Design of Farm Machinery

Prof. Hifjur Raheman
Agricultural and Food Engineering Department
Indian Institute of Technology Kharagpur
Week – 10

Lecture 50 : Geometry of knife section and model for estimating load causing failure of stem

Hi everyone, this is Professor H. Raheman. I welcome you all to this lecture on the Design of Farm Machinery related to the geometry of knife sections and models for estimating the load causing failure of stems. The concepts that will be covered are the geometry of knife sections, then models representing the plant stem, then estimation of the load causing failure of the stem in bending, and then a few numericals related to design.

I have shown you the cross-section of a blade. You can see in the first part, I have indicated some angles. So, I will discuss those angles.

Usually, we say the knife is fine, what does that mean? When I say the knife is fine, it means it has a small bevel angle. So, if you look at the figure, the bevel angle is indicated there, phi bk - the top surface making an angle with that edge. When the knife is blunt, that means it has a large bevel angle. Then the other parameter I mention is : the knife is sharpened. So, sharpness is defined by the radius r ek, the radius I have indicated here, the radius at the tip. A sharp knife has a small radius, while a dull knife has a larger radius. So, there is a difference between a sharp blade and a fine blade. So, fine refers to the bevel angle, and sharpness refers to the radius at the tip. Then, initial penetration of the knife into the plant material is aided by the knife's rake angle. If the rake angle is greater, then penetration of the point of the knife into the plant stem will be easier.

Then the third angle, which is nothing but the clearance angle, is the angle formed between the bottom edge of the knife and the xy plane. So, we have the bevel angle, we have the rake angle, and we have the clearance angle. So, the summation of these angles should be equal to 90 degrees. Just a minute - these three angles, which I mentioned, the summation of these should be equal to 90 degrees. The oblique angle is the angle which the knife edge makes with the y-axis - suppose this is the angle, this is the oblique angle. The direction of travel - the knife edge making with the direction of travel—that is called φ_{ok}.

Now, when I said it is a straight cut, that means the edge of the knife is parallel to the direction of travel. So, φ_{ok} becomes 0, and an oblique cut is one in which φ_{ok} is not equal to 0, meaning you will have gradual cutting. In a straight cut, at a time, the entire stem diameter is in touch with the knife, whereas in the case of oblique cutting, the stem diameter gradually comes in contact with the knife. So, that is why the force requirement in the case of an oblique cut is lesser than the force requirement of a straight cut. So, I have written here that oblique cutting reduces the peak cutting force because the plant material is sheared progressively rather than all at once, as in the case of a straight cut.

Then, one more angle I have defined is the chip angle, which is defined as $\varphi_{bk} + \varphi_{ck,}$ the summation of these two. This is called the chip angle. Then, one more angle is the clip angle, φ_{cl}, the angle between the knife and the counter shear. So, this is nothing but φ_{ok}, the oblique angle of the knife, plus φ_{oc}, the oblique angle of the counter shear, the summation of that is called the clip angle. Now, similar angles are available with respect to the knife, whatever angles we have defined, similar angles are there for the counter shear or the ledger plate. So, I have denoted the bevel angle for the knife as φ_{bk}, whereas the bevel angle for the counter shear is φ_{bc}. Rake angle for the knife is φ_{rk}, rake angle for the counter shear is φ_{rc}. So, the suffix k and c will denote, k for knife, c for counter shear or the ledger plate.

In addition to the angles we discussed related to the knife and the counter shear, there will be two angles associated with the plant stem. One is with respect to the y-z plane, and the other is in the x-z plane. That means, the angle the stem axis makes with the z-axis and its projection in the y-z plane is called the tilt angle. So, one angle is the tilt angle, and the other is the slant angle. The slant angle is the angle between the stem axis and the z-axis and its projection on the x-z plane.

Now, after knowing the angles, the next thing is determining what angle should prevent the sliding of plant material during cutting. So, one possibility is when φ_{ok} (the oblique angle) is not zero, and the plant material is not yet in contact with the counter shear. So, the possibility exists that the plant material may slide along the edge of the knife before or while being cut. So sliding is expected if the oblique angle is greater than φ_{okmax}. That means, we cannot take φ_{okmax} more than this expression: $\tan^{-1}(f_{ek})$, where f_{ek} is the coefficient of friction between the plant and the knife.

So, f_{ek} can be defined as the coefficient of friction, which is the ratio of the lateral force (parallel to the knife edge imposed by the plant) to the normal force imposed by the plant. So, lateral force divided by normal force is the coefficient of friction f_{ek}. Therefore, the maximum value of φ_{ok} should be $\tan^{-1}(f\ ek)$. So, you must keep it less than this to prevent sliding under this condition. The condition is the plant is not in contact with the knife, but there is still a possibility of slippage.

