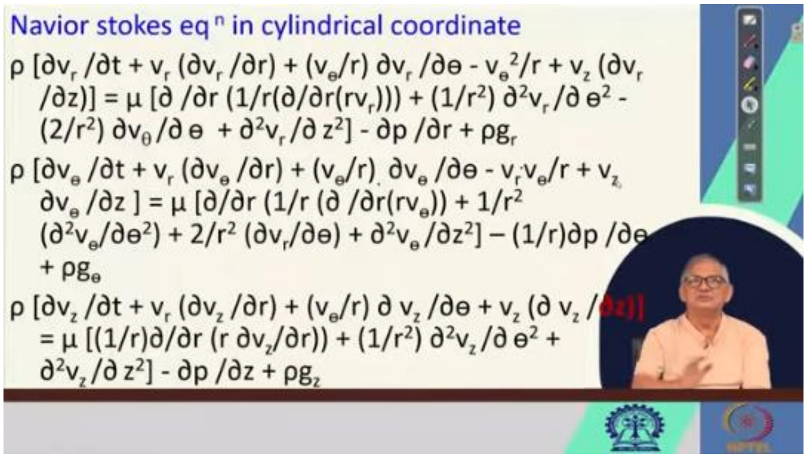
LECTURE 14 : PROBLEMS AND SOLUTIONS WITH THE HELP OF NAVIER STOKES EQUATION

Good morning, my dear students and friends. For the last many classes, we have been handling the equation of motion, right? Its derivation in the Cartesian coordinate, we have done. We are now dealing with the equation of motion in the cylindrical coordinate, that is, r Θ z, (r, theta, z).

Everywhere we are giving this volume element because otherwise, it will be out of place from our mind. That our volume element is like this, this was for Cartesian coordinate. Obviously, for cylindrical coordinate, it will be quite different, right? Which we have shown in your equation of motion, equation of continuity development, right? And there we have seen how different it is from Cartesian coordinate, right. So, what we have arrived at till now is that Navier-Stokes equations for the Cartesian coordinates, Navier-Stokes equations for the cylindrical coordinates, right.



And we were discussing the individual terms, what they physically mean, right. So, if we go back to that. This was our Navier-Stokes equation in cylindrical coordinate, right. So, we were discussing individual terms like what is del vr del t, right? This is a differential term. So, it implies that what is the rate of change of v_r, that is, the velocity component



in the direction r, v_r, what is the rate of change of velocity v_r with respect to time Similarly, obviously, v_r does not need to be explained because v_r is the velocity component in the r direction. And the next differential form is del v_r del r, meaning what is the change of velocity v_r in the direction. r, right? Plus, we have seen some term which are v_Θ by r, v_Θ is the velocity component in the Θ direction, right. And v_Θ by r is a number, and then the differential that is del del Θ of v_r, meaning

What is the rate of change of v_r with respect to Θ, right? Then v_Θ square by r is Θ square is known, that is the velocity component in the Θ direction, r is the radius, right. Then this is plus v_z, the velocity component in the z direction, times del del z of v_r. Again, what is the rate of change of v_r with respect to z, right?

