Traction Engineering
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Lecture 09
Characterization of Shear Stress and Shear Strength in Different Soil Conditions

Hi everyone. This is Professor H. Raheman from Agricultural and Food Engineering Department, IIT Kharagpur. I welcome you all to this NPTEL online course on Traction Engineering. And this is the lecture number 9, where I will try to cover Characterization of Shear Stress and Shear Strength in different Soil Conditions.

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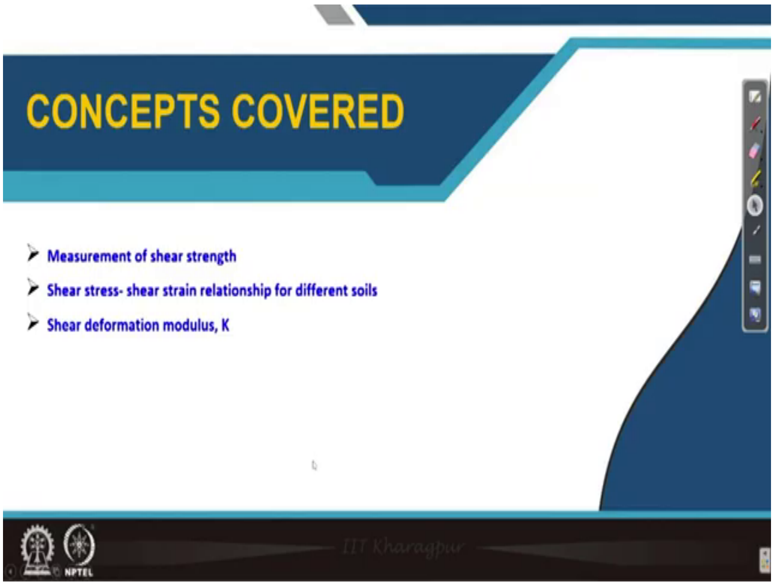

As the shear stress is important in deciding the tractive force which is developed, so that's why you are going to take up this lecture, where the concept to be covered are measurement of shear strength, how we are going to measure the shear strength of soil. Then shear stress-shear strain relationship for different soils, then how to find out deformation modulus K?

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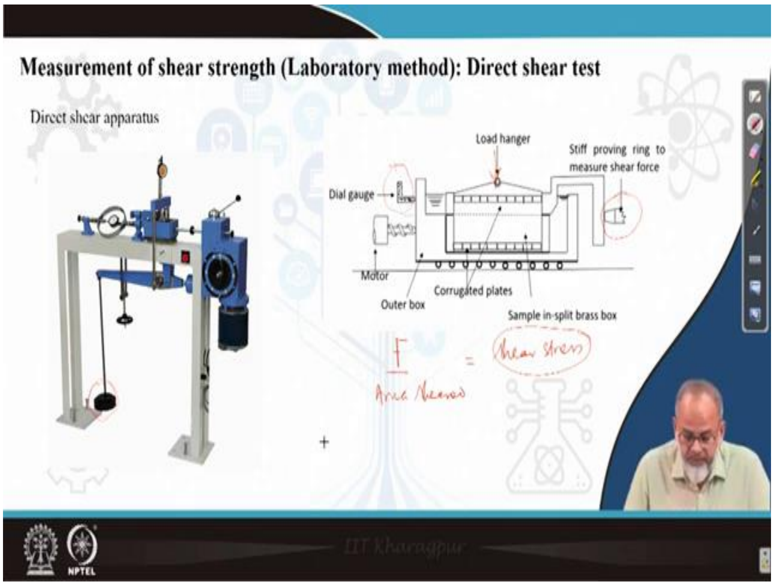

So, to start with how to measure shear strength of soil. You can measure in the laboratory or you can measure in the field directly, so the apparatus which is used for measuring shear strength, both in the laboratory as well as in the field, is called direct shear apparatus. And this direct shear apparatus, which is used in the laboratory is shown in this figure, where you can see the soil, whose strength or shear strength is to be determined. That has to be ~~keep~~, kept in a box, so basically this is the, inside this there is a box, shearing box, which comprises of two halves and they are of square cross section, 6 centimeter by 6 centimeter and the depth is also 6 centimeter. And the soil to be tested is to be kept inside this box and then the box is kept inside a container and the container, one end of the container is attached to the motor for giving the tangential force. The other end is connected to a proving ring, so that will measure the tangential force which is applied. So basically the shear box is shown in this on the right side figure, it comprises of two halves, two equal halves, in fact, and then the soil to be put. Initially, you have to put, keep these two halves intact, so you have to put some pins, so that the box is not displaced, then on the bottom of the box, there will be a corrugated or serrated plates. So this plate is provided, so that there will be no slip of soil during tangential force application. Then similarly there will be a serrated plate at the top, so then this lower portion of this shear box is resting on a container, where the container is almost fixed and is rolling over this or sliding over these rollers, which are provided at the bottom. So we apply a tangential force after removing the pins, which are initially given to a, which are initially put in that shear box, then we try to apply a force through the help of a motor.

