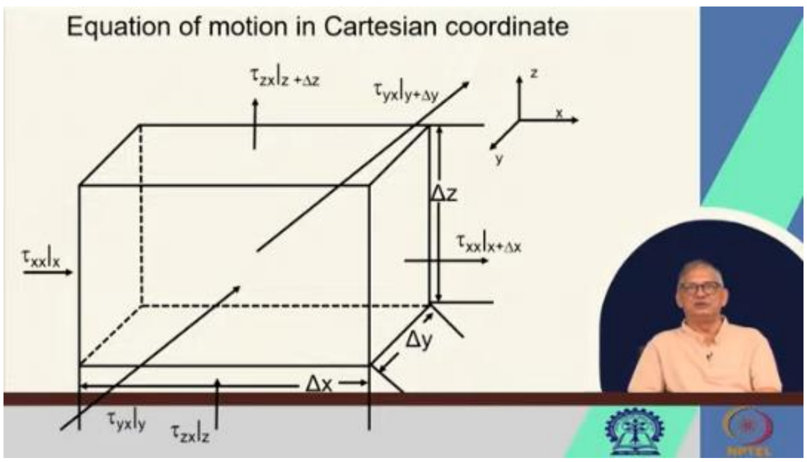
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture11

LECTURE 11 : EQUATION OF MOTION TO BE CONTINUED

Good morning, my dear students and friends. We are doing the equation of motion. Right. In the previous class, we could not complete it because we said that the equation of motion is a really big time-consuming derivation. So, we left off where we started from there. OK.

So, for recapitulation, we said that we are doing the equation of motion for Cartesian coordinates because it is the simplest of the lot. Why did I say 'of the lot'? 'Of the lot' means the other two coordinates, which are cylindrical and spherical, that is r, θ, z for cylindrical and r, θ, φ for spherical. So, that is much more complicated and even more time-consuming, for which we are developing



the equation of motion under Cartesian coordinates x, y, z, right. So, we also said that momentum, it is based on momentum balance. So, two types of momentum transfer occur: one by the bulk flow, I will give the example of waves of rivers or sea, right. So, there, the bulk flow occurs from one point to another point.

The equation of motion -:

A momentum balance is approached

Rate of momentum in - Rate of momentum out + sum of force acting on system = rate of momentum accumulation

By convection or bulk flow

Rate of x comp. of moment^{um} on face x and x + Δx = $\rho v_x v_x |_x \Delta y \Delta z$ and $\rho v_x v_x |_{x+\Delta x} \Delta y \Delta z$

Rate of x comp. of moment^{um} on face y and y + Δy = $\rho v_y v_x |_y \Delta x \Delta z$ and $\rho v_y v_x |_{y+\Delta y} \Delta x \Delta z$

Rate of x comp. of moment^{um} on face z and z + Δz = $\rho v_z v_x |_z \Delta x \Delta y$ and $\rho v_z v_x |_{z+\Delta z} \Delta x \Delta y$

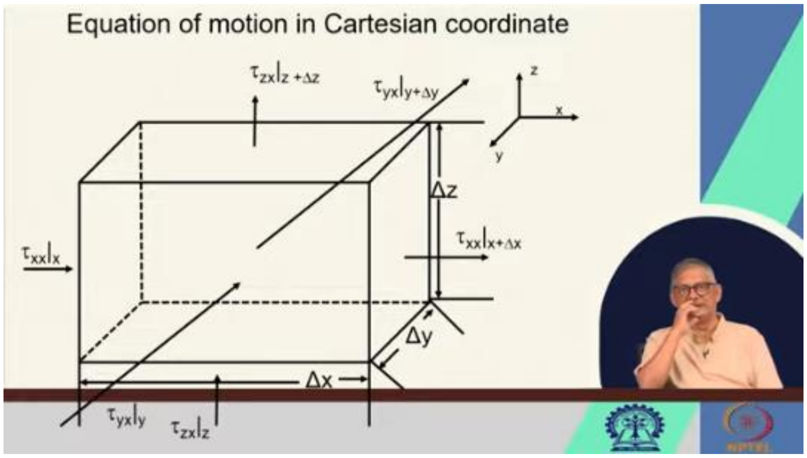
And, because of the molecules within that bulk, they also have some movement, right, in all directions, of course, and that is called molecular transport, which is represented by tau, τ, right. So, in this diagram, we see that we have shown the face x and x plus Δx, where the τ_x can be said to represent the normal force, right, because it is directly impacting on the face x and x plus Δx, right. But the other two, as we see, τ_{xy} or yx, rather τ_{yx} and τ_{yx} plus Δy that is or that can be said to be a shear component, shear force, and similarly, that τ_{zx} at z and τ_{zx} at z plus Δz. These are the shear components. of the x component of velocity, right. So, we came to this up to this, we derived, right, where we said that this is del del t of ρ v_x, this is equals to minus del del x of ρ v_x v_x plus del del y of ρ v_y v_x plus del del z of ρ v_z v_x, right, plus τ_{xx} del del del y of τy and del del z of τ_{zx} plus the other forces like pressure force and gravitational force.