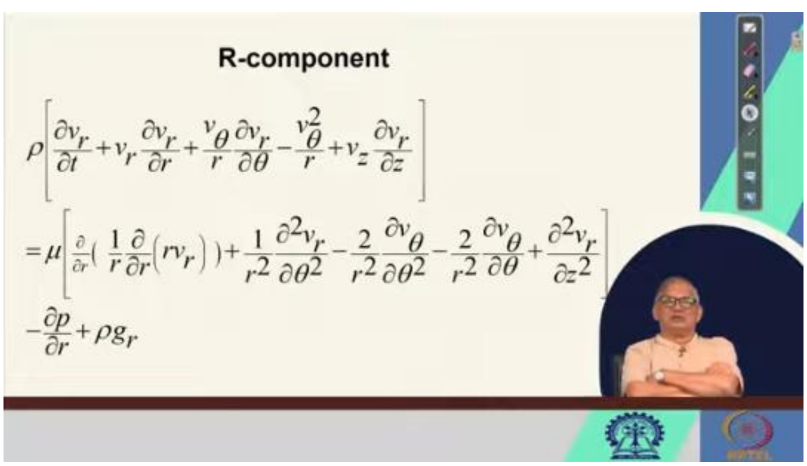
This is equal to mu times del del r of 1 by r del del r of r v_r. So, many derivatives are there, it is a double derivative. So, mu times, obviously, that is viscosity, we have said earlier, and this is with respect to that, what is the rate of change of 1 by r del del r of r v_r, right? r v_r is a number, v_r is known, the velocity component in the r direction, r is the radius, known, right. What is the rate of change of, what is the rate of change of r v_r in the r direction times 1 by r, this is 1, and that is, what is the rate of change of this parameter with respect to r is del del r of 1 by r del del r of r v_r, right.

1 by r square del 2 v_r del Θ square, 1 by r square is a number. So, del 2 v_r del Θ square again, it is nothing but we have shown earlier that, this is del del r, del del

Θ of del v_r del Θ, right, that is, what is the rate of change of the first one is del v_Θ del r del v_r del Θ, that is, what is the rate of change of vr with respect to v_Θ, right, and again, this parameter, what is the rate of change of this parameter with respect to Θ, that is the double derivative of v_r v_Θ, ok. Similarly, 2 by r square del v_Θ del Θ, 2 by r square is the number.

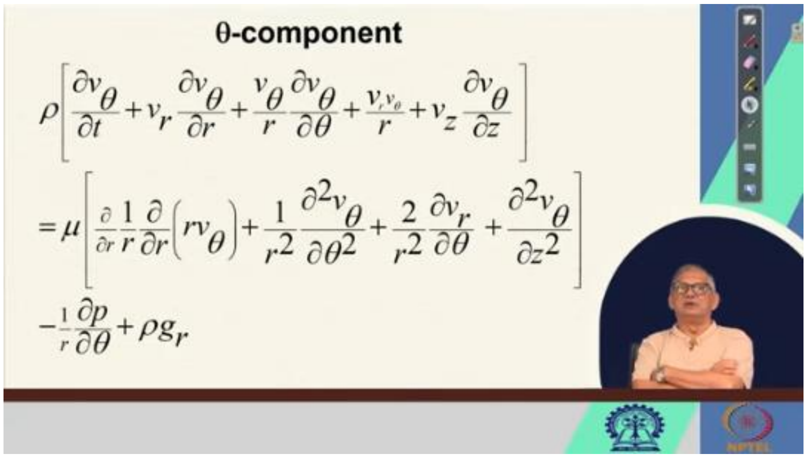
So, del v Θ del Θ meaning that, what is the rate of change of Θ with respect to v_Θ. And, del to v_r del z square means, what is the rate of change of v_r with respect to z, and again, this parameter, what is the rate of change of this parameter with respect to r, once more. right, and del p del r is the pressure term, and rho g_r is the gravitational term. Similarly, all other terms in the Θ component and z components are there, and this can be explained in this fashion in a similar way, right. Now, we go to the

It can be better written like the r component of velocity in Cartesian rather than cylindrical coordinates, and this is known as the Navier-Stokes equation. So, in the Navier-Stokes equation, what is the r component? We have written clearly here: rho times del vr del t plus v_r del v_r del r plus v_Θ square by r del v_r del Θ minus v_Θ square by r plus v_z del v_r del z equals mu times del v_r. Del del r of 1 by r del del r of r v_r, right, plus 1 by r square del 2 v_r. Del Θ square minus 2 by r square del v_Θ del Θ square minus 2 by r square del v_Θ del Θ plus del 2 v_r del z square minus del p del r plus del g_r. This is the r component.