So by rotating the, by switching on the motor, the motor will try to give a force to this lower box and this is transmitted to the upper one and the resistance is measured with the help of a dial gauge, which is mounted on the right corner. So the displacement also can be measured by putting a dial gauge. So for measuring the displacement, we have to put a dial gauge at the left corner, this one.

And for measuring the force, which is applied tangential force that we have to provide a proving ring with the dial gauge, so this dial gauge and the right corner like here, so this dial gauge and, this proving ring is, they are to be calibrated initially before putting into the system. So now we apply a constant strain and what we measure is the tangential force. So tangential force if you denote it by F divided by the area which is sheared. So, that will give you shear stress.

So, now we have to find out what is the area which is sheared. So the shearing takes place gradually, at a constant rate,. So here this shearing is taking place along this dotted line. So, this has to be conducted at several normal weights that means you have to have some arrangement, like here we have indicator, that is a load loading yoke is there. So, when you put weight on the left hand side there, with the help of leverage it is going to apply a load on the top of this shear box, with the help of a roller or a ball. So, that the load is more or less uniformly distributed over the entire surface.

So, the shear stresses are to be measured for different loading conditions. It can vary the weight and then find out shear stress versus normal stress curve. And we can also plot shear stress versus the shear displacement. So then what we will do with this data which we obtained from this test that we are going to discuss.

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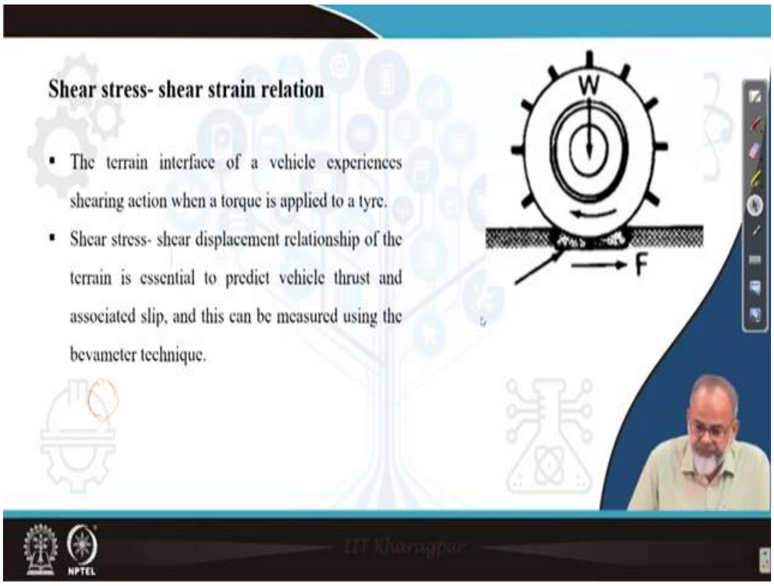

So the data will be utilized for finding out the shear stress. Shear displacement data will be utilized for finding out what is the maximum stress which is occurring or which is causing the failure of the soil. So this test setup which we discuss, it has a disadvantage, disadvantage in the sense, many disadvantages.