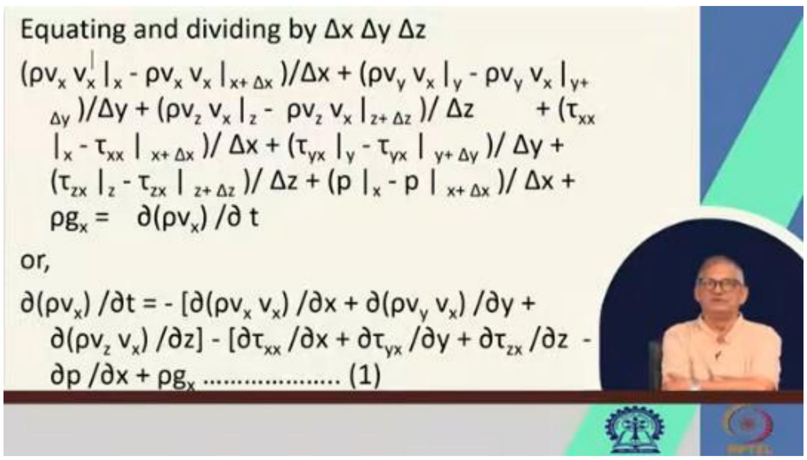
By molecular transport -:

Rate of x component of momentum on face x and x+Δx = $\tau_{xx} |_x \Delta y \Delta z$ and $\tau_{xx} |_{x+\Delta x} \Delta y \Delta z$ respectively.

Rate of x component of momentum on face y and y+Δy = $\tau_{yx} |_y \Delta x \Delta z$ and $\tau_x |_{y+\Delta y} \Delta x \Delta z$ respectively.

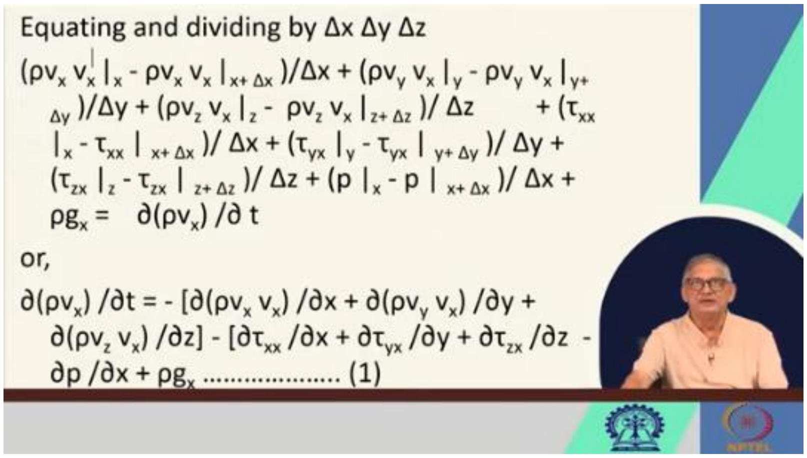
Rate of x component of momentum on face z and z+Δz = $\tau_{zx} |_z \Delta x \Delta y$ and $\tau_{zx} |_{z+\Delta z} \Delta x \Delta y$ respectively.