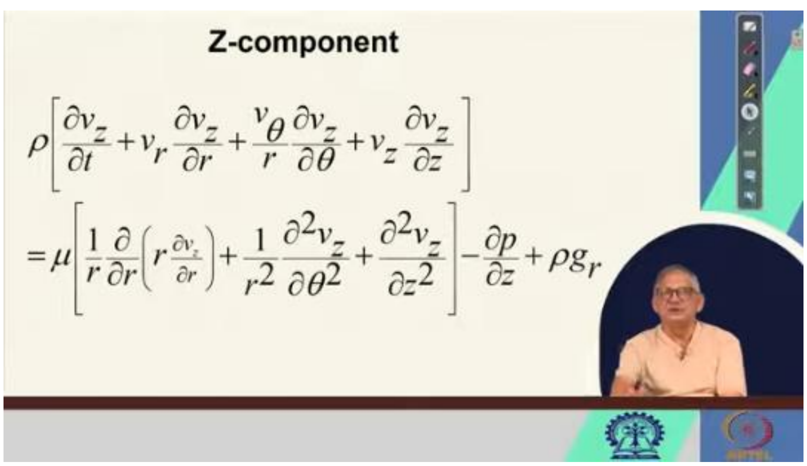


Viscosity velocity is known as the Navier-Stokes equation in the r component of velocity. Similarly, the Θ component can be said as rho del v_Θ del t plus v_r del v_Θ del r, v_r times del v_Θ del r. Plus v_Θ by r times del v_Θ del Θ plus v_r v_Θ by r plus v_z del v_Θ del z equals mu times del del r of 1 by r del del r of r v_Θ plus. 1 by r square del

2 v_Θ del Θ square plus 2 by r square del v_r del Θ plus del 2 v_Θ del z square is minus rather 1 by r del p del Θ plus (rho) ρ g_r. So, this is the

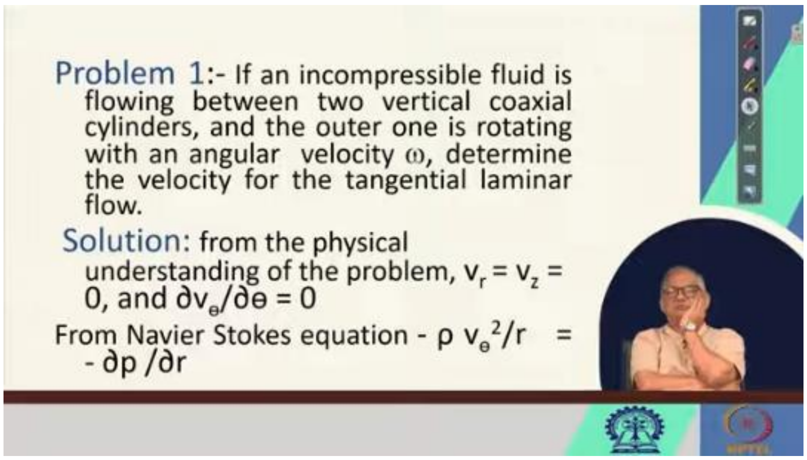


Θ component of the velocity in terms of the Navier-Stokes equation. And the z component can be said to be rho times del v_z del t plus v_r del v_z del r plus v_Θ by r del v_z del r or rather del v_z del Θ. Plus v_z del v_z del z equals mu times 1 by r del del r of r del v_z del r plus 1 by r square del 2 v_z del del Θ square plus del 2 v_z del z square minus del p del z. ρ g_r is the z component of momentum according to the Navier-Stokes equation in the cylindrical coordinate r, Θ, z, right. I have not given spherical coordinates because that is even more complicated.



That is even more complicated, which is why I have not given it here, and in most cases, in all kinds of problem solutions, we do not need it so much, right. If we look at a problem using the Navier-Stokes equation solution, we can do this in this class by first understanding and then solving it. What is given? It is given that if an incompressible fluid is mentioned, the moment it is said incompressible fluid should strike in your head that Incompressible means the density is constant; density is not variable.

