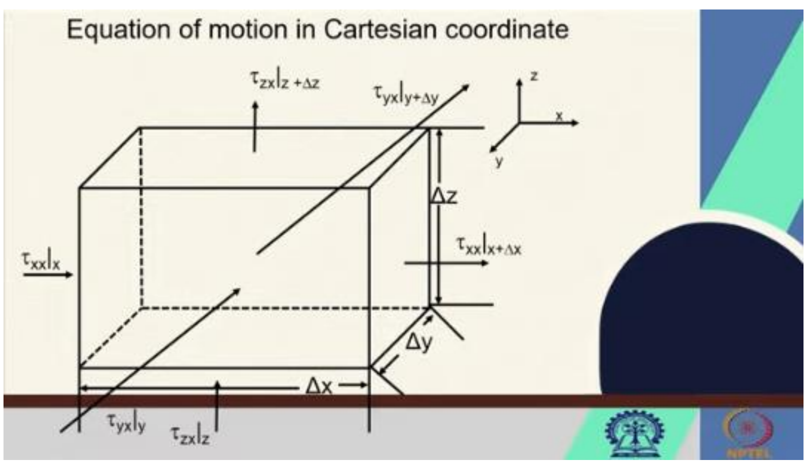
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

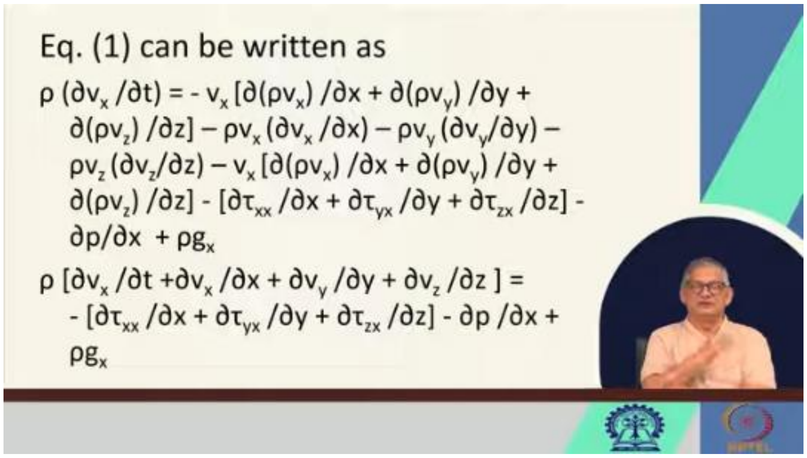
Lecture12

LECTURE 12 : NAVIER STOKES EQUATIONS

Good morning, my dear friends and students. We have done equation of continuity. And now we are doing equation of motion and equation of motion is lengthy one takes time we have said number of classes it will be required. So, we have done expansion of first equation in that is x component that we have done ok, not all, but partially we have done. Now, we will go to that ok and this was our this was our again that this was this was

control volume right and we are expanding, we are taking expansion and rearrangement all these for finding out equation of motion, ok. Now, in the previous class if you remember we had gone up to this, ok. From here we came that rho times del v_x del t plus del v_x del x plus del v_y del y plus del v_z del z. that was equals to minus del del x of tau x_x plus del del y of tau y_x plus del del z of tau z_x minus del p del x plus rho g_x this was x component momentum acting in all directions, right.





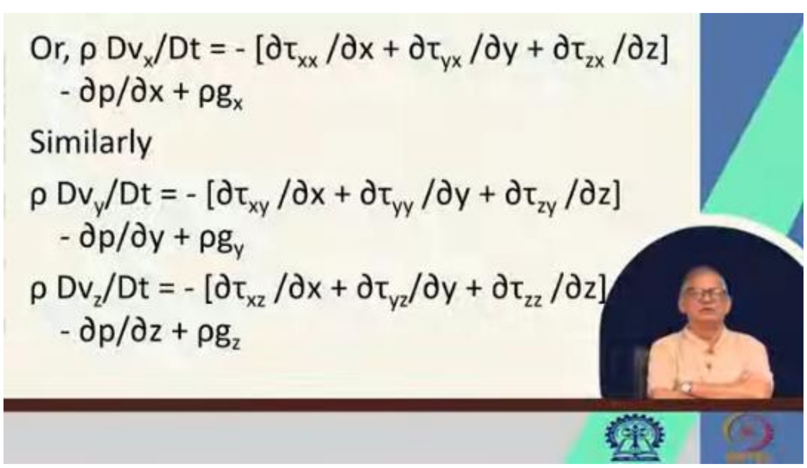
Equating and dividing by $\Delta x\ \Delta y\ \Delta z$