So, they were del P del x and $\rho\, g_x$. So, these are all added, ok. One more thing to highlight here again that, in the τ, you see the direction is first, τ_y, τ_x, τ_z, that is the direction. and the component of the velocity that is the second, that is τ_{xx}, τ_{yx}, τ_{zx}. So, remember, if you remember this that the direction is first, and the second is the component, then we will not make a mistake.

ok. Then, similar to this derivation of equation 1, if we see the y and z components, so then we can write that was x component momentum. acting in all directions. So, this is y component of momentum acting in all directions. So, in a similar fashion of equation 1, we can write del del t of $\rho\, v_y$ say here the velocity component is v_y is equal to del del x of $\rho\, v_x\, v_y$



∴Sum of the convective and molecular transport terms:

$(\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x})\Delta y\Delta z + (\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y})\Delta x\Delta z + (\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})\Delta y\Delta x - (\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x})\Delta y\Delta z + (\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})\Delta x\Delta z + (\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})\Delta x\Delta y$

del del y of ρ v_y v_y plus del del z of ρ vz v_y minus that is the molecular transport del del x of τ_{xy} del del y of τ_{yy}. Now, this τ_{yy} can be said to be normal force, right, like the τ_{xx} earlier. So, plus τ_{zy} or rather plus del del z of τ_{zy} minus del p del y plus ρ g_y is equal to rather, this is the second component of velocity in the y velocity component in all directions, right. And in the same way, we can write the third one as del del z, as del del t del ρ vz that is equal to del del x of ρ v_x vz plus del del y of ρ v_y del del z of ρ vz vz. So, here also that okay when it is coming to τ, then I should say minus del del x of τ_{xz} plus del del y of τ_{yz}. del del z of τ_{zz}. So, this τ_{zz} can also be said to be the normal force right. So, minus del P del z plus ρ g_z right. So, then we have in our hand all three velocity components acting in all directions and they are equation of motion derivation part is the first one, okay. Now, if we look at equation 1, that it was del del t of ρ v_x on the left side. Similarly, for equation 2, it was del del t of ρ v_y and for z component just now, we saw that it was del del t of ρ v_z. Now, if we expand that del del t of ρ v_x, then we get again in the same u v method which we said earlier.

Other terms,

Pressure force:- $(p|_x - p|_{x+\Delta x})\Delta y\Delta z$

Gravity force- $\rho g_x \Delta x\Delta y\Delta z$

Accumulation -:

Rate of accumulation of x component of momentum

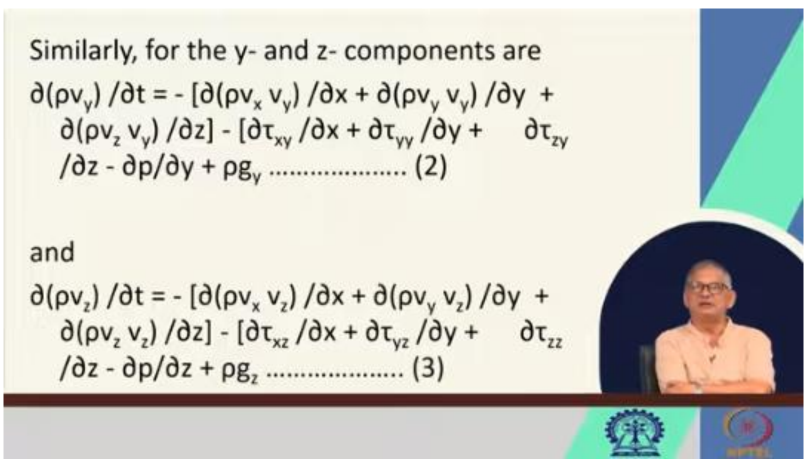
$= (\partial(\rho v_x) / \partial t)\, \Delta x\, \Delta y\, \Delta z$

Equating and dividing by $\Delta x\, \Delta y\, \Delta z$

$(\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x})/\Delta x + (\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y})/\Delta y + (\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})/\Delta z + (\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x})/\Delta x + (\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y})/\Delta y + (\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z})/\Delta z + (p|_x - p|_{x+\Delta x})/\Delta x + \rho g_x = \partial(\rho v_x)/\partial t$

or,

$\partial(\rho v_x)/\partial t = - [\partial(\rho v_x v_x)/\partial x + \partial(\rho v_y v_x)/\partial y + \partial(\rho v_z v_x)/\partial z] - [\partial\tau_{xx}/\partial x + \partial\tau_{yx}/\partial y + \partial\tau_{zx}/\partial z - \partial p/\partial x + \rho g_x$ ……………….. (1)



If you want, I can also show, okay. I did not do beforehand, okay. It can be that del del x of u v is u into del del x of v plus v into del del x of u that is the expansion of u v. Similarly, here also we are expanding it del del t of ρ v_x. So, one is u say ρ and v is say v_x, right. So, ρ times del del t of v_x.