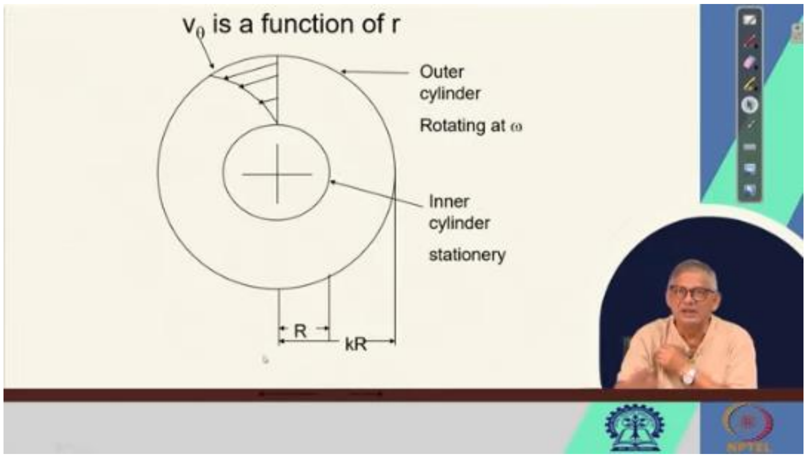
So, if an incompressible fluid is flowing between two vertical coaxial cylinders. So, two vertical coaxial cylinders like this is a vertical coaxial cylinder and say this is another vertical coaxial cylinder, right. Coaxial means the same axis, right? Vertical we know this is vertical. Horizontal would have been this, right, like this one and this one another. So, but it is vertical.



So, it is like this. Similarly, we can say we can tell the problem. If an incompressible fluid is flowing between two vertical coaxial cylinders and the outer is rotating with an angular velocity omega. Determine the velocity for the tangential laminar flow, right.

I repeat, if an incompressible fluid is flowing within two vertical coaxial cylinders and the outer one is rotating. with an angular velocity omega, determine the velocity for the tangential laminar flow, right. Had it been not tangential, but the, but the, what we say, normal tangential laminar flow, instead of that if we would have said normal laminar flow, then it would have been v_r, but here we have said tangential laminar flow. So, we need to know v_Θ. How?

Like this, the problem can be described in terms of a pictorial view, right. We have two coaxial cylinders, right, we have two coaxial cylinders. So, one is this coaxial cylinder and another is this coaxial cylinder, right. We are told that the outer one is rotating with an angular velocity of omega. So, which one is the outer one?



The outer one is this one. It is rotating. This one is rotating with an angular velocity of omega, right? As this is said, right. We are asked what is the tangential laminar velocity to find out which one is that. So, our coaxiality is this. This is the axis, this is the first radius, and this is the second radius, ok. If this is termed as the first radius R, this can be termed as a multiple of R, say K R. So, in this case, definitely from the

figure, it appears that R is less than KR. That means K is greater than 1, right. So, we can say that we have to find out the velocity component with Θ, right. So, we have to find out the velocity component with respect to Θ. So, from the physical understanding of the problem, what do you understand? We have seen in the Navier-Stokes equation, there are many, many, many terms like the velocity components v_r, v_Θ, v_z, right, then the derivatives del v_r, del z, etc.

So, from the physical understanding, what is the physical understanding? That the inner radius, the inner cylinder is stable, constant, not rotating, but the outer cylinder is rotating. With an angular velocity, and the fluid is in between them, right? This we have shown, ok. Again, I show once more so that it becomes easier. This is the two cylinders, coaxial cylinders, right. The outer cylinder is rotating with an angular

with an angular velocity, sorry, with an angular velocity of omega, right. So, we can say that v_r is v_z is equal to 0. We go back to the initial whether the internal this one is fixed and it is laminar. So, the layers of fluid are flowing like this, right? Layers of fluid are flowing like, No, no, no cross, no cross, because if it is crossing, that means there is some turbulence.