$(\rho v_x\, v_x|_x - \rho v_x\, v_x|_{x+\Delta x})/\Delta x + (\rho v_y\, v_x|_y - \rho v_y\, v_x|_{y+\Delta y})/\Delta y + (\rho v_z\, v_x|_z - \rho v_z\, v_x|_{z+\Delta z})/\Delta z + (\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x})/\Delta x + (\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})/\Delta y + (\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})/\Delta z + (p|_x - p|_{x+\Delta x})/\Delta x + \rho g_x = \partial(\rho v_x)/\partial t$

or,

$\partial(\rho v_x)/\partial t = -\,[\partial(\rho v_x\, v_x)/\partial x + \partial(\rho v_y\, v_x)/\partial y + \partial(\rho v_z\, v_x)/\partial z] - [\partial\tau_{xx}/\partial x + \partial\tau_{yx}/\partial y + \partial\tau_{zx}/\partial z - \partial p/\partial x + \rho g_x$ ……………….. (1)

From there we also came to this point that this is nothing, but rho times that substantial time derivative that is capital D apparent. So, capital D v_x capital Dt is equals to minus of del del x of tau x_x plus del del y of tau y_x plus del del z of tau z_x minus del p del x plus rho g_x. This was for x component of momentum from equation 1 right. We also made for y component of momentum and gave the number equation 2 and also we made z component of momentum



and that we gave equation 3. So, keeping this equation that is from equation 1 which we have come to this point, we can say in a similar fashion we also can arrive at the y component of momentum and z component of momentum and this comes to rho capital D v_y d t right capital D v_y d t is equal to minus del del x of tau x_y plus del del y of tau y_y plus del del z of tau z_y minus del p del y. plus rho g_y right and this is for the y component of the momentum. Similarly, for the z component of momentum we can write rho times d or capital D Operand D v_z dt capital D v_z capital Dt is equal to minus del del x of tau x_z plus del del y of tau y_z plus del del z of tau z_z minus del g_z this was or this is for the z component of momentum right. In a exactly the way we have done for from rather equation 1 and expanded and then rearranged everything. So, we came to the top equation that is rho times capital D Operand D v_x / dt equal to minus del del x of tau x_x plus del del y of tau y_x plus del del z of tau z_x minus del p del x plus rho g_x this is for from equation 1 that is the velocity component v_x acting in all directions. Similarly, velocity component v_y acting in all direction we have in a similar way we have written right and that is rho times capital D Operand Capital D v_y / D t equal to minus del del x of tau x_y plus del del y of tau y_y del del z of tau z_y right and the pressure and gravitational force component that is minus del P del y plus rho g_y right. This was for y component of velocity.

Similarly, z component of velocity it could be written as rho capital D v_z capital D t is equal to minus del del x of tau x_z plus del del y of tau y_z plus del del z of tau z_z minus del p del x or rather del p del z plus rho g_z. So, all three components of momentum we have come to this substantial time derivative form ok. Now, for the right term. right hand side term that is del del x of tau x_x or del del y of tau y_x etc. all these tau terms.

Let us define tau as tau x_x is nothing, but minus mu del v_x del x right. You see it is minus mu del v_x del x is tau x_x. Similarly, for tau y_x we can write minus mu del v_x del y, velocity component is v_x. acting in the y direction.

So, that is why minus mu del v_x del y right and tau z_x that we are writing minus mu del v_x del z right. You see everywhere that v_x component of velocity that is in the differential form, whereas the direction, whereas the direction that is coming in the differential to be differentiated with right. similar way we can define tau x_y is minus mu del v_y del x right. The second term of tau we have said to be equal to the velocity component and the first term we said to be the direction.

