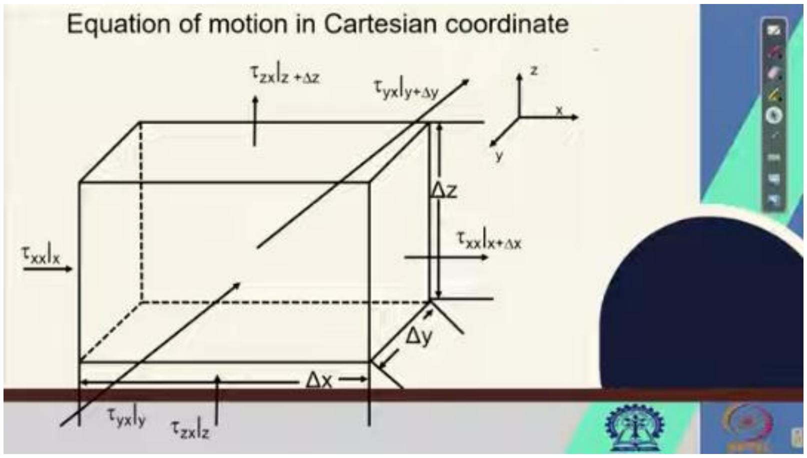
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture10

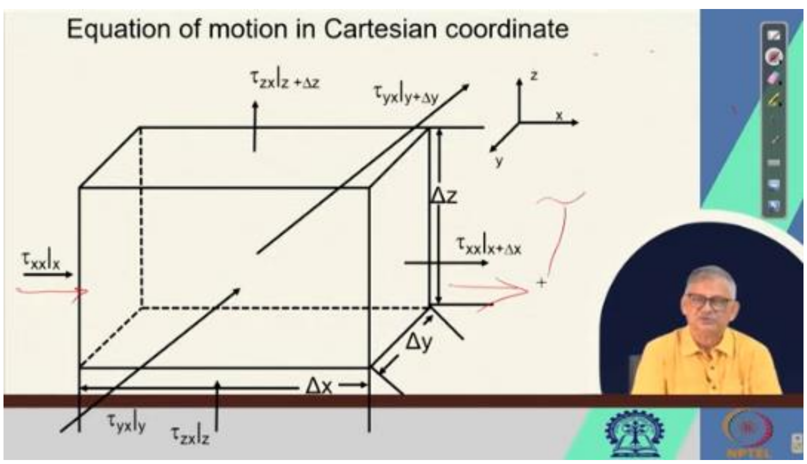
LECTURE 10 : EQUATION OF MOTION IN CARTESIAN COORDINATE

Good evening, my dear friends and students. We have finished equation of continuity. We said in the previous class that we would go to the equation of motion, right. So, we come to the equation of motion, and we also said that it is a very complicated System, because here we have not dealt with momentum, earlier, right.

Now, we are dealing only with momentum, not mass. We have dealt with the mass. Now, here it is not only mass or momentum; it is also two types of momentum, right. That we will explain what those two types are and what their implications are, right. So, we will derive the equation of motion in Cartesian coordinates. So, if we look at the equation of motion in Cartesian coordinates, that is



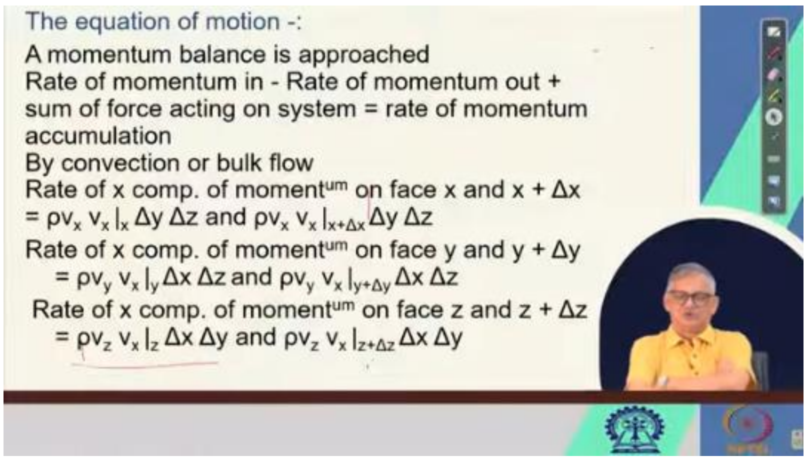
Our volume element is Δx Δy and Δz, right. As you see, we have drawn the volume element in the x-y-z coordinate, and we have also shown two things: one is this one, right? A new term which we have not come across is τ. This symbol is called tau, τ, right. So, this is the shear component, right. This is the shear force or the obvious force, but shear force can also be present even when it is



Impacting like this. So, it is direct impact force if it is shearing over one over the other, then another one like that, like that, OK. So, you have taken the Cartesian coordinate that is Δx (delta x) Δy and Δz right. So, these three coordinates, and as we have shown, the center will be our focus where whatever is coming in will be accumulated. So, it is minus and again it will be plus, right.