So, all the layers like this, like this, like this, like this, as close or as good as you can make it up, ok. So, they are rotating with an angular velocity of omega, right? We then can say that from the physical understanding, we say vr is equal to v_Θ v_r. What is that? That velocity component in the r direction. What is that?

This is the r direction. So, the velocity component in the r direction is not there because these are all in the laminar region, right? So, it is not that this velocity is getting changed, right? Had it been that this velocity here is getting changed, then it would not have been laminar. So, we come to that this is a laminar flow and velocity component of v_r and v_Θ and v_z, there is no vertical, right? Had it been like this, sorry, had it been like this, that cylinder. So, we do not have any z component.

Again, had it been a z component profile, then there would have been mixing of the liquid, which we are not saying it is under laminar condition, right. So, We can say that the velocity components v_r and v_z are equal to 0. Right, that is what we have said that from the understanding of the problem, v_r is equal to v_z is equal to 0, and also del v_Θ del Θ. What is that del v_Θ del Θ?

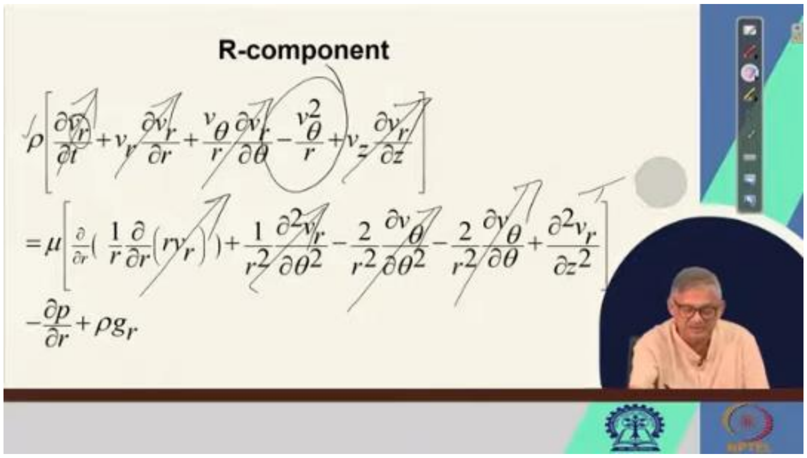
That del v_Θ del Θ means the rate of change of Θ with respect to Θ or the rate of change of v_Θ with respect to Θ. Now, Θ means it is moving like this, right? It is moving like this. So, this is the Θ. Is there any change in v_Θ, v_Θ with respect to Θ? Whatever it was here is also there, right? There is no change of velocity v_Θ. So, v_Θ is not changing with the velocity v_Θ.

So that means, v_Θ is a del v_Θ del Θ is 0, right? So, with this del v_Θ del Θ is 0, v_r is v_z equals to 0. So, we can proceed to the Navier-Stokes equation. What is that Navier-Stokes equation? If we take the r component,

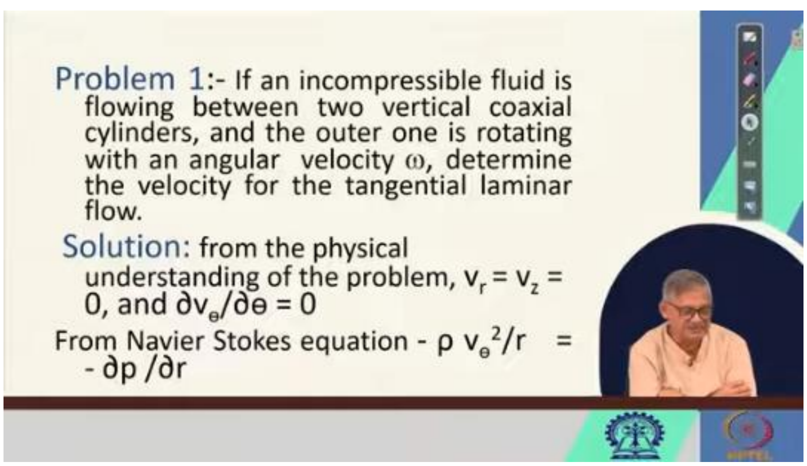
If we take the r component, the first one here it is said that rho we have not said anything, but we have said v_r is equal to 0. If v_r equals to 0, then what we get? v_r is equal to 0 that means, this term is out, v_Θ by r del v_r del v_Θ square by r we have not said, right, minus v_Θ square by r is there because v_Θ is 0 or not, we do not know.