Similarly, for the y- and z- components are

$\partial(\rho v_y)/\partial t = -[\partial(\rho v_x v_y)/\partial x + \partial(\rho v_y v_y)/\partial y + \partial(\rho v_z v_y)/\partial z] - [\partial\tau_{xy}/\partial x + \partial\tau_{yy}/\partial y + \partial\tau_{zy}/\partial z - \partial p/\partial y + \rho g_y$ ……………….. (2)

and

$\partial(\rho v_z)/\partial t = -[\partial(\rho v_x v_z)/\partial x + \partial(\rho v_y v_z)/\partial y + \partial(\rho v_z v_z)/\partial z] - [\partial\tau_{xz}/\partial x + \partial\tau_{yz}/\partial y + \partial\tau_{zz}/\partial z - \partial p/\partial z + \rho g_z$ ……………….. (3)

tau x_y means tau x is the direction and tau y is the velocity component. So, here we are also getting minus mu del del x of v_y. tau y_z equal to or rather tau y_y equals to minus mu del v_y del y and in the same fashion we can write tau z_y equals to minus mu del v_y / del z. So, all terms we can express in terms of mu right by definition mu. that is viscosity. So, tau x_z is equal to minus mu del v_z del x tau y_z equals to minus mu del v_z del y and tau z_z equals to minus mu del v_z del z right.

From (1)

$\partial(\rho v_x)/\partial t = \rho\,(\partial v_x/\partial t) + v_x\,(\partial\rho/\partial t)$

From equation of continuity

$\partial\rho/\partial t = -[\partial(\rho v_x)/\partial x + \partial(\rho v_y)/\partial y + \partial(\rho v_z)/\partial z]$

$\therefore \partial(\rho v_x)/\partial t = \rho\,(\partial v_x/\partial t) - v_x[\partial(\rho v_x)/\partial x + \partial(\rho v_y)/\partial y + \partial(\rho v_z)/\partial z]$

The right hand side of eq^n (1) can be written as,

$-[\partial(\rho v_x v_x)/\partial x + \partial(\rho v_y v_x)/\partial y + \partial(\rho v_z v_x)/\partial z] - [\partial\tau_{xx}/\partial x + \partial\tau_{yx}/\partial y + \partial\tau_{zx}/\partial z] - \partial p/\partial x + \rho g_x$

So, if we substitute mu terms in the tau term. So, then we get tau x_x or del del x of tau x_x equals to minus mu del 2 v_x del x square right del 2 v_x del x square. If you remember our our original term was tau x_x del del x of tau x_x right.

The first term can be expanded as

$-\rho v_x\,(\partial v_x/\partial x) - \rho v_y\,(\partial v_x/\partial y) - \rho v_z\,(\partial v_x/\partial z) - v_x[\partial(\rho v_x)/\partial x + \partial(\rho v_y)/\partial y + \partial(\rho v_z)/\partial z]$

So, we have we have replaced it with minus mu del del x of v_x because it is x_x right. This tau y_x you have used del del y of v_x minus mu and this is del del rather del del z of v_x right. So, same fashion we have done here tau x_x is minus del del x of tau x_x equals to minus mu. del 2 v_x del x square right.

Eq. (1) can be written as

$$\rho\ (\partial v_x /\partial t) = -\ v_x [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z] - \rho v_x (\partial v_x /\partial x) - \rho v_y (\partial v_y/\partial y) - \rho v_z (\partial v_z/\partial z) - v_x [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z] - [\partial \tau_{xx} /\partial x + \partial \tau_{yx} /\partial y + \partial \tau_{zx} /\partial z] - \partial p/\partial x + \rho g_x$$

$$\rho\ [\partial v_x /\partial t + \partial v_x /\partial x + \partial v_y /\partial y + \partial v_z /\partial z] = -\ [\partial \tau_{xx} /\partial x + \partial \tau_{yx} /\partial y + \partial \tau_{zx} /\partial z] - \partial p /\partial x + \rho g_x$$

It was del v del x. So, now, it is del 2 v_x del x square and similarly del del y of tau y_x is del 2 v_x del y square. You remember, we are getting, mind it that we are getting one negative from the definition of tau in terms of viscosity minus and we already had originally one minus. So, these two minuses are cancelling out. So, del del y of tau y_x we can write as del 2 v_x del v_y square or rather del 2 v_x del y square. and tau z_x the del z tau z_x is equal to del 2 v_x del y square. In the same fashion other taus also you can write del tau x_y or del del x of tau x_y is equal to del 2 v_x del del 2 v_y del x square. del del y of tau yy is del 2 vy del y square and del del z of tau yz is equals to del 2 v_z del z square. And same way we can write del x of tau x z as del 2 vx del x square or del 2 v_z del x square.