The second condition of slippage is when the plant is in contact with both the knife and the counter shear, but it is expected to slip. So, in that case, the clip angle should be greater than the following maximum. So, this is the maximum we have to follow, which is equal to $\tan^{-1} (f_{ek} + f_{ec})/(1 - f_{ek} \times f_{ec})$. That means, f_{ek} is the coefficient of friction between plant and knife, and f_{ec} is the coefficient of friction between plant and the counter-cutting edge of the ledger plate. So, while deciding the angles - clip angle or the oblique angle - we should take these factors into consideration. That means, we have to take the coefficient of friction into consideration: the coefficient of friction between knife and plant, and between counter shear and the plant.

I will show you a typical force-displacement curve when the plant material is cut by a knife. So, this is a straight cut. So, what you can observe is initially it starts rising, and it rises at a faster rate, reaches a peak, then it goes down. The entire - this is nothing but your stem diameter. Now, this has been divided into three segments. The first segment is when the blade or the knife just comes and touches the plant - that is the point here. As the knife moves, that means, it enters into the stem - our pressure requirement, the force requirement, will increase, and the crop will not allow the knife to enter. So, it will oppose the liquid present in the cells will oppose the movement of the knife. So, that is why the force will rise. So, the knife will try to compress. So, at a point, the stem will initially fail, and some compression still continues. Further, there will be a rise, and in section C, finally, the stem has failed, and the plant stem has completely compressed. Since the knife is continuously moving, the force will drop down. So, this is how cutting takes place. So, this is for a straight cut. Now, if it is an oblique cutting, the nature of the curve will be little different. That means, you may not get this much higher peak force, you may get some force like this and the duration will be again increased. This is a gradual cutting. So, the rate of rise will be slow and it will be for a longer period. So, that is the difference between a straight cut and oblique cut.

And to find out the knife edge force, we have to multiply the projected frontal area of the knife edge with the pressure which is imposed on the knife edge by the plant. So, the approximate frontal area if you can calculate so that can be represented by A ek, frontal area of the knife per unit width of the knife that will be equal to the radius at the tip, $r_{ek}(1 + \cos(\emptyset_{bk} + \emptyset_{ck}))$. $\emptyset_{bk}$ is bevel angle, $\emptyset_{ck}$ is clearance angle. So, r_{ek} is the radius of the knife, is in millimeter and a frontal area will be in millimeter square per millimeter of width.

Now, if you look at the grass stem cross section - if you look at you can see the grass stem is comprising of nodes and internodes. Nodes are more or less solid whereas, internodes are hollow. So, if you, if you carry out cutting here it becomes easier, if you carry out cutting here, some more force will be required. Now, or if you take a forage crop you may get this type of structure nodes and internodes or you may get some structures like cell fibers are there throughout the stem at different locations. So, how to represent the models of this stem?

So, the simplified models which are drawn to represent the actual stress actual cross section of the hollow stem for analyzing stem strength in bending is shown. This is one where the center one is hollow one. This is one where the center one is hollow, but the cell fibers are just like reinforcement they are uniformly distributed like this. So for analyzing or mechanical analysis we represent the stem by these two models. That means, a concentric cylinder model and a model with reinforcement rods. Then the strength is determined by the amount of structural fiber. These are nothing, but structural fiber more the structural fibers higher will be the strength of the stem. And if you denote it by the outer diameter then what will happen now - you can see this inner one is hollow that means, we are estimating for a higher cross-sectional area. So, that may give you some erroneous results, but since it is difficult to locate this, so, what we do is generally for all calculations we take the total diameter. Now, let us see what is the bending strength which is required while carrying out cutting, because bending strength will try to oppose the force which is applied by the knife. So, if you want to find out what is the force required to cause failure of the stem, then we have to follow the stress due to bending. Then stress due to bending should be higher than the ultimate stress - ultimate stress of the plant stem, so that cutting can take place.

So, if you take that into consideration, there are two possibilities of cutting either you use a counter shear or you may not use a counter shear. there are two possibilities of cutting either you use a counter shear or you may not use a counter shear. So, in both the cases you are applying a load which is normal to the axis of the stem. So, but the situations are different. Situations are different in the sense when a counter shear is not present then the loading which is applied and then the stem will be treated as cantilever beam. Because it is supported at the ground level and we are applying load at a certain height, where we try to carry out cutting. So, we consider that to be the case of a cantilever beam.

