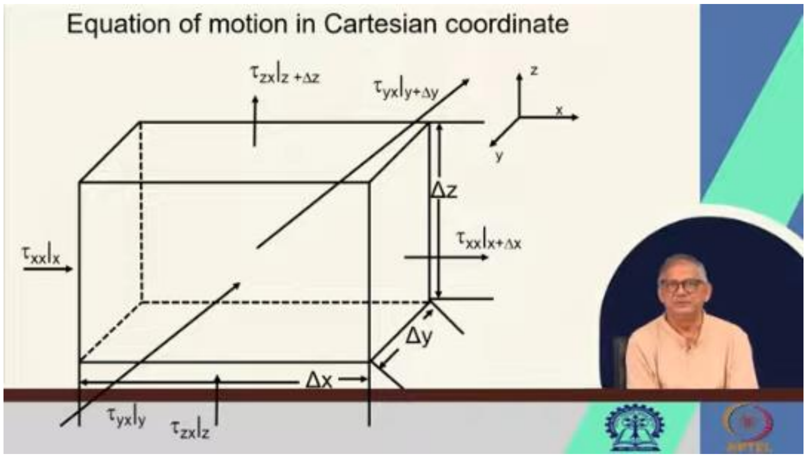
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture13

LECTURE 13 : NAVIER STOKES EQUATIONS TO BE CONTINUED

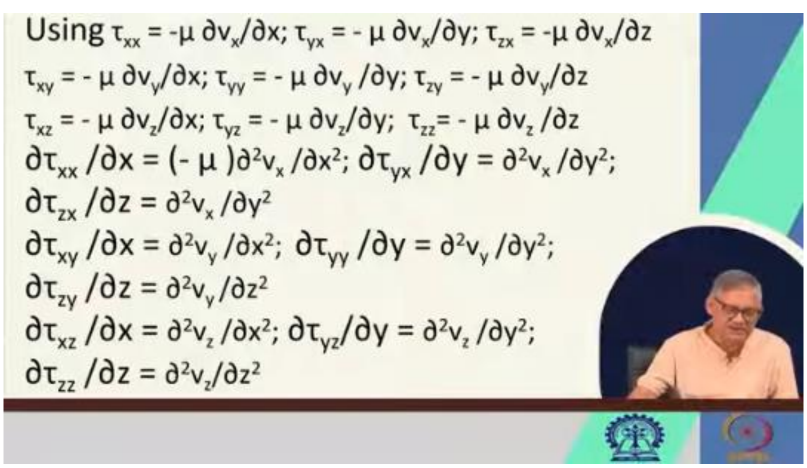
Good morning, my dear students and friends. We are continuing with the derivation of the equation of motion for the flow of fluids, right. Because this equation of motion is considered to be one of the vital equations in the flow of fluids, right. So, again, we come back to the original pictorial view of the Cartesian coordinate volume element.



So, on this volume element, we have worked, and based on the transfer of momentum, we are repeating that we said there are two types of momentum transfer: one is the bulk transfer, and the other is the molecular transfer, right. And bulk transfer, we have said to be (rho) ρ v_x v_x or rho times v, which is momentum transfer by bulk and by molecular transport, we have said it to be τ (tau), τ_{xx}, τ_{xy}, etc. We also defined this momentum transfer τ's in terms of viscosity, like τ_{xx}, we have defined as minus mu del del x of v_x, right, or del v_x del x, which is the τ_{xx}, we have defined. Similarly, τ_{yx}, τ_{zx}, τ_{xz}, τ_{xy}, τ_{yz}, everything, we have defined in terms of mu. And then we substituted them into the equations. If you remember, we had given three names to the equations, three numbers rather: one was for the x-component momentum, we had given equation 1. For the y-component momentum, we had given equation 2, and for the z-component momentum, we had given equation number 3, right.