Or, $\rho\ Dv_x/Dt = -\ [\partial \tau_{xx} /\partial x + \partial \tau_{yx} /\partial y + \partial \tau_{zx} /\partial z] - \partial p/\partial x + \rho g_x$

Similarly

$$\rho\ Dv_y/Dt = -\ [\partial \tau_{xy} /\partial x + \partial \tau_{yy} /\partial y + \partial \tau_{zy} /\partial z] - \partial p/\partial y + \rho g_y$$

$$\rho\ Dv_z/Dt = -\ [\partial \tau_{xz} /\partial x + \partial \tau_{yz}/\partial y + \partial \tau_{zz} /\partial z] - \partial p/\partial z + \rho g_z$$

del del y of tau y_z as del 2 v_z del y square and del del z of tau z_z equals to del 2 v_z del z square. all taus we have converted to obviously everywhere mu's were there ok. So, that mu we have not written every time here, right, but when we rewrite it we will take mu also and this is known by replacing tau with the viscosity, right. And this is called the Navier Stokes equation.

Using $\tau_{xx} = -\mu\ \partial v_x/\partial x$; $\tau_{yx} = -\ \mu\ \partial v_x/\partial y$; $\tau_{zx} = -\mu\ \partial v_x/\partial z$

$\tau_{xy} = -\ \mu\ \partial v_y/\partial x$; $\tau_{yy} = -\ \mu\ \partial v_y /\partial y$; $\tau_{zy} = -\ \mu\ \partial v_y/\partial z$

$\tau_{xz} = -\mu\, \partial v_z/\partial x$; $\tau_{yz} = -\mu\, \partial v_z/\partial y$; $\tau_{zz} = -\mu\, \partial v_z/\partial z$

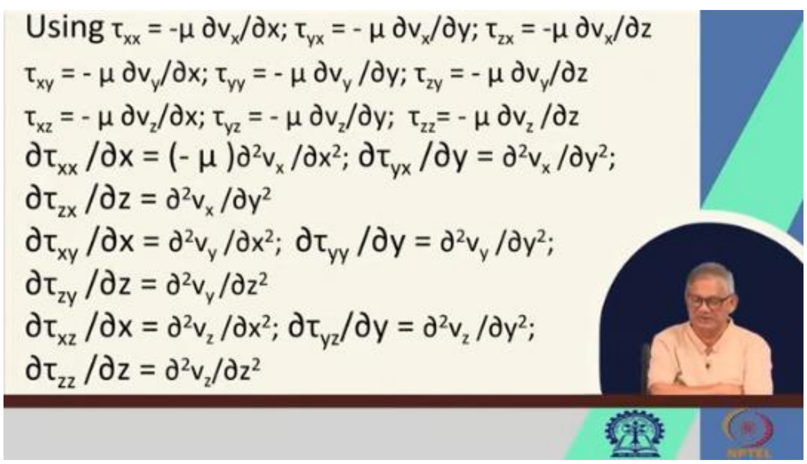
$\partial \tau_{xx}/\partial x = (-\mu)\partial^2 v_x/\partial x^2$; $\partial \tau_{yx}/\partial y = \partial^2 v_x/\partial y^2$; $\partial \tau_{zx}/\partial z = \partial^2 v_x/\partial y^2$

$\partial \tau_{xy}/\partial x = \partial^2 v_y/\partial x^2$; $\partial \tau_{yy}/\partial y = \partial^2 v_y/\partial y^2$; $\partial \tau_{zy}/\partial z = \partial^2 v_y/\partial z^2$

$\partial \tau_{xz}/\partial x = \partial^2 v_z/\partial x^2$; $\partial \tau_{yz}/\partial y = \partial^2 v_z/\partial y^2$; $\partial \tau_{zz}/\partial z = \partial^2 v_z/\partial z^2$

The set of equations which we are arriving at, they are known as Navier-Stokes equation and this Navier-Stokes equation is one of the pioneer equation in fluid flow. In flow of fluid wherever it be that Navier-Stokes equation is basic of all the equations that is why it is so important, right. If we do not understand, you can raise your question, but again to make it understandable, once again, we write that, we have defined taus or molecular transports in terms of viscosity from the definition of viscosity.