Now, when a counter shear is present or the ledger is present then there is a support, support at the ground level, support at the top. The support is in fact, given by the knife and it is the counter shear which will cut. So, that is considered as a simple supported beam. So, the two conditions, when there is a counter shear then it will be treated as a simple supported beam, when there is no counter shear then it is treated as a cantilever beam. Now, in both the cases, the direction of loading is radial that is means is perpendicular to the stem axis. So, now, the stress which is induced that should be greater than the ultimate strength of the stem, then only cut - the plant will be cut otherwise plant will not be cut. So, now, if you apply the flexural formula. So, stress - ultimate stress we are taking should be equal to (bending moment, M/I - this moment of inertia) × y. Now, this has been represented in a different way that means, the stress, M is the bending moment is nothing, but the radial load which you applied, which is denoted as F_{bu} × length, length - the span - cantilever span or the height of cut you can say. So, F_{bu} ×L is nothing, but your moment and moment of inertia is I. So, assuming the stem section to be either solid section or is a hollow section. So, you can find out moment of inertia. Now, if I substitute in this equation M. So, what will happen? $S_u = F_{bu} \times L \times Y/I$. Y is nothing, but the stem radius that means, d/2. So, I can write as $F_{bu} = S_u \times I/(L \times d/2)$. So, this d/2 is represented as C, radius from neutral axis. So, this has been given like this. So, why I have taken S_u is because we want that ultimate stress because we want that the plant should fail, the stem should fail. So, that is why you have taken S_u value. So, from here if you know this S_u value and assuming the cross section to be either solid or hollow, we can find out what should be the force required to cause failure of the stem. So, if you consider it to be a solid circular section, then $I = \frac{\pi d^4}{64}$ and if you consider that it is a hollow section with a thickness of t, then the moment of inertia, $I = \frac{3\pi d^3 t}{32}$, where d is the diameter, t is the thickness, wall thickness.

Now while carrying out cutting whether it is a simple supported condition or whether it is a cantilever condition each will be associated with a deflection. That means, the stem is going to deflect and the deflection is given as, $\delta = \frac{F_r L^3}{C_b EI}$. Delta is the deflection F_r is the radial load which is applied that means, which is nothing but F bu, just now we calculated and L is the span height of cut C_b is the coefficient it depends whether if it is a cantilever beam then the value is 3 if it is simple supported beam the value is 48. Now, E is the modulus of elasticity and I is the moment of inertia So, if you know these values or if you calculate these values, then you can find out what is the force required to cause failure and the corresponding deflections. So, this will help you in designing what should be the knife speed, so that you can exert that much of force on the plant stem, so that plant is going to fail. So, I will take up some design problems which will further clarify these equations.

So, the design problems are that during cutting of a grass stem of 3 millimeter diameter with a knife without counter shear, it is loaded horizontally at a distance of 30 millimeter above the soil surface. Based on the entire stem cross-section, the modulus of elasticity is 1600 Newton per millimeter square and the ultimate tensile strength is 35 Newton per millimeter square. Calculate the horizontal force that would cause bending failure and the horizontal deflection of the stem at the point of failure.

So, whatever equation just now we discussed the same has to be applied for finding out the horizontal force we defined that $F_{bu} = \frac{I}{Cb}\frac{S_u}{L}$, C b into L this is our equation.

Now ultimate strength is given as 35 Newton per millimeter square. So, S_u is given as 35 Newton per millimeter square. I, you have to calculate, $I = \frac{\pi d^4}{64}$ assuming to be a solid circular section then moment of inertia is nothing, but pi d 4 by 64. Diameter is given. So, we can calculate $I = \frac{\pi \times 3^4}{64}$. Now, C_b, C_b is nothing, but radius radius of the stem which is equal to 3 by 2, 1.5 millimetre. Now, L is nothing, but 30 millimetre. So, 30 millimetre. Now, you substitute in this equation. So, you will find out F_{bu} will be equal to 3.09 Newton.

Now, what will be the deflection - horizontal deflection? So, delta will be equal to this $\delta = \frac{F_{bu}L^3}{C_b EI}$ moment of inertia. So, what is F_{bu}? This is the force which we just calculated. The length of the span can be - this is a cantilever without a counter shear, which means this is a cantilever condition. So, L is nothing but 30 millimeters, and C_b value is 3 for cantilever. C_b value will be 3. E is the modulus of elasticity, which is nothing but 1600 Newton per millimeter square, and the moment of inertia you just calculated. This moment of inertia will come around 3.976. So, if we substitute in this, then we will find out delta will be equal to 4.376 millimeter.

So, let us now see if, instead of without counter shear, you provide a counter shear, how it is going to affect. So, the question is when you try to apply with a counter shear – so, now the model will be similar to your simply supported beam. Now, the simply supported beam means one end, the knife is trying to push the stem, and this is being supported by the counter shear and the ground - this is the ground. So, actually, this and this point - these are the supports. The counter shear is going to create that failure. So, what you have to do is this: the length now will be reduced to - initially it was 30 millimeters - now it will reduce to $30 - 10$, because the gap between the knife and the ledger plate should not be more than the diameter of the stem; it is lesser than that. I have taken the gap as d, that means this distance we have taken as d. So, now, new L, that is denoted as L', becomes $30 - d$. Now, you put in this formula. So, F_{bu} - now the radial force which will cause failure to the plant will be in case of with counter shear.