Similarly, for the y- and z- components are

$\partial(\rho v_y)/\partial t = - [\partial(\rho v_x v_y)/\partial x + \partial(\rho v_y v_y)/\partial y + \partial(\rho v_z v_y)/\partial z] - [\partial\tau_{xy}/\partial x + \partial\tau_{yy}/\partial y + \partial\tau_{zy}/\partial z - \partial p/\partial y + \rho g_y$ ……………….. (2)

and

$\partial(\rho v_z) / \partial t = - [\partial(\rho v_x v_z) / \partial x + \partial(\rho v_y v_z) / \partial y + \partial(\rho v_z v_z) / \partial z] - [\partial \tau_{xz} / \partial x + \partial \tau_{yz} / \partial y + \partial \tau_{zz} / \partial z - \partial p / \partial z + \rho g_z$ ……………….. (3)

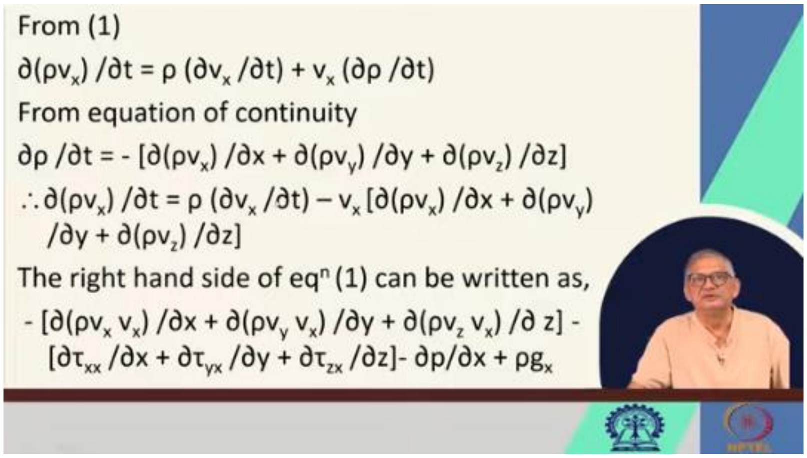
Now v is that v_x. So, v_x times del del t of ρ, right. And now if we go back to our equation of continuity, I said while doing that, that this will be required in future. So, now it is being utilized. We have seen that in equation of continuity del ρ del t, right, del ρ del t is equal to minus del del x of ρ v_x plus del del y of ρ v_y plus del del z of ρ v_z, right.

We can say that del del t of ρ v_x on expansion is ρ times del v_x del t, right. Here you Can you find out something which is not logical in sign? It will be corrected afterwards. See here also.

We are ok, taking it as now, ρ times del del t of v_x, this is plus v_x del del x of ρ v_x. Similarly, for y, del del y of ρ v_y plus del del z of ρ v_z. So, what we are doing is, from the equation of continuity, that is del ρ del t is equal to minus del del x of ρ v_x plus del del y of ρ v_y plus del del z of ρ v_z, this we have taken from the equation of continuity. So, we are writing that del ρ v_x del t, right, del ρ v_x del t, this is equal to ρ on expansion into del v_x del t. Actually, it should be plus v_x del del x of ρ v_x, right, plus as usual whatever was there del del y of ρ v_y plus del del z of ρ v_z. So, we have made only one expansion and There is a mistake here, it should have been plus, right.

What did we do? We have expanded this del del x of ρ v_x, right, as from here, obviously, this part remains. So, here we have this part, we have expanded this is, It was minus of that whole thing. So, if we put minus at the beginning, then it should have been ok, right.

So, it is, that is why, we have taken it out, right. So, it is like from here minus it remains here and This becomes plus. So, del del t of ρ v_x is ρ del del t of v_x plus v_x into del ρ v_x del del x of ρ v_x and other two terms remain intact that is Del del y of ρ v_y plus del del z of ρ v_z like this, ok.