And, the definition of viscosity says that tau x_x is nothing, but minus mu del del x of v_x, right. On the same fashion we have defined tau yx as del del y of v_x right minus mu del del y of v_x. Similarly, for the third term tau z_x we have defined it as minus mu del del z of v_x, right or del v_x del z that we can say ok. This is for x, similarly for x_y that is tau x_y we defined it as minus mu del del x of rho v_y.



tau y_y minus mu del del y of v_y. and tau z_y is minus mu del v_z del y rather the other way round minus mu del v_y del z. Just not to make mistake or mixing up with the definition, I am repeating, that is why I am repeating that you must make it that in the tau there are two subscripts below together one is may be x another may be y. one may be x another may be z or one may be z another may be y, whatever it be the first component of the subscript, what you are using that denotes the direction. And the second component denotes

velocity component, right. So, we have to be very careful while defining that. which goes where? That is why you see tau y_x we have written minus mu del del y of v_x, right or del v_x del y. So, minus mu you see that v_x is on the second part. So, it is the velocity component.

So, it is v_x. and y is the direction. So, it is with respect to y derivation is or derivative is with respect to y. So, vx with respect to y. if it is made to a derivative, then it is minus mu del del y of v_x, right on the same fashion all the terms we have described right. So, I repeat there that like tau x_y, we have defined as minus mu del v_y del x

sorry minus mu del v_y del x tau y_y is minus mu del v_y del y and tau z_y as minus mu del v_y del z right. On the same fashion we have said the tau y_z rather tau x_z is minus mu del v_z del x, tau y_z is minus mu del v_z del y and tau z_z is minus mu del v_z del z. So, this definition of mu or in terms of viscosity of tau that is the force component it can normal force or it can be tangential force.

Normal forces are like, tau x_x tau y_y tau z_z are normal forces and others are all shear forces, right. Why also we have said that if the velocity is acting like this and if it is the force So, it is hitting directly. So, it is the normal force whereas, all other forces are may be tangential like this or like this. So, any such these are called shear force, right.

like with the scissor you cut that is also a shear force you are cutting through a shear force you are not making any impact or any other. So, that is a shear force you are applying, ok. So, based on this we have substituted all the defined forces taus and we have arrived at tau x_x rather tau x del del x of tau x_x, is minus mu del 2 v_x del x square. Why del 2 v_x del x square?

Here you see this is already del del del x of tau x_x. Now tau x_x we have defined to be So, there is two derivative del del x of del v_x del x, right that is why it is called del 2 v_x del x^2, right. Similarly, we can say del del y of tau y_x to be del 2 v_x del y square and the third one is del 2 v_x del y square to be tau

no this is yeah it should be del 2. del del x del del z of tau z tau z_x is del 2 v_x it should be del z square right it should be del z square. There are there are some typo mistakes maybe because it being similar. So, by doing cut and paste some mistakes could be there that is why of course, I did not correct it the reason being I wanted you also to find out where is the if there is any error.

So, that error I am pointing out. here the velocity component is a or v_x and the direction is z. So, it is coming del del v_x right. So, that is what we are writing del 2 v_x del z square not del y square this is in this ok, where this is in this. here it is.

So, we should make correction in the right place. So, it is del del z of tau z_x. So, that is del del z of again del del z of tau del del z of not tau del del z of del del z of v_x right. we can write from mathematics that it is del 2 v_x del x del y square, right. In the next class of course, I will also write here because I did not install that writing pad.

In the next class I will show you also how it is because in the next class we will come the equation that equation of motion leading up to Navier Stokes equation right. So, we write all the terms in terms of tau like del del x of tau x_y as del 2 v_y del x square, del del y of tau y_y as del 2 v_y del y square and del del z of tau y zis del 2 v_y del z square, right. Similarly, del del x of tau x_z as del 2 v_z del x square and del 2 y_z or del del y of tau y_z is del 2 v_z del z square or del y square.

and del z of tau y_z del z of tau z_z is del 2 v_z del z square, ok. Now, the time is up for this class we will come to the next class with all these equations which are known as Navier-Stokes equation, that is a pioneer equation, right and everywhere it is known. Thank you very much. Thank you.