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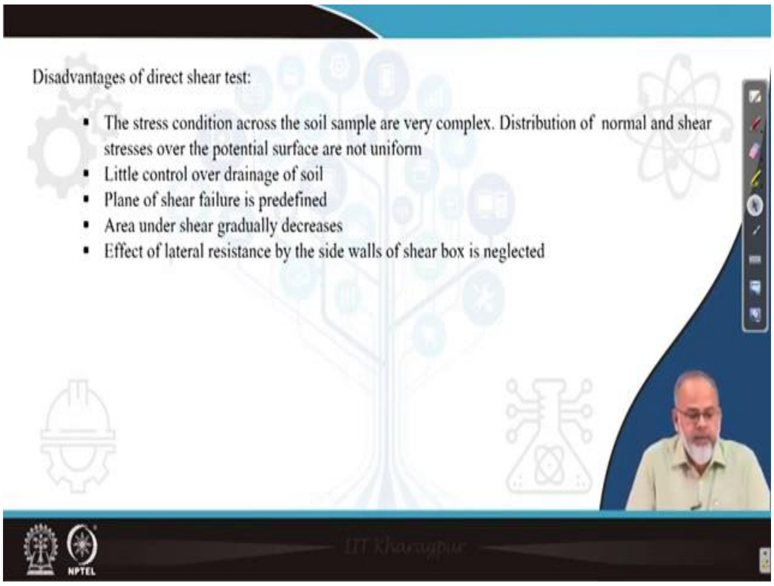

The first disadvantages among the 1, 2, 3, 4, 5 is, the stress conditions across the soil sample are very complex. So, distribution of normal and shear stress over the potential surface are not uniform. There is little control over drainage of soil, second disadvantage. And the third disadvantage is, the plane of shear failure is predefined, that means we are forcing the soil to fail in a particular plane, here the plane is horizontal.

Then area under shear that is going to reduce with time, so it is not constant, but in our calculation when we take the shear stress divided by area that we are taking as constant, so that is another difficulty. Then effective lateral resistance, because the soil is confined to a box, so there may be a side wall effect, while measuring the shear force. So these are some of the disadvantages. But if you want to find out shear strength in the control condition, then in the laboratory, this is the best method by which you can find out.

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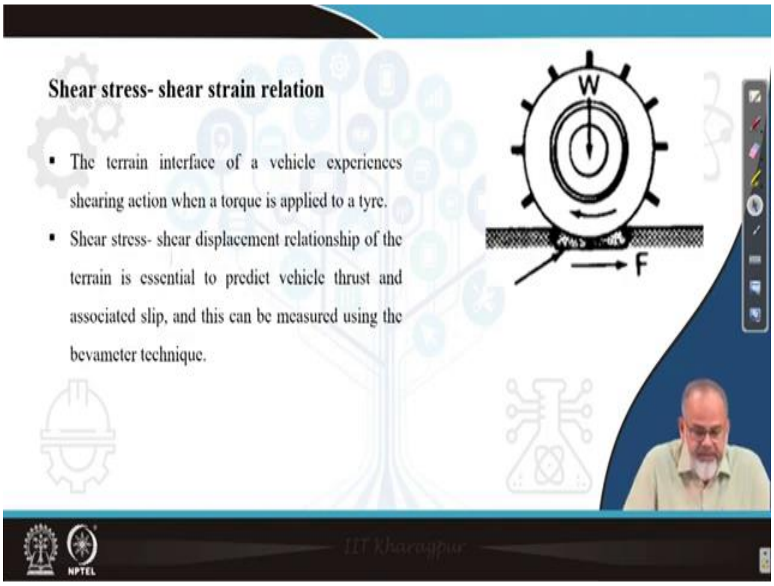

Now the shear stress, why you are interested because the shear stress which is developed at the bottom, that is at the interface of the wheel and soil, that is going to create the tractive effort. So shear stress at different displacement has to be measured or has to be identified or we have to find out the relationship between shear stress and shear displacement.

So that we can find out how much will be the thrust, which will be developed at different displacement values. So there are different ways which we talked about this to find out the shear stress, one is your bevameter technique, where you carry out plate sinkage test and shear annulus test to find out the shear strength of soil.