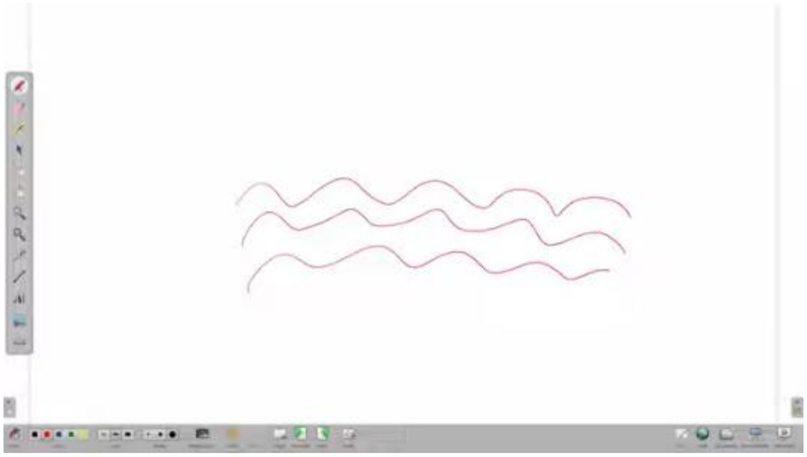
So, we develop the equation of motion for the Cartesian coordinates. I do not know; yeah, it is a little long. So, I do not know whether it will take one class or two classes; whatever it be, it does not matter; let it take its own time. Our intention is to make it clear to us so that. We do not face any difficulty, right.

So, here's one thing we have to remember: That for developing the equation of motion, we have taken momentum balance, unlike mass balance in the continuity equation, right. Here, we have taken. Momentum balance and momentum: We have said that ρv is the momentum. Obviously, we are not deleting or we are not segregating the velocity components individually right like v_x, v_y, v_z they are obviously remaining. Now, the question is whether we should do the

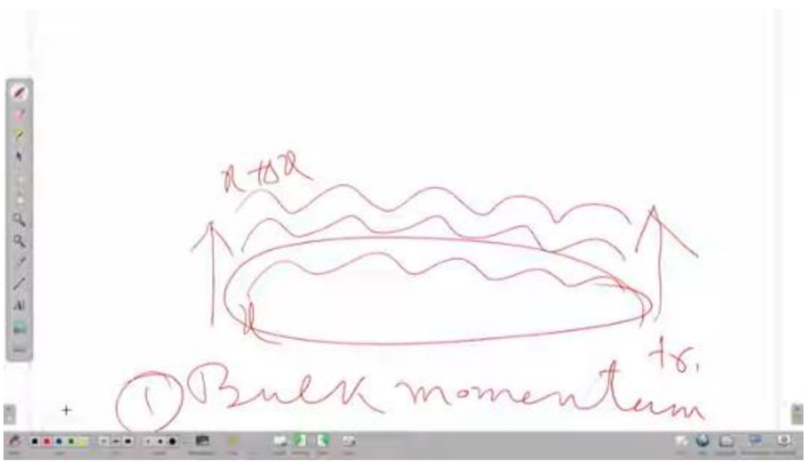


balance of momentum; then, the governing equation we can write as: the rate of momentum in minus the rate of momentum out plus the sum of the forces acting on the system is equal to the rate of momentum accumulation. I repeat; the governing equation is Rate of momentum in - Rate of momentum out + Sum of the forces acting on the system = Rate of momentum accumulation. Now, momentum is transferred by two ways: one by bulk flow; what do you mean? Imagine; I hope all of us have seen the ocean—any ocean, wherever.

So, there we have seen what? These kinds of waves are there. One wave like that, another wave like this; waves are there. Right, and these waves are moving in one direction, that is, towards the shore. So, what is happening a bulk of fluid right if getting transported from this point of say x to this point say x plus delta x right.