Not given in the question, I just wanted to give you the importance what is the role of countershear. And how the design criteria are changed. So, F_{bu} will be equal to S_u does not change S_u remains as 35 Newton per millimeter square. I, moment of inertia also does not change it remains 3.976 that is $I = \frac{\pi d^4}{64}$. C_b value is nothing, but it - it also does not change. So, that is 1.5. L value will change that becomes 30 minus d, d is taken as 3 millimeter. So, instead of 30 you can take 27. So, the force required to cause failure will be 3.435 Newton. Initially, we are getting 3.09 Newton, now we are getting a little higher force.

Now, what is the corresponding deflection? Now, deflection if you put in that formula, deflection is nothing, but $\delta_r = \frac{F_{bu}L^3}{C_b EI}$. Here again C_b value will change. The previous case when there was no countershear, it was a cantilever. So, the value of C_b is 3, now the value of C_b is 48, E remains same, I remain same, L will change. L will change to 27 millimetre, F_{bu} will also change to 3.435 Newton. So, deflection we are getting very very low 0.2214 mm. So, this is the difference in calculation when a countershear is provided and when the countershear is not provided. So, the only thing is you have to take care of the span, the length at which you are trying to cut.

Now, impact cutting is to be used to cut forage crops with stem diameters of 15 millimeters at a height of 30 millimeters above the ground. The mass of the plant above the cut is 0.1 kg. Assume that cutting occurs when the pressure ahead of the knife edge reaches 25 Newtons per square millimeter. The knife has a bevel angle of 20 degrees, zero clearance angle, an edge radius of 0.3 millimeters, and a width of 25 millimeters. Calculate the minimum knife speed required for impact cutting. If the CG height of the cut material above the plane of cutting is 15 centimeters and the bending resistance is 1.8 Newton meters.

Now, the first thing is you have to calculate what is the force which is applied. For calculating the force which is applied, if you look at this figure, F_x is the force which is applied. Then you have to find out - what is given is the pressure, which is given as 25 Newtons per square millimeter. And to find out the force from this, we need to know the area of the blade. So, the area of the blade, $A = r_{ek}(1 + \cos(\emptyset_{bk} + \emptyset_{ck}))$. Here, φ_{bk} (bevel angle) is given as 20 degrees, and the clearance angle is 0. So, we can calculate A - will be equal to 14 - sorry, 0.5819, this is per unit width. Now, the width is given as—the width of the knife is given as 25 mm. So, you just multiply by 25, that will give you the area - the total area. So, 14.547 square millimeters.

Now F_x will be equal to this area multiplied with the pressure. So, 14.547 into pressure, pressure is given as 25 Newton per millimeter square. So, that way we will get 363.69 Newton. This is the force F_x. Now, bending resistance F_b is given as 1.8 Newton. So, F_x we have calculated F_b is given 1.8 Newton.

Now, to find out the knife velocity, $V_k = \sqrt{\frac{(F_x - F_b)Z_c \times d}{(Z_c + Z_{cg})m_p \times 1000}}$. So, directly it will give you in meter per second. So, d is in millimeter So, F_x value we have computed, F_b value is given, Z_c value is given as 30 and Z_{cg} value is given as 15. So, m_p value, this will be 15 centimeter that means, 150 millimeter. So, this is 30 + 150. Now, mass of the plant is given as 0.01 kg So, if I put this is 1000 not 100, 1000. So, now, I put 363.69 minus 1.8 into Z_c is nothing but 30 mm, then diameter is 15 mm divided by 30 + 150, The mass of the plant is given as 0.01 kg, sorry, 0.01 kg directly into 1000 square root. This will be the equation. So, the result will be in meters per second. This is what we have to calculate. The main thing here is what should be the force which is applied? So, for calculating that force, we need to know the area because the pressure is given. If the force is given directly, then we can multiply and find out what the total force acting is. Since pressure is given, you have to find out the area and then multiply the pressure to find out the force. Now, once you know the force, then the rest of the things are given. The data is given. So, you can see how, utilizing that equation, you can find out what should be the knife speed so that cutting can take place. So, that is all.

The references are: these are some of the references, and in conclusion, we can say we discussed the different angles which are associated with a knife as well as the countershear and how to represent the plant stem models. Then, we discussed how to find out the force required to cause failure of the plant and what are the different conditions, when there is countershear present or when no countershear is present and what are the corresponding deflections? So, this will help you in designing either a vertical conveyor reaper or a combined harvester cutting unit.

Thank you.