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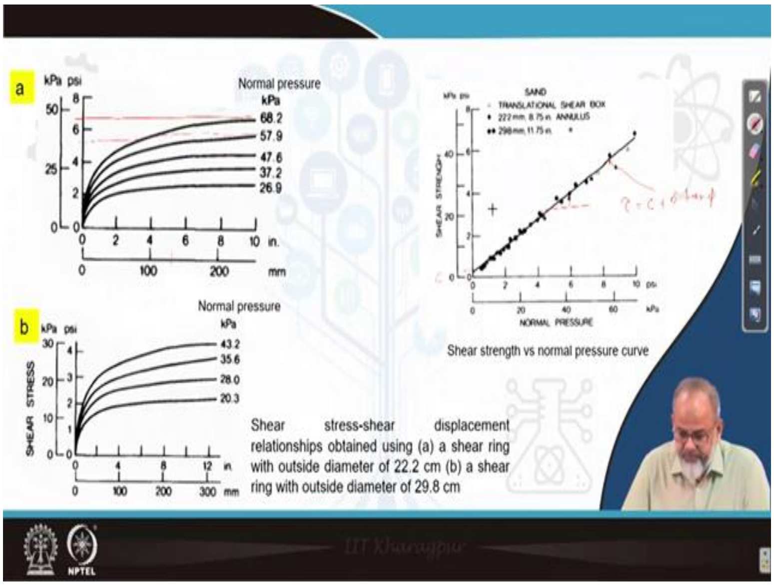

Now we will see what are the data which you obtained from shear stress - shear displacement relationship, which we obtained by using a shear ring of different outside diameter. At the left side figure if you look at, this test has been conducted using a shear annulus with an outer diameter of 22.2 centimeter and this is carried out in sandy soil and the normal pressure which you applied is starting from 26.9 kPa to 68.2 kPa.

Now if you look at the curve, so initially the shear stress is rising at a very faster rate, then after that if we increase in shear displacement it is almost nearly constant, we can say. So, this is the trend which is observed for all those four or five normal pressures which you carried out. Now when we try to carry out the same test with a different shear annulus apparatus, different means the outside diameter has been increased to 29.8 centimeter. And the normal pressure which you varied is from 20.3 kPa to 43.2 kPa. We see the same nature of curve that means initially it is increasing at a faster rate, then it is slowing down. So what we conclude from here is, whether you use a smaller annulus or whether you use a bigger annulus that is not going to affect the shear strength measurement.

But one thing is sure that the shear strength or the shear stress is dependent on shear displacement and normal pressure. So, from these curves if you try to find out the maximum shear stress values. So, maximum shear stress means, I will just draw a line like this, horizontal line, so that will give you maximum shear stress, that means beyond that the soil will start to lose that means fail.

So, if I take those maximum values and try to plot with respect to normal pressure. What you can observe is, the curve which you get in the right side, so these are the points which are representing the maximum shear stress and corresponding a particular normal pressure. Now if you best fit this line and extend it to touch y-axis you will find out at the y-axis the intercept is, it has some intercept, and it has some slope.

So now if you look at the Coulomb's equation,

$$\tau = c + \sigma tan\varphi$$

So now if you fit this equation to this line, then the intercept is nothing but your cohesion and the slope is nothing but your angle of internal friction. So this is how we find out shear strength at different normal pressure and from there, we try to find out the values of c and φ for a given soil.

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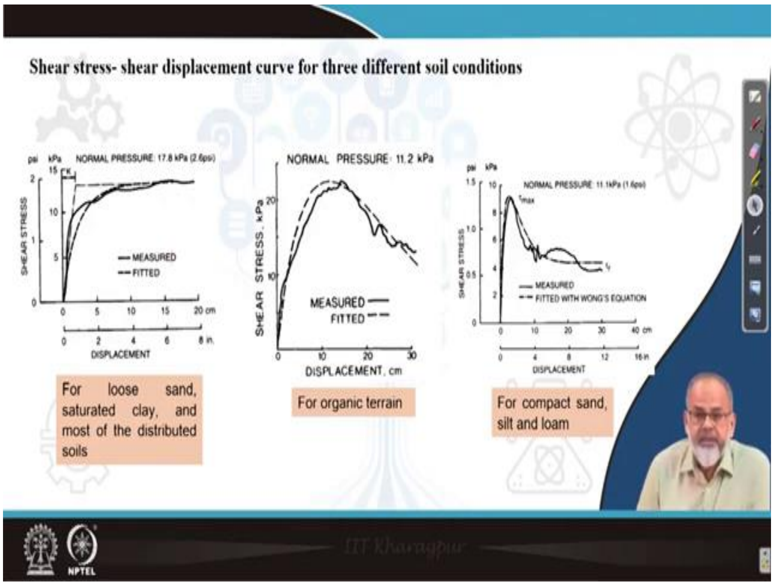

Now from the experimental data, what you observed is there are three basic characteristics of shear strength and displacement curves. Three basic curves we can see. One is your for loose sand, saturated clay and disturbed soil, one nature is it. Then for organic terrain we get another nature and for compaction, silt and loam we get another nature of the curve. This is the variation of shear stress-shear displacement.