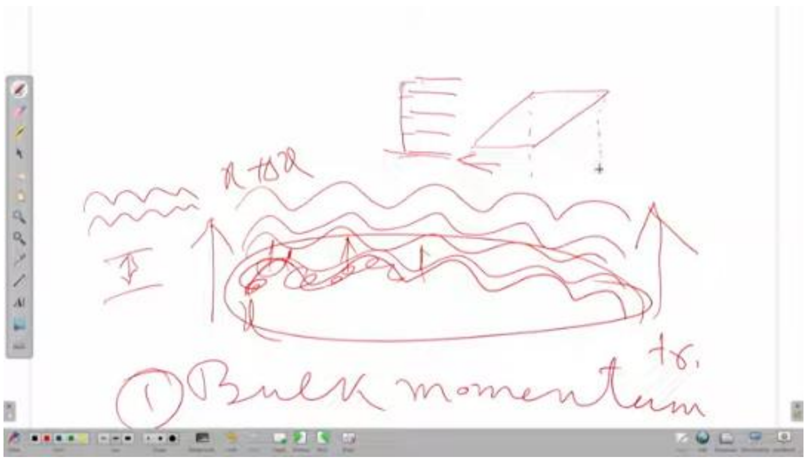


A bulk of fluid is being moved from this x point to the x plus Δx point. So, by that, bulk momentum is getting transferred, right. So, we call it bulk momentum transfer. Now, this is one kind of momentum transfer where the entire mass is moving from one point to another point. But if you see microscopically that each wave, this is one wave, and this is another wave.



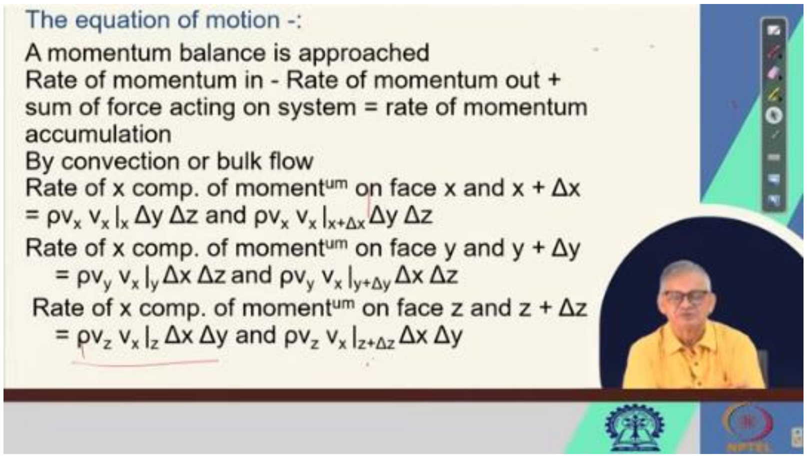
So, this wave is transferring energy to this point, some to this point, some to this point, because the molecules in that wave are having to-and-fro motion within a mean free path, right. So, what will happen? This wave is getting some transfer from that wave. I do not know whether you have seen playing cards. If you have seen them, you have many cards there.

One line, because these are the cards. Now, if you take a little and give some force to the lower one, then these cards will take a shape like this, right? These cards will take a similar shape, like this, because. The one which we have shifted here, which was originally in this line, right. Now, this was the original; it has shifted to this.



So, it becomes like that. So, that means you have given a force to the lower layer, which it has transmitted to its next layer, which it has transmitted to its next adjacent layer, and so on. But by the time it reached the top, it might have reduced or vanished altogether. So, this type of exchange of momentum is called this type of exchange of momentum, unlike bulk, which is a whole process involving the motion of molecules, right. So, that molecular momentum transfer is known as τ, right.

So, that is called molecular momentum transfer, and it also has a definition, which will come afterwards, right. So, if we look at it only by the convection of bulk flow because we said that the whole mass of the wave is going to the next point. So, it is the bulk movement of the fluid that is water in the sea, right. So, there we can say that rate of x-component because we have said already in the figure that x, y, and z are the three directions or three components, right.