Then, the right-hand side of equation 1, which we had, let us go back to that. This was our right-hand side. So, the left-hand side was del del t of ρ v_x, this is equal to minus del del x of ρ v_x v_x plus del del y of ρ v_y v_x plus del del z of ρ v_z v_x, right, minus del del x of τ_{xx}, plus del del y of τ_{yx}, del del z of τ_{zx} minus the pressure term del P del x and the gravitational term ρ g_x, right. So, from here, as we have just shown, from here we can say that the right-hand side of that equation 1. We write minus del del x of ρ v_x v_x plus del del y of ρ v_x v_y, plus, sorry, del del y of ρ v_y vx, plus del del z of ρ v_z v_x minus del del x of τ_{xx} plus, del del y of τ_{yx} plus del del z of τ_{zx} minus del P del x plus ρ g_x. This was on the right-hand side of equation 1, right. We have already done something on the left-hand side, that is del del t of ρ v_x on expansion, and then, we have come to one point, right. So, taking the right-hand side, we can say. That the first term can be expanded. The first term is what? del del x of ρ v_x v_x, right.

From (1)

$\partial(\rho v_x) /\partial t = \rho (\partial v_x /\partial t) + v_x (\partial \rho /\partial t)$

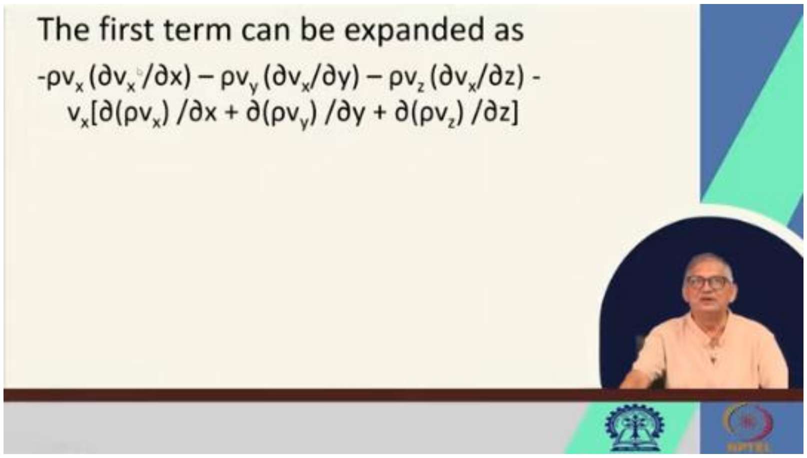
From equation of continuity

$\partial \rho /\partial t = - [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z]$

$\therefore \partial(\rho v_x) /\partial t = \rho (\partial v_x /\partial t) – v_x [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z]$

The right hand side of eq^n (1) can be written as,

$- [\partial(\rho v_x v_x) /\partial x + \partial(\rho v_y v_x) /\partial y + \partial(\rho v_z v_x) /\partial z] - [\partial\tau_{xx} /\partial x + \partial\tau_{yx} /\partial y + \partial\tau_{zx} /\partial z]- \partial p/\partial x + \rho g_x$



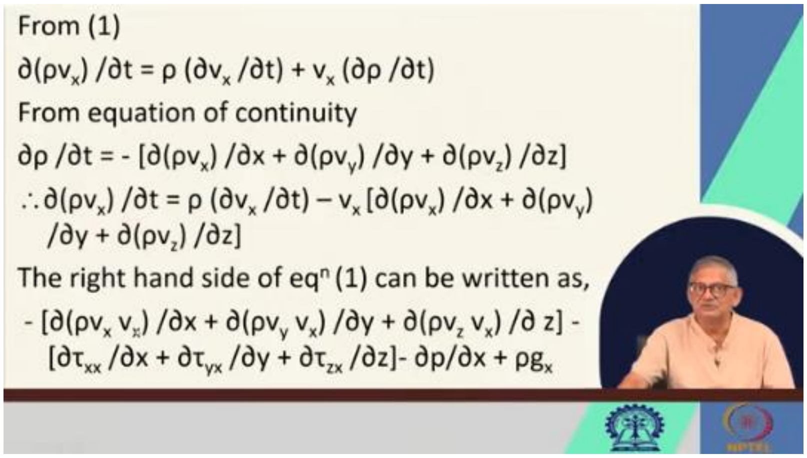
So, there, if we take ρ v_x as u and v_x as v, then minus ρ v_x del del x of v_x, now the plus is out because we have taken all negative outside. So, ρ v_x del del x of v_x, right. Similarly, the other term for v_y, del del y of ρ v_y, del del rather ρ v_y times del y of v_x minus ρ v_z del del z of v_x, right.