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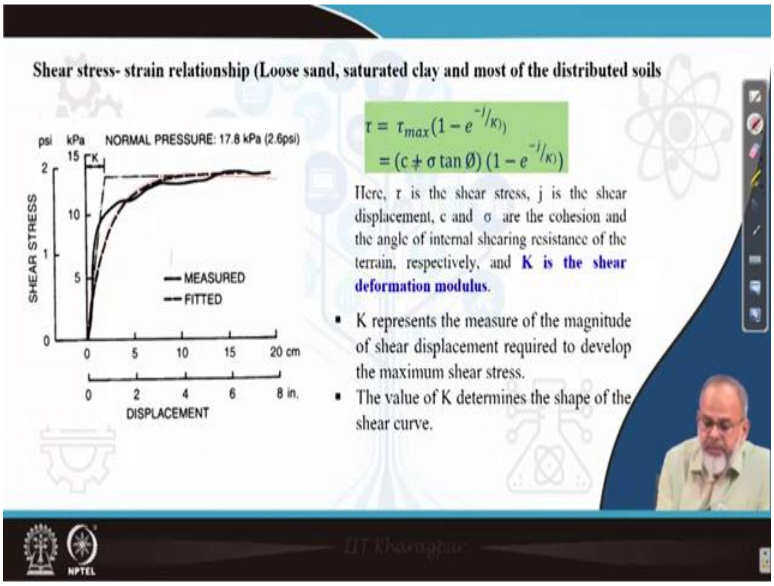

So now we will go into details of this one, how the shear stress and shear strain relationship is observed in case of loose sand, saturated clay and in most of the disturbed soils. So, the nature of the curve is like this, the x axis is your displacement and the y axis is your shear stress. Now if you look at this, initially, this is rising at a very faster rate, then after some time it is almost stable.

So now if you best fit that one, the curve will be exponential, like this. So, this exponential curve can be given by an equation which is given here,

$$\tau = \tau_{max} \times \left(1 - e^{\left(-\frac{j}{k}\right)}\right)$$

Where τ is a shear stress at a displacement j and τ_{max} is the maximum shear stress and j is the shear displacement and K is the shear deformation modulus. And this K, in fact, represents the major of the magnitude of shear displacement which is required to develop the maximum shear stress.

The value of K is going to determine as the shape of the shear curve. Now how to find out this value of K? Now if you look at this graph on the left side, what you can see is, I have drawn two tangents to the original curve, one is a tangent at the 0 point, at the origin, the other one is a tangent to the maximum shear stress that means it is basically a horizontal line and they will intersect at a point.

So distance of that point from the vertical axis is taken as K, which is shear deformation modulus. Now if you look at the equation, then tau max can be represented by

$$c + \sigma tan\varphi$$

that means

$c + \sigma tan\varphi$

so

$$\tau = (c + \sigma tan\varphi) \times \left(1 - e^{\left(-\frac{j}{k}\right)}\right)$$

The value of K may be represented by the distance between the vertical axis as I told and between the vertical axis and the intersection point.

And the slope of the shear curve at the origin can be found out by taking this dτ/dj. If you differentiate at j is equal to 0, displacement is equal to 0, then, from that equation we found that

$$\frac{\tau_{max}}{K} \times e^{-j/K}$$

Now, at j is equal to 0, this is simply comes down to τ_{max}/K.

If you cannot do this exercise, then the other way of finding out K is, you have to take the one third of the shear displacement where, shear displacement which is 95 percent of the maximum shear stress, which is 95 percent. Suppose the shear displacement is maximum here, this is the maximum shear displacement. So 95 percent of 15, so somewhere here, now one third of that one, see this displacement value will give you directly the value of K.

The unit of K is in millimeter. But, in actual practice though I have drawn a very nice curve and fitted an equation, but in actual practice the shear curves are not that smooth, so in order to get an optimum value that will minimize the errors in representing this curve that can be utilized, that can be obtained by this equation,

$$K = \frac{\Sigma\left(1 - \left(\frac{\tau}{\tau_{max}}\right)\right)^2 j^2}{\Sigma\left(1 - \left(\frac{\tau}{\tau_{max}}\right)\right)^2 j\left[ln\left(1 - \left(\frac{\tau}{\tau_{max}}\right)\right)\right]}$$

So by this you can get an optimum value of K which will minimize the error in explaining the behavior of the shear stress-shear displacement curve.