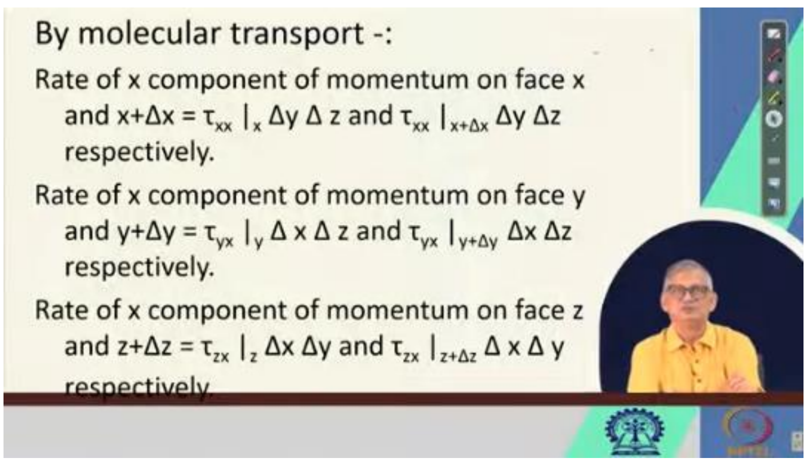
So, rate of x component of momentum acting on the face x it has three faces x and x plus delta x, y and y plus delta y and z and z plus delta z. These three, we have said. So, x component momentum acting on the face x and also at the face x plus delta x that can be written as equal to rho v_x that is the momentum in the x direction that is v_x component rho v_x into v_x at the face x into its area that is del y del z at the face x and at the face x plus delta x it is rho v_x v_x at the face x and x plus delta x into area delta y delta z, right. Similarly, if we say that rate of x component of momentum acting on the face y plus delta y, y and y plus delta y right.

So, that will be x component momentum acting on the y and y plus delta y, x component momentum x is v_x this way right y is this way right. So, z was the third. rate of x component of momentum acting on the face y and y plus delta y that is equal to rho v_y that is in the y direction rho v_y of whose x component into v_x at the face y times the area what is the area other than y that is delta x delta z. and rho v_y at the face y, rho v_y into v_x rather at the face y plus delta y into area del x del z right. So, we can say rate of x component of momentum acting on the face z and z plus delta z direction should be

The z-component and x-component momentum acting on the z and z + Δz direction. So, it is the rho v_z that is z direction momentum acted by the x component that is v_x at the face z and area delta x delta y. and the z plus delta z point, there it will be rho v_z into v_x at the face z plus delta z into delta x delta y right. So, these are the x-component momentum acting in all three directions (x, y, and z) out of bulk momentum transfer is correct.

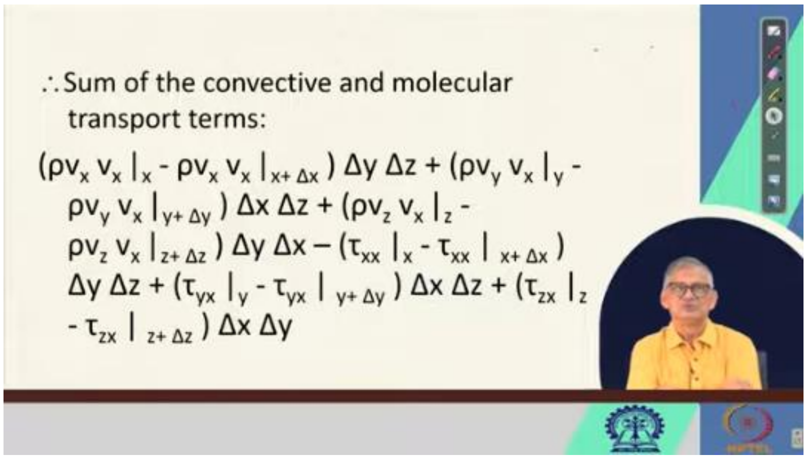
Now, another momentum transfer method is by molecular transport. So, for that. Rate of the x-component of momentum acting on the face between x and x + Δx is termed as τxx, because this is normal, right. Our face is this x, and momentum is heating like this. So, this is the normal force, right.

is called τ_{xx}. So, τ_{xx} at the point x is τ_{xx} at the point x into area del y del z and τxx at the point x plus delta x is τ_{xx} at x plus delta x into area del y del z right. for the rate of x component of momentum acting on the face y and y plus delta y will be τ_{yx}, τ_{yx}. So, the direction is y and the component is x that is why this τ_{yx} earlier it was direction was x. and the component is also x. So, it is x x. Now, its direction is y, component is x. So, yx. So, τ_{yx} at the face y into area is delta x del z and τ_{yx} into rather τ_{yx} at the face y plus delta y into area delta x delta z. These are the two x component momentum acting on the face y plus y and y plus delta y. Similarly, x component of momentum acting on the face z and z plus delta z that can be