So, this is also 0, right? So, this is also 0, this is also del 2 v_r del Θ square, here is 0. So, this is also 0 and 2 by v square 2 by r square del v $_\Theta$ del Θ square we already said del del Θ of v_Θ is 0. So, del 2 v_Θ del Θ square is also 0 and this is also 0 and del v_r del z square is also 0.

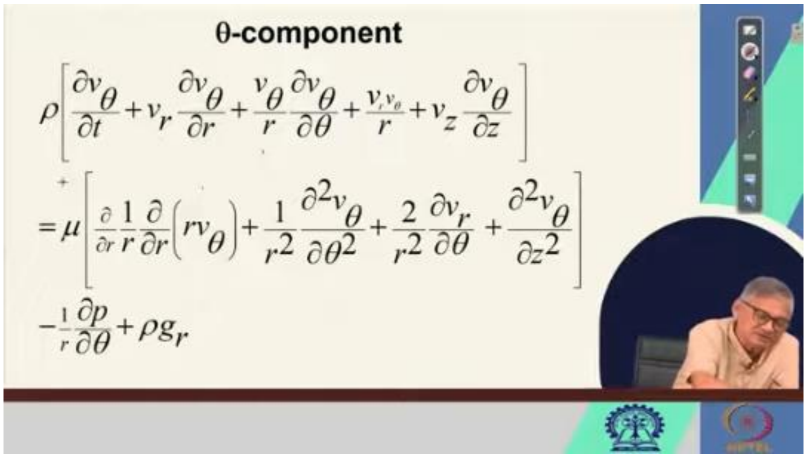
So, we get ultimately what? Ultimately we get from this in the Navier-Stokes equation 1 as this, right. We get this that minus rho v_Θ square by r is equal to minus del P del r. We are neglecting here gr because that is not very high. So, g_r we are neglecting.



Then, from the first equation of the Navier-Stokes equation, we get minus rho v_Θ square by r is equal to del P minus del P del r. This is the first equation. Next, we get, from the second equation, again we go back to the Θ component. In the Θ component, again we see whatever we have been said from there: del v_Θ del t, right, del v_Θ del t. Since it is a laminar flow, and we can assume it to be a quasi-steady state.