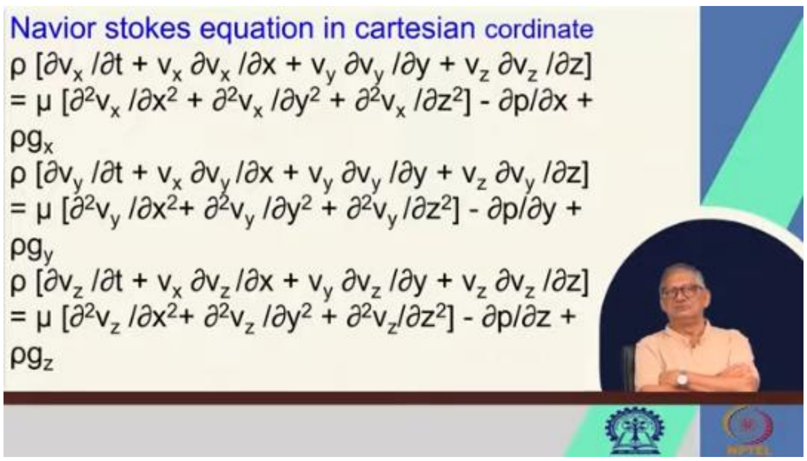
So, if we remember all these and ultimately, we arrived at substituting the values of τ's and doing some rearrangements. We arrived at the equations. And these equations, we said to be the well-known fluid flow equations, and maybe one of the pioneer equations. In the name of the Navier-Stokes equation. Anywhere, in any book, anything, you will get these Navier-Stokes equations, right.

So, we go back to that. We get that here. We finished in the last class, we substituted all the τ's in terms of viscosity, and now from there, we come to the expanded form of Navier-Stokes equation in the Cartesian coordinates, right. So, Navier-Stokes equations in the Cartesian coordinates we can write to be rho times del v_x del t plus v_x del v_x del x or del del x of v_x, you can also say plus v_y del del y of v_y plus v_z del z of v_z, this is equal to mu,



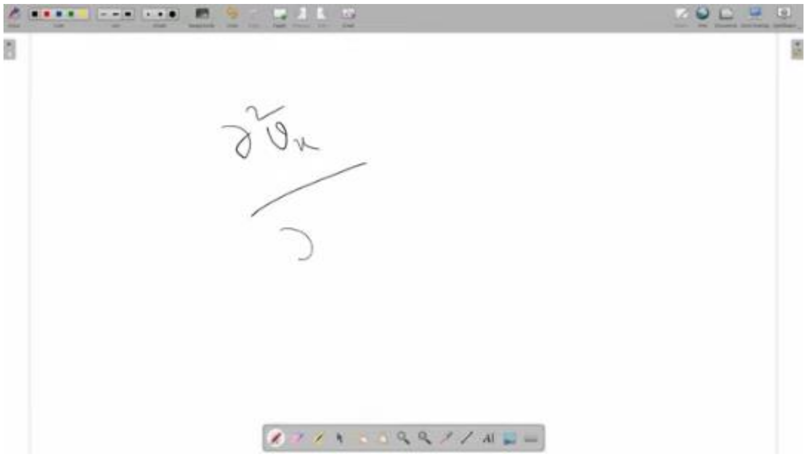
that has become common and the negative sign has disappeared because here we had one negative and with mu also, we faced one more negative. So, those two canceled out. So, mu del 2 v_x del x square plus del 2 v_x del y square plus del 2 v_x del z square minus del p del x plus rho g_x, right. So, here you see there is a similarity to remember, right.

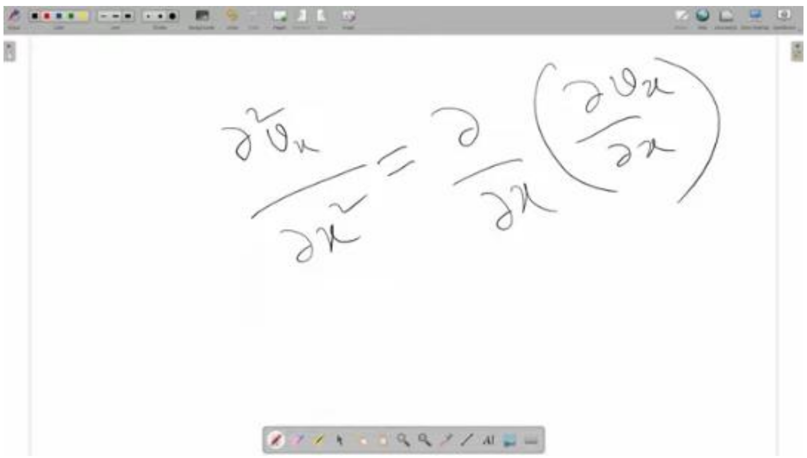
The equation starts with rho times del v_x del t plus v_x del v_x v_x rather del v_x del x, right here it is nothing, but the velocity component in the x direction, y direction and z direction. So, del vx del t that was the velocity component in the x direction. If you remember, in the beginning we had said, partial time derivative, right.