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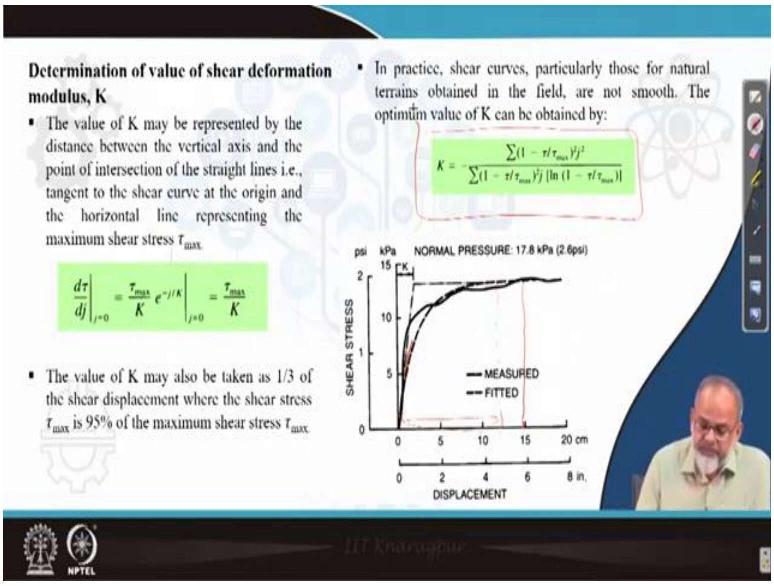

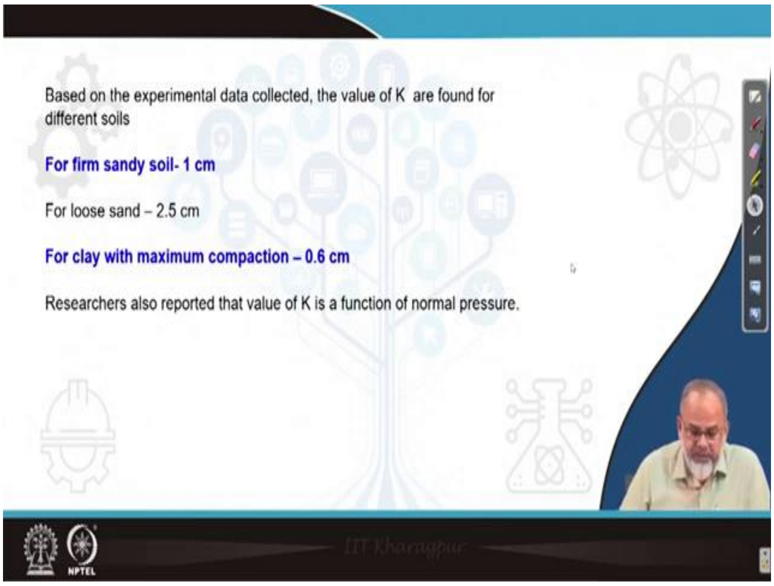

Next is another cell condition, before that we can, some experimental values are available for K. So, those values can be taken as for firm sandy soil, K value is 1 centimeter, for loose sand it is 2.5 centimeter and for clay with maximum compaction, it is 0.6 centimeter. So researchers they have also reported the value of K to be a function of normal pressure. So these are some of the information which are available for K.

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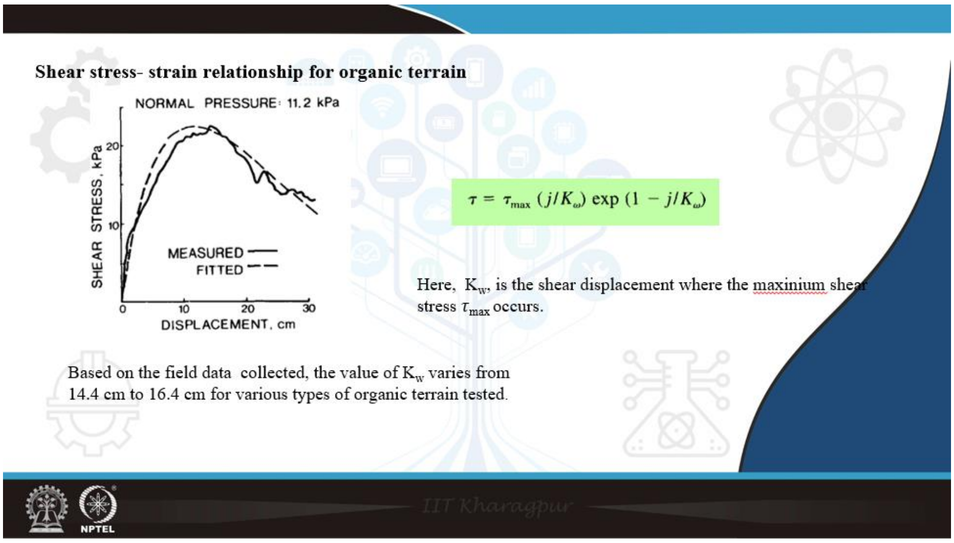