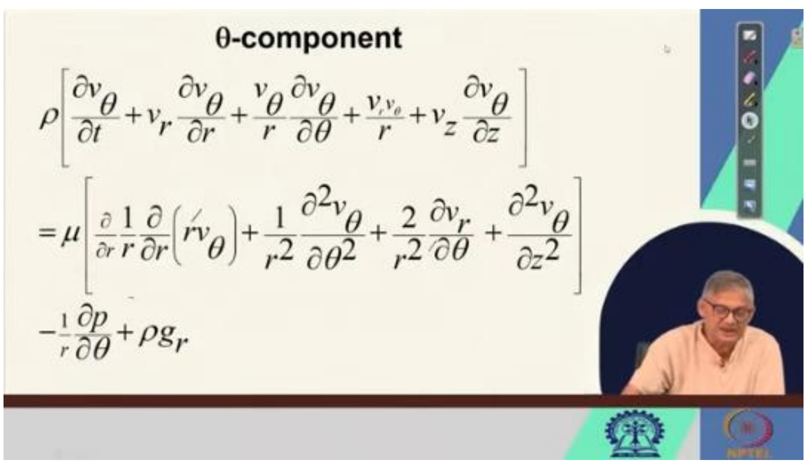


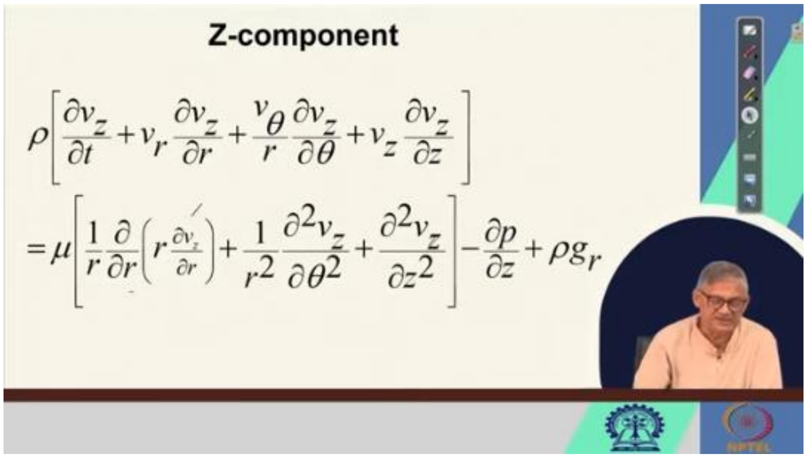
So, del v_Θ del t is 0, right, here is 0, this is 0, then del v_Θ del Θ, already we said to be 0. Then, v_r v_Θ by r, v_r is 0, so this is 0. v_z is also 0, so this is 0. Then, you have mu times del del r of 1 by r. del del r of r v_Θ here, this is OK, but here v_Θ del del Θ of v_Θ is 0.



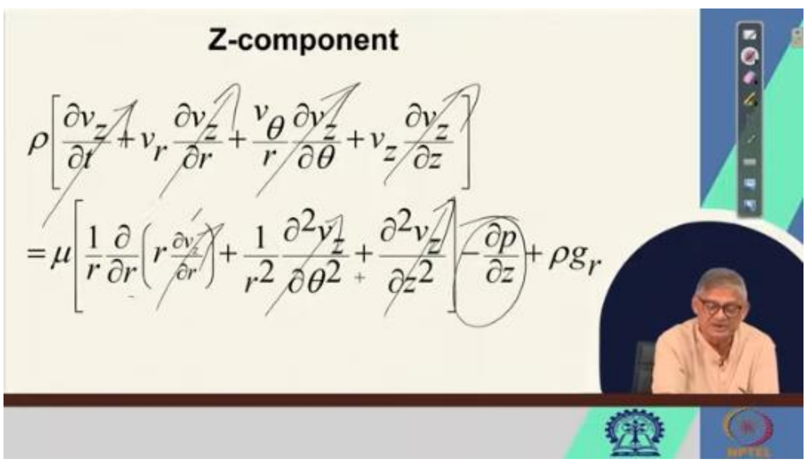
So, this is normal derivative is also 0, v_r is also 0, right. Then, what do we get? We get del 2 v_Θ del z square to be equal to that del 2 v_Θ del z square, yeah, this is and del P del Θ is there, (rho) ρg_r we are eliminating. So, we get that

that equation, the second one is 0 is equal to del del r of 1 by r del del r of r v_Θ, right. And the third one, that is Z component, here we get again from del v_z del z del v_z is 0. So, this is 0, this is 0, here both del v_z del r and v_r are 0.





So, del v_z del Θ is 0, del v_z del z is also 0, mu 1 by r del del r of r del v_z del r del v_z del r means whatever. The rate of change of r with the rate of change of v_r v_z with respect to r, v_z is 0. So, this is also 0, del 2 v_z del Θ square is 0, del 2 v_z del z square is also 0. So, we get minus del p del z plus rho g_r.