This is the first term minus all you see v_x are taken out v_x into del del x of ρ v_x. del del y of ρ v_y plus del del z of ρ v_z. Let me clear it out again, taking this, right, taking this that minus. del del x of ρ v_x v_x. This we are expanding right, as u v. So, in that case ρ v_x becomes 1 u and v_x becomes 1 v. Similarly, ρ v_y becomes 1 u and v_x becomes 1 v for expansion. And ρ v_z becomes 1 u and v_x becomes 1 v, right. So, the first term if we expand then it is ρ v_x times del del x of v_x, right.

The first term can be expanded as

$$-\rho v_x (\partial v_x /\partial x) - \rho v_y (\partial v_x/\partial y) - \rho v_z (\partial v_x/\partial z) - v_x[\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z]$$

So, you are not keeping now v_x. Separate because in all other cases that is why we have taken it if you see the next equation right, I will go to that. So, similarly here also ρ v_y as u. So, ρ v_y into del del x of. v_x and similarly ρ v_z times del del x del, sorry, this was del del y of v_x and this is del del z of v_x. This is one. Now, the other term where everywhere you see v_x times this, right.



So, that is why v_x we have taken common. So, v_x. times del del x of ρ v_x, here v_x times del del y of ρ v_y, here v_x times del del z of ρ v_z. That is what explicitly we have done here, right. We have taken out that negative. So, it is minus ρ v_x times del del x of v_x minus ρ v_y del del y of v_x.

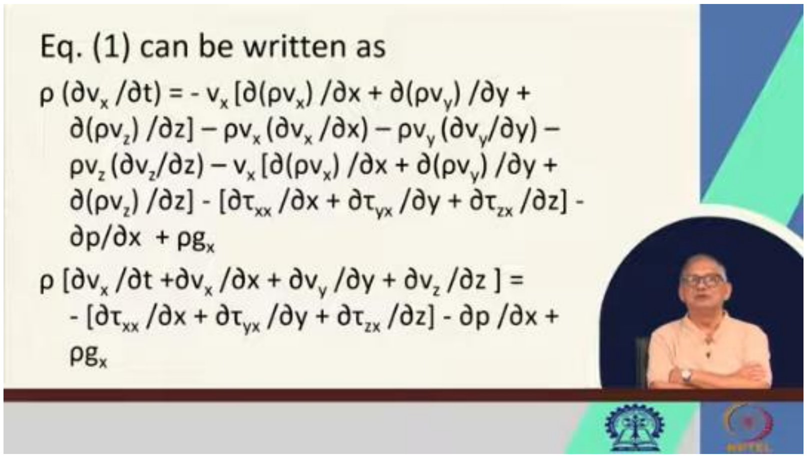
And minus ρ v_z del del z of ρ v_x and this minus again we have taken out v_x common because everywhere it is coming. as v. So, v into that part. So, v_x into del del x of ρ v_x plus del del y of ρ v_y plus del del z of ρ v_z, right. So, this is the one expanded form. So, we can rewrite that equation 1.

Eq. (1) can be written as

$$\rho (\partial v_x /\partial t) = - v_x [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z] – \rho v_x (\partial v_x /\partial x) – \rho v_y (\partial v_y/\partial y) – \rho v_z (\partial v_z/\partial z) – v_x [\partial(\rho v_x) /\partial x + \partial(\rho v_y) /\partial y + \partial(\rho v_z) /\partial z] - [\partial \tau_{xx} /\partial x + \partial \tau_{yx} /\partial y + \partial \tau_{zx} /\partial z] - \partial p/\partial x + \rho g_x$$

$$\rho [\partial v_x /\partial t +\partial v_x /\partial x + \partial v_y /\partial y + \partial v_z /\partial z] = - [\partial \tau_{xx} /\partial x + \partial \tau_{yx} /\partial y + \partial \tau_{zx} /\partial z] - \partial p /\partial x + \rho g_x$$