Next thing is, we will try to find out what is the shear stress shear displacement in case of organic terrain. Organic terrain means the top cover is your vegetative cover is there on the top and on the bottom there is, there will be saturated peat or saturated clay. So if that is the condition of soil then the shear stress-shear displacement curve will look like this, which is shown in the left side of the figure, left side of the slide.

So, it is initially rising to a maximum value, in fact, it can get a hump here, then it is going down. So, now if I best fit this one, so the best fit curve is given by this dotted line and the equation to that is given as

$$\tau = \tau_{max}\left(\frac{j}{K_w}\right) \times e^{\left(1-\frac{j}{K_W}\right)}$$

Here K_w is the shear displacement, where the maximum shear stress occurs.

And the value on the field data, based on the value of field data, K_w values they vary from 14.4 centimeter to 16.4 centimeter for various type of organic terrains, which are tested. So, this curve has been drawn for a normal pressure of 11.2 kPa, so the moment you change the normal pressure you will get another curve, but the nature of the curve will remain nearly same, that means initially it will rise, rise to a peak and then slowly it will go down.

So the peak is raised, so long as the vegetative cover on the top soil is offering resistance, the shear stress is increasing. The moment, the vegetative cover fails, then there is no more resistance, the bottom peat is not going to offer same resistance, so that is why the shear stress value goes down.

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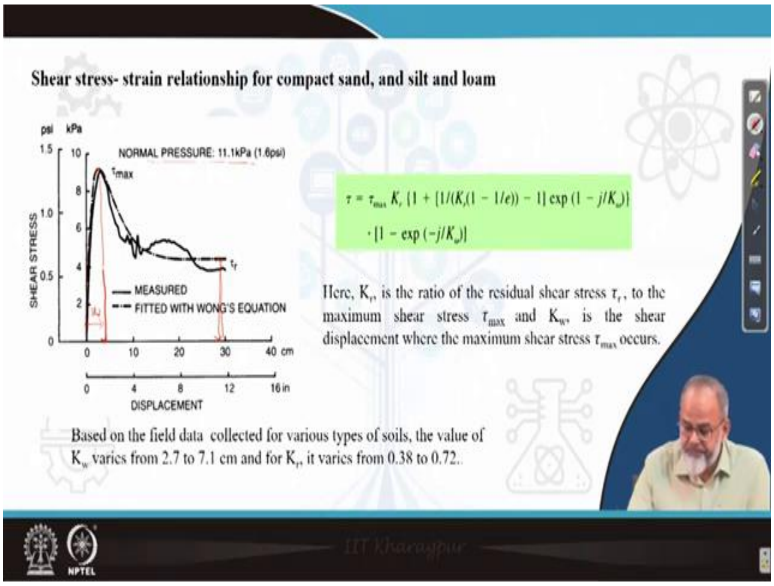

So next relationship is what is observed for compact sand and silt and loam. Whether it is a compact sand, whether it is a silt soil, whether it is a loamy soil, for all these three conditions you will get or you will experience a curve like this, which is shown in the left side of the slide, the curve is like this. Initially, it rises at a very faster rate, then it reaches to a peak, after that it goes down and then remains almost static.

So, this kind of curve can be best fitted as

$$\tau = \tau_{max}\left\{1 + \left[\left({}^{1}/_{K_r}\right) \times \left(1 - {}^{1}/_{e}\right)\right] e^{\left(1 - \frac{j}{K_W}\right)}\right\} \times \left[1 - e^{\left(1 - \frac{j}{K_W}\right)}\right]$$

Here two things, two values are there, one is K_r and K_w. So, what is K_r? K_r is the ratio of residual stress to the maximum shear stress. If you look at the figure, the residual stress corresponds to this one.