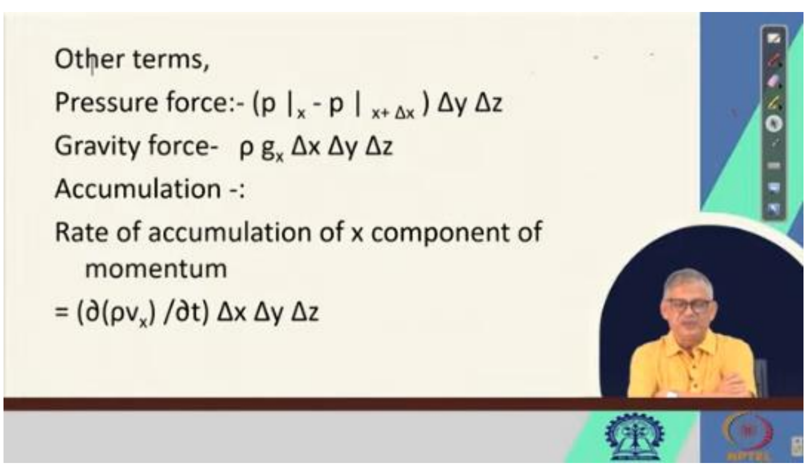


equal to τ_{zx} at the phase z, into the area that is delta x delta y and τ_{zx}, z is the direction x is the component right, z is the direction x is the component. So, τ_{zx} at z Into the area delta x delta y. So, we have all momentum transfers, one by bulk momentum transfer, that is the net mass of the component, x component acting in all three directions, and the molecular transport. Of the x component acting in all three directions, this we have identified. Now, if we also take into account some of the convective and molecular terms.

So, then we can write because these are the two ways by which the momentum was transferred: one by bulk convective transfer. And another method was the molecular transfer. So, we can write rho v_x into v_x at x minus rho v_x into v_x at x plus delta x into area del y del z plus rho v_y into v_x at y minus rho v_y into v_x at y plus delta y into area delta x delta y, I said del x del z plus rho v_z into v at z, minus rho v_z into v_x at z plus delta z into area delta y delta x minus τ_{xx} at the face x minus τ_{xx} at the face x plus delta x plus into area rather del y del z plus τ_{yx} at the face y. minus τ_{yx} at the face y plus delta y into area that is delta x delta z plus τ_{zx} at the face z minus τ_{zx} at the face z plus delta z into area del x del y right.

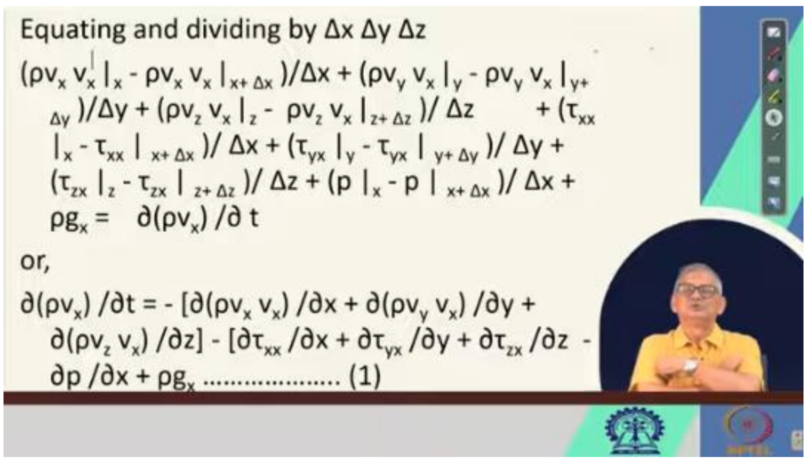


So, now if we simplify We will do the simplification after because if you remember, the governing equation was "rate of momentum in by convection" minus "rate of momentum in" or "rate momentum in by convection," rate net momentum in or out, or net rather. By molecular transport, plus some of the forces acting upon it, is equal to the rate of momentum accumulation, right. So, if we take other terms other forces, other forces are pressure force that is P at the face x for x component minus P at the face x plus delta x into its area that is del and also the gravitational force acting on it.



What is the gravitational force? Re sorry gravitational force will be rho g x times the volume element delta x delta y delta z. It is taking the g at the point x right. Though the volume element is very very small, But we are considering all the components. So, rho into g_x into delta x delta y delta z is the gravitational force.