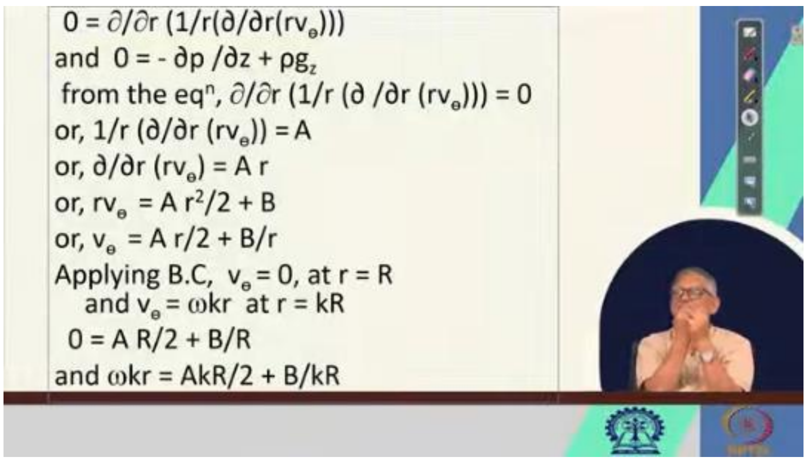


Again, we will try not to take the gravitational terms because they are not significant in a low area, small height, or small system, ok. Then we get The third one is 0 equals to minus del P del z plus rho gr. So, from the three equations, understanding the problem, we have come to identify the terms. That is the most fundamental, identifying the terms.

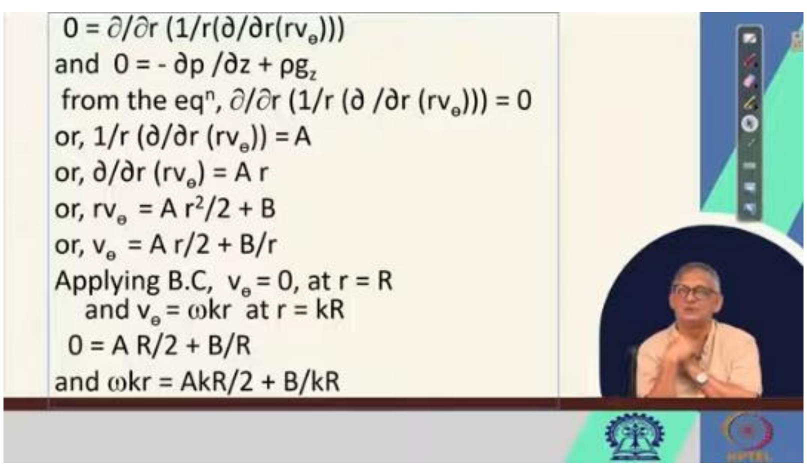
I am not saying that you memorize the equation of continuity; it is very difficult. At least you can refer to it, right, and understand the terminologies. We are said in this that v_r and v_z equal to 0. And also, we have said that there is no del v_Θ del Θ, that is no change of Θ v_Θ with respect to Θ, right. So, in this case, wherever vr's were there either directly or in the differential form, all got 0.

Wherever v_z were there either directly or in the form of derivative, all were 0. Only v_Θ is not said to be 0. But anything with respect to v_Θ having v_r or v_z is also 0. And we have been said that del v_Θ del Θ is also equal to 0. So, from this information, understanding the problem, we have given the drawing, and we have found out three equations.

One for the r component, one for the Θ component, and one for the z component. So, v_r, v_Θ, v_z, we have three equations. We have one unknown, which is v_Θ. So, one equation is good enough to solve it. So, out of these three, obviously, we will choose that



which will lead to a solution, isn't it? It is not that we will arbitrarily try all the equations and find a solution. That will not help because if that is done, then we will have to do trial and error again and again. Obviously, we will pick the one which is the best solution, okay? So, today, now this time is up.



So, thank you all for joining this class. We will solve the problem again in the next class. So, this is the application of the Navier-Stokes equation. How you are applying it to a given problem, right? Thank you very much.