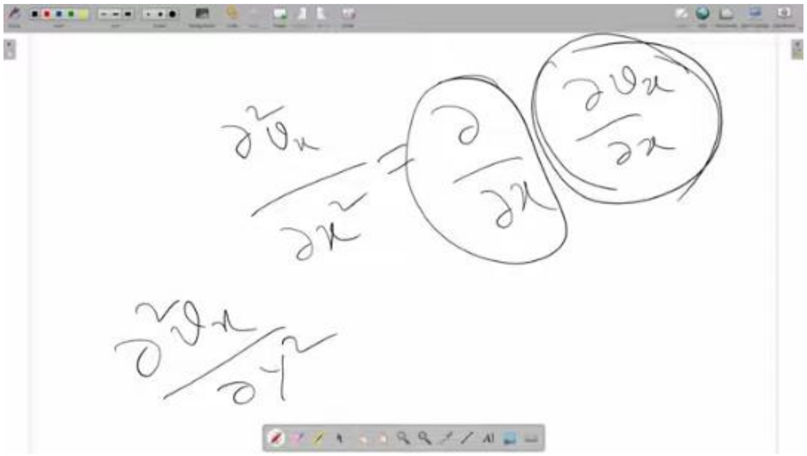
So, this is partial time derivative of velocity v_x, right. So, rho times del v_x del t plus v_x times del v_x del x plus v_y times del v_y del y plus v_z times del v_z del z and this is equal to mu that is the viscosity of the fluid times del 2 v_x del x square plus del 2 v_x del y square plus del 2 v_x del z square. So, all v_x components in all three directions, right. It is del 2 v_x del x square means, it is a double derivative, right. It can be said to be del del x of v_x, right, del del x of v_x, right.

So, del 2 v_x del x square, this we can write as del del x of del del x of v_x or del v_x del x, right? This is what the double derivative is, right. So, del del x of del v_x del x, this is for del 2 v_x del x square. Similarly, if we write del 2 v_x del y square,



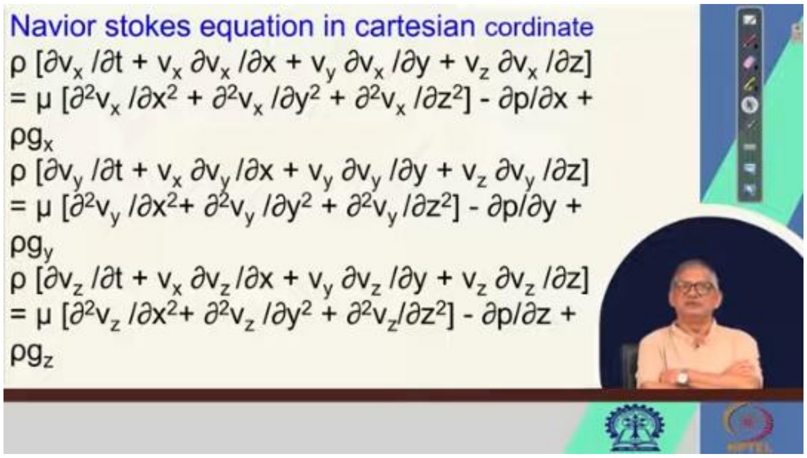


then what we can write is this is equal to del del y of del v_x del y, right. So, it is a double derivative, first with y and then with v_x, right, del del y of del v_x del x. It is said that the first equation is in terms of the v_x component, and that is rho times del v_x del t plus v_x del v_x del x plus v_y times del v_y del y plus v_z times del v_z del z, right. This is equal to mu del 2 v_x del x square plus del 2 v_y del y square, sorry, rather del 2 v_x del y square plus del 2 v_x del z square, minus del p del x plus rho g_x. This is the x component of momentum. The y component of momentum can be written as rho times del v_y del t. You see, in the first case, it was del v_x del t, now it is del v_y del t,