If you remember, we had ρ times del del t of v_x that is equal to minus v_x del del x of ρ v_x plus del del y of ρ v_y plus del del z of ρ v_z minus ρ v_x times. del del x of v_x minus del del y of ρ v_y. Or rather this, since we have taken minus common. So, v_x we have taken, yeah, minus ρ v_x del v_x del x minus ρ v_y del v_y del y minus ρ v_z del v_z del z. Now, all minus you have taken out minus v_x. into del del x of ρ v_x plus, because minus we have taken out, plus del del y of ρ v_y plus del del z of ρ v_z, right, del del z of ρ v_z. And we still have that



molecular transport terms that is minus del del x of τ_{xx} plus del del y of τ_{yx} plus del del z of τ_{zx} and the pressure terms del p del x plus ρ g_x, right. So, we can rewrite, right, ρ times del v_x del t plus del v_x del x plus del v_y del y plus del v_z del z, this is equal to del del x of τ_{xx} plus del del y of τ_{yx} plus del del z of τ_{zx} minus del P del x plus ρ g_x, right. So, here you see this was our

this was our left side, ρ del v_x del x. So, we have taken ρ common here, ok. And here you see minus v_x del del x of ρ v_x plus v_y del del y of ρ v_y. plus del del z of ρ v_z, right. This is 1 and here minus v_x del del x of ρ v_x plus del del y of ρ v_y plus del del z of ρ v_z, ok.

One sign problem is appearing because this term and that term they are canceling, right. And remaining is this, ρ is taken common del v_x del t, right. This is taken common and it is going to that side. So, ρ is taken out. So, del v_x del x, del v_x del x plus del v_y del y.

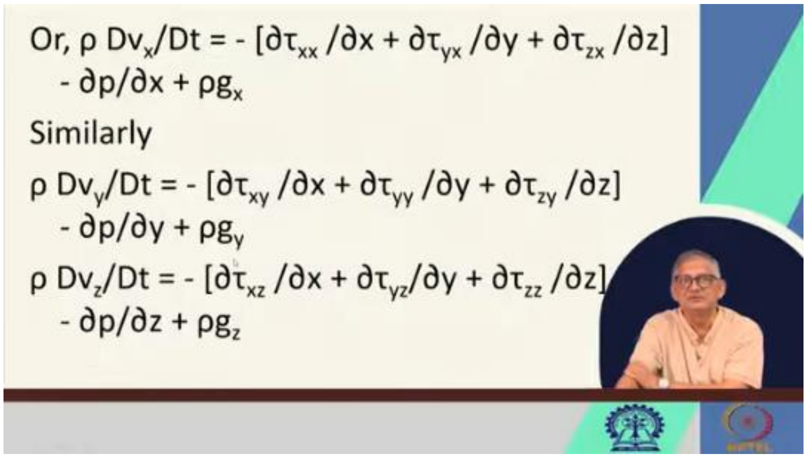
Plus del v_z del z, right, and remaining this part del del x of τ_{xx} del del $_y$ of τ_{yy} is yx and del del z of τ_{zx} right, and del p del x for the pressure and ρ g_x for the gravitational force, ok. Then we can rewrite because this part is similar with respect to capital D, right. So, we can write ρ del or rather capital D v_x / Dt is equal to minus del τ_{xx} del x plus del τ_{Yx} del y.

Or, $\rho\, Dv_x/Dt = -\,[\partial\tau_{xx}/\partial x + \partial\tau_{yx}/\partial y + \partial\tau_{zx}/\partial z] \quad - \partial p/\partial x + \rho g_x$

Similarly

$\rho\, Dv_y/Dt = -\,[\partial\tau_{xy}/\partial x + \partial\tau_{yy}/\partial y + \partial\tau_{zy}/\partial z] \quad - \partial p/\partial y + \rho g_y$

$\rho\, Dv_z/Dt = -\,[\partial\tau_{xz}/\partial x + \partial\tau_{yz}/\partial y + \partial\tau_{zz}/\partial z] \quad - \partial p/\partial z + \rho g_z$



Plus del τ_{zx} del z minus del p del x plus ρ g_x, right. This we arrive at from that equation 1, ok. Now, time is up; we will continue in the next class.

Thank you.