Then the other term which remains is the rate of accumulation of the x-component of momentum. Now, the rate of accumulation of the x-component of momentum is definitely the product of the volume element and the rate of change of momentum. With time, right. So, that is for the x component del del t of rho v_x into delta x delta y delta z. So, del del t of rho v_x into delta x delta y delta z. Now, we are coming for equating. So, if we equate all these terms.

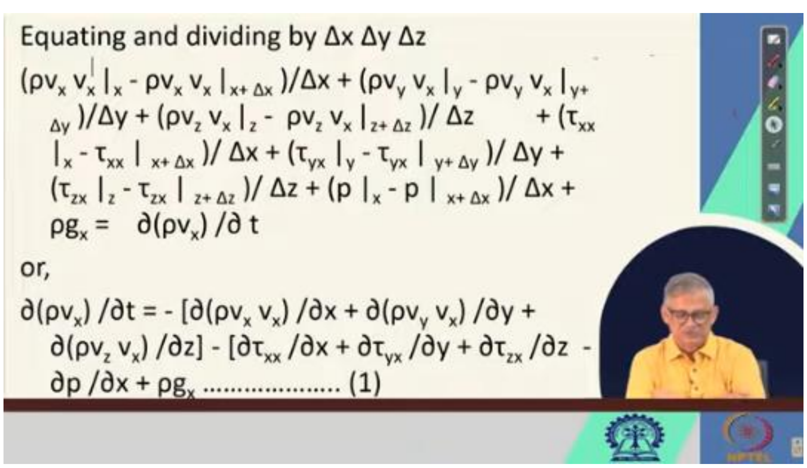


and now divide by delta x delta y delta z. In earlier accumulation term also we have seen there is a volume element delta x delta y delta z and in the pressure terms also we have seen that area terms were there for the x component delta y delta z was there right. if we divide all the terms by delta x delta y delta z that is the entire volume. Then we get rho v_x into v_x at x minus rho v_x into v_x at x plus delta x over delta x plus rho v_y into v_x at x plus delta rho v_y into v_x at y minus rho v_y into v_x at y plus delta y divided by delta y because we have divided by delta x delta y delta z. So, wherever del y del z is there that cancels out and delta x remains in the first term. delta x delta z in the second term is cancelling out, del y remains.

Similarly, for the third term, Δz will remain, and Δx Δy will cancel out. So, this minus rho v_z rather plus rho v_z into v_x at z minus rho v_z into v_x at z plus delta z over delta z this is for the bulk momentum. plus now the molecular momentum that is τ_{xx} minus τ_{xx} at x minus τ_{xx} at x plus delta x over delta x plus τ_{yx} at y minus τ_{yx} at y plus delta y over delta y plus τ_{zx} at z minus τ_{zx} at z plus delta z over delta z. plus p at x minus p at x plus delta x over delta x plus rho g_x is equal to del del t of rho v_x here we have taken off the delta x delta y and delta z term right. So, you can get.

from the definition of derivative, right rho v_x into v_x at x minus rho v_x into v_x at x plus delta x right. This as one, but now we are writing the last term that is del del t of rho v_x right. This we are equating. del t of ρv_x is equal to you see everywhere it is higher minus lower. So, by the definition of derivative, we know that "higher minus lower" divided by the distance or difference is the derivative, right.

So, here it is. rho v_x to v_x at x plus delta x it is at a negative and rho v_x at into v_x at x right it is positive. So, it means that the derivative will become negative. So, minus del del x of rho v_x into v_x plus del del y of rho v_y into v_x plus del del rho v_z into v_x minus again the same thing same logic applies that higher minus lower divided by the distance or difference that is del del x of τ_{yx}, τ x plus del del y of τ_{yx} plus del del z of τ_{zx} minus. So, the pressure term $\partial p/\partial x$ here since delta y delta z was the area. So, del x became in the denominator. So, it becomes $\partial/\partial x$ and the gravitational term, where we had one volume term. So, here it is ρg_x, and we take this as Equation 1. So, with this, we can say that we have come to a point where the equation of motion, which we have already said is a very big 1. And we have come to that equation of motion up to forming equation (1) right. So, with this, we can thank you, and we will meet again in the next class and continue with the equation of motion, as this will take some time. Because, as you have seen, it is so complicated—so many terms are there—and this applies to both expansion and contraction, it will become another approach, or not just another approach, that we also have to discuss, right?



Thank you, all.