right. And other terms, you see in the first case, it was v_x times del v_x del x, now it is plus v_x times del v_y del x, right, because it is the y component of momentum, right, plus v_y as it was, but it is del v_y times del v_y del y, right. Plus v_z times del v_y del z. del v_x times del v_x del x plus v_y times del v_x del y, it should be corrected. So, it is then the first term for the v_x component is rho times del v_x del t plus v_x times del v_x del x plus v_y times del v_x del y plus v_z times del v_x del z.

This is equal to mu del 2 v_x del x square plus del 2 v_x del y square plus del 2 v_x del z square minus del p del x plus rho g_x. It is the first equation. For the y component, it was, it is rather, rho times del vy del t plus, you see that the changes from the x component to the y component, the x component, it was del v_x del t, now for the y component, it is del v_y del t, right, plus v_x remains the same, it was del v_x del x, it is del v_y del x, right, because our velocity component is v_y. So, this is also plus v_y, v_y remains the same, it was del v_x del y, it is now del v_y del y, and it was v_z plus v_z, it was del v_x del z, it is del v_y del z. It is equal to mu del 2 v_y del x square plus del 2 v_y del y square plus del 2 v_y del z square minus del p del y plus rho g_y, right. This, whether it is correct or not, you can go back to the previous one,

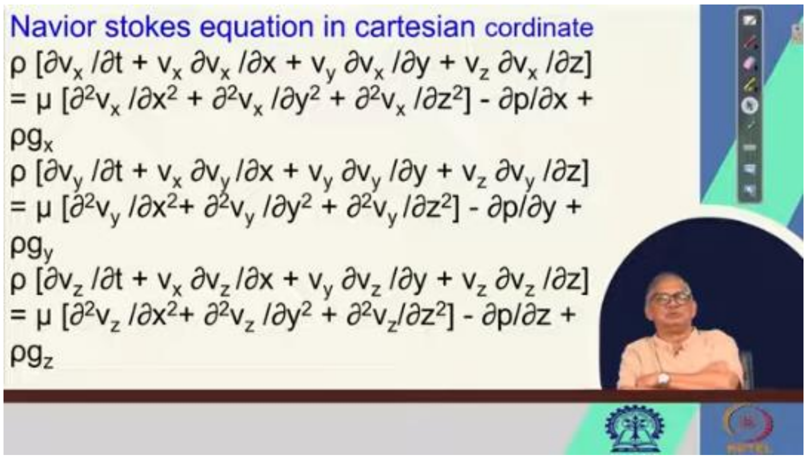


right, where you had substituted mu in terms of τ, right. So, whether these double derivatives are correct or not, you can also have a cross-check. And the third component, that is the v_z component, is like this: that rho into del v_z del t, again, you see it was del v_x del t, or del v_y del t for the x and y components. Now, for the z component, del v_z del t, right, plus it was v_x everywhere you see v_x times first one del v_x del x, second one del v_y del x, third one del v_z del x, right, plus the first x component was v_y times del v_x del y, then for the y component it was del v_y times del v_y del y, and for the z component, it is del v_y times del v_z del y,

right, plus again for the x component it was v_z times del v_x del z, for the y component it was v_z del v_y del z, and now for the z component plus v_z del v_z del z, right, and this is equal to mu del 2 v_z del x square plus del 2 v_z del y square plus del 2 v_z del z square minus del p del z plus rho g_z. So, for the z component, instead of making similarity from the x component and y component to the z component, let us come to the z component only, and that tells, rho times del v_z del t plus v_x del v_z del x plus v_y del v_z del v_z del y. v_z del v_z del z. This is equal to mu del 2 v_z del x square, plus del 2 v_z del y square, plus del 2 v_z del z square minus del p del z plus rho g_z, right. So, these are the

x, y, and z components of momentum, right, in the Cartesian coordinate. So, finally, I read out all the equations; there are similarities in all the equations except the velocity component. So, for the x component of velocity, it is rho times del v_x del t plus v_x del v_x del x plus v_y del v_x del y plus v_z del v_x del z, this is equal to mu del 2 v_x del x square plus del 2 v_x del y square, plus del 2 v_x del z square minus del p del x plus rho g_x is the x component momentum.