That means after the curve has become almost constant, so that stress is called the residual stress and the maximum stress is corresponding to this peak value, that is your maximum stress. So, ratio of this τ_r residual stress to maximum stress tau max is denoted as K_r and K_w is the shear displacement where the maximum shear stress occurs that means this value, K_w. So, if you know K_r and K_w then at any value of j that is shear displacement, we can find out what is the corresponding shear stress if you know the maximum shear stress value. And the nature of the curve which I indicated is for a normal pressure of 11.1 kPa, and if you change the normal pressure then peak will rise but the nature will remain same.

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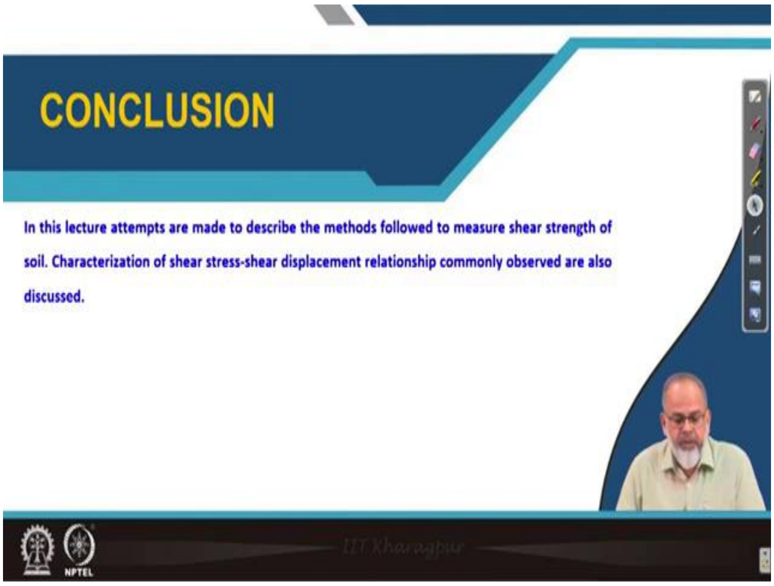

So, in this lecture I tried to give the method which is followed for measuring shear stress in the laboratory and then the nature of the curves which are generally observed, nature of the curve means nature of shear stress-shear displacement, which are observed in nature that are discussed. Then that will help you in characterizing the shear stress-shear displacement relationship and these relationships can be utilized to find out the shear stress.

If you are interested in deciding or in determining the shear stress values, which will be utilized later on for measuring the strength of soil, for measuring the strength of soil, as well as for developing the tractive effort.

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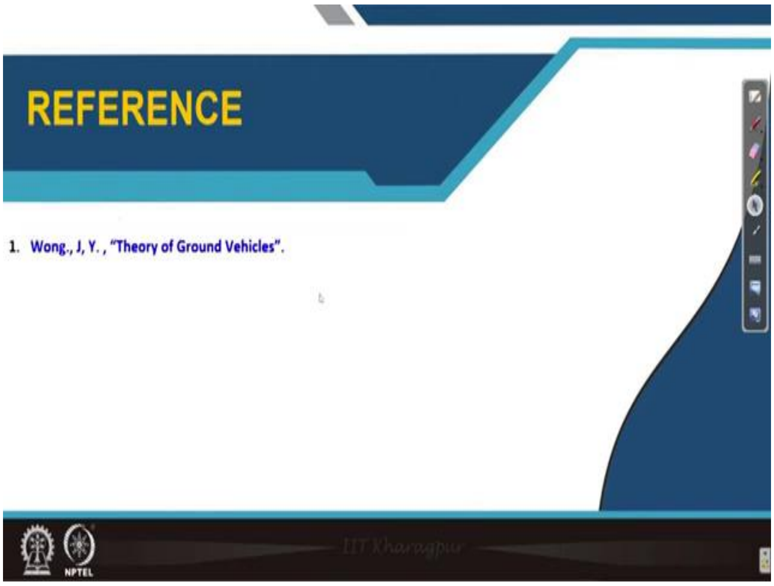

So the reference you can take as 'Theory of Ground Vehicles' by J. Y. Wong.