Similarly, for the y component momentum, rho times del v_y del t plus v_x del v_y del x plus v_y times del v_y del y plus v_z times del v_y del z is equal to mu del 2 v_y del x square plus del 2 v_y del y square plus del 2 v_y del z square. minus del p del y plus rho g_y, and the third one is for the z component, it is rho times del v_z del t plus v_x del v_z del x plus v_y del v_z del y plus v_z del v_z del z, equal to mu times del 2 v_z del x square plus del 2 v_z del y square plus del 2 v_z del z square minus del p del z plus rho g_z, ok. Now, from the Cartesian coordinate, we go to the Navier-Stokes equation for cylindrical coordinates.



You see, it is much more different from the Cartesian coordinate, right. It is in r Θ z, but even though r Θ z. The terms which we can see are a little more different than the similarity we got in x, y, z components. In Cartesian coordinate, ok. So, here the coordinate system is r Θ z. So, in that case, the first one is the r component, the r velocity component, that is v_r. In that case, we can write rho times del v_r del t plus v_r times del v_r del r plus v_Θ by r del v_r del Θ minus v_Θ square by r plus v_z del v_r del z. So, this is equal to mu times del del r of 1 by r del del r of r v_r, right, You see, mu times del del r of 1 by r del del r of r v_r, right, plus 1 by r square del 2 v_r del Θ square minus 2 by r square del v_Θ plus del 2 v_r del z square minus del p del r plus rho g_r, right. This is for the r component. We will repeat if required, because if time is not permitting, then we will also come to the next class where again we will repeat this.

So that this is kept in mind because, the equation of motion in the form of the Navier-Stokes equation is very important. Subsequently, when we handle these equations for solving problems, we will show that, yes, it is very important, ok. For the Θ component, it is rho times del v_Θ del t plus v_r times del v_Θ del r plus v_Θ by r del v_Θ del Θ minus v_r v_Θ by r plus v_z del v_Θ del z is equal to mu times del del r of 1 by r del del r of r v_Θ plus

1 by r square del 2 v_Θ del Θ square plus 2 by r square del del Θ of v_r plus del 2 v_Θ del z square minus 1 by r del del Θ of p plus rho g_Θ. This is for the Θ component. Similarly, for the z component, it is rho times del del t of v_z plus v_r del del r of v_z plus v_Θ by r times del v_z del Θ plus v_z del v_z del z equal to 1 by r del del r of

rho v_z del r plus 1 by r square del 2 v_z del Θ square plus del 2 v_z del z square minus del v_z plus rho g_z. You see, it is much more different than that of the Cartesian coordinate system, right. Because many new terms, or in a new way, have come up. So, while developing the equation from the cylindrical coordinate, which we did for the equation of continuity. How the difficulty is, OK. That is why we have taken the final form of the equations, which are also very, very

Not complicated, at least terms-wise, many, many terms are there, right. Because we understand that the terms like del del t of v_r, what is, what does it mean? It means the rate of change of vr with time. The rate of change of v_r with time. Similarly, the next term v_r for the r component of momentum.

Similarly, the next one is vr times del vr del r means v_r, the velocity component of r, right, v_r, this is the rate of change of velocity of the r component with respect to the direction r, right, that is what del v_r del r. Similarly, there is a term, which is nothing but a simple term, like v_Θ by r, into again the velocity component v_r, how it is changing with Θ, that is del v_r del Θ, right. Now, obviously, we have come to the end of this class, time is up. So, we will come to the explanation of individual terms in the next class, and till that time, we get acquainted with the terms, OK. So, I thank you all again. We will meet and explain all these terms in the next class.

Thank you.