Traction Engineering
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Lecture 40: Tutorial - 8

Hi everyone, this is Professor H. Raheman from Agricultural and Food Engineering Department. I welcome you all to this traction engineering course. This is lecture 40 where I will try to give a tutorial on how to compute slip angle, understeer coefficient and verify the slide slope stability of a tractor while moving across a slope.

So, the concepts will be a computation of lateral force and slip angles developed by a tractor. Then we will be finding out under steer coefficient to decide the steering condition whether it is under steer, over steer or neutral steer. Then we will try to verify the stability of the tractor while moving across a sloppy land. So, I will discuss three problems.

The first problem is a 6-16 tractor tyre has a cornering stiffness of 133.30 kilo Newton per radian and a camber stiffness of 229 Newton per radian. The camber angle at which the tyre is fitted to the axle is 4^{o}. If both the cornering force and camber thrust are acting in the same direction and they are linearly related to slip and camber angle respectively, then determine the total lateral force developed for a slip angle of 3^{o}. So, what we have to do in this problem is, we have been given the data related to cornering stiffness of front that is tyre 6-16 and the camber stiffness.

So, we know that total force which is developed side force developed is dependent on the lateral force due to slip angle $F_{s\alpha}$ plus lateral force due to camber angle $F_{s\gamma}$. So,

$$F_s = F_{s\alpha} + F_{s\gamma}$$

I have put a plus sign because I have indicated that they are acting in the same direction. So, that is why I have put a positive sign. If I have indicated that they are acting in opposite direction then obviously, I have to put a minus sign depending on their directions. Now what is given is, they are linearly related obviously, these slope, the slip angle is 3^{o} and the camber angle is 4^{o}. So, obviously, at a smaller angle, we know that a side force is linearly related to the cornering stiffness.

So, what you can write is

$$F_{s\alpha} = C_{s\alpha} \times \alpha = 133.33 \times 3 \times \frac{\pi}{180}\ kN$$

Similarly, for finding out the side force due to camber angle, we have to take the camber stiffness. So, $F_{s\gamma}$ which is the side force of the lateral force developed due to camber angle. So, I can write as

$$F_{s\gamma} = C_{s\gamma} \times \gamma = 229 \times 4 \times \frac{\pi}{180}\ N$$

So, what we get is in Newton, what we have got here is in kilo Newton. So, change the units then add it that will give you the total lateral force which is developed by a 6 16 tractor tyre with the given data. So, basically you have to see that what is the relationship since it has been given that it is linearly related. So, it becomes easier for you to calculate the lateral force. And the second thing which has to be seen is whether the directions of this lateral force that is due to slip angle and the lateral force which is developed due to camber angle whether they are in the same direction or they are in the opposite direction. So, that has to be seen then only you can easily calculate what is the total lateral force.

The next problem is related to the steering. A two-wheel drive tractor which has a total weight of 18.2 kilo Newton, it has a wheel base 1.85 meter with a static weight distribution of 30 percent at the front axle and 70 percent at the rear axle. The cornering stiffness of each of the front tyre is given and that of the rear tyre is given. So, what you are been asked to find out is draw conclusions for the slip angle of front and rear tyres ok. So, what you have to find out is first the understeer coefficient. For finding out the understeer coefficient, what you have to do is, k us is nothing, but weight coming on the front wheel divided by c, the cornering stiffness

$$K_{us} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$$

So, now in this data which are given we have to find out these values then only you can compute k us. So, what data is given, cornering stiffness is given for the front tyre and the rear tyre. So, these values are given.

So, $C_{\alpha f}$ is given as 38.92 and $C_{\alpha r}$ is given as 36.25 and W_f and W_r we have to find out. So, how to find out W_f is the weight coming on each of the wheel. So, the axle weight now you have given the distribution with a total weight. So,

$$W_f = 0.3 \times 18.2/2$$

for the rear axle,

$$W_r = 0.7 \times 18.2/2$$

So, you know $C_{\alpha f}$ you know W_f you know $C_{\alpha r}$ you know W_r. So, if you substitute in this equation that will give you a value. So, if the value is positive that means, this is an understeer condition. If value is negative this is an oversteer condition. If the value is 0 then it is neutral steer. So, we have to draw conclusion about slip angles.

So, slip angles are nothing, but α. So, α is nothing, but F by c s F, lateral force by C_{sf} so that means, what is the lateral force developed by C_{sf}. So, first we have to find out K_{us}. Once you know the K_{us} value then we will find out whether it is a neutral steer, understeer or oversteer. Now, if it is neutral steer the turning radius, the other component which I ask is turning radius is constant. If it is understeer then turning radius will be decreasing and if it is oversteering that means, when the K_{us} value is negative it is oversteering that means, the turning radius is increasing.

So, that is the conclusion you have to find out. Now, he is asking the determine the speed above which the tractor exhibit directional instability. So, when we find out K_{us} that will indicate whether you are in understeer oversteer condition. If you are in understeer condition, then you have to calculate the $V_{characteristic\ speed}$ which is nothing. But in this case, what we observed is, we are getting a K_{us} value which is negative that means that means it is an oversteering condition. So, oversteering condition means we have to find out the critical speed. So, critical speed, $V_{critical}$ will be equal to $\sqrt{\frac{g \times L}{-K_{us}}}$.

So, since K_{us} you got a negative value. So, when you substitute here, you will find out the velocity some value. So, roughly around it is coming 13.1 m/s so that means, this is the critical speed beyond which if you exceed then there will be directional instability. Suppose the data which are given that will lead you to a value of k us positive.

So, what will happen if is a positive value that means, the turning radius is decreasing. So, you have to calculate b characteristics which is nothing, but $\sqrt{\frac{g \times L}{K_{us}}}$. So, that will give you a value which will indicate the limit beyond which if you exceed that is going to give you directional instability. I hope I have clarified.

Then the third problem will be related to the side slope stability. A tractor fitted with 6 16 tire in the front axle and 13.6 28 tire in the rear axle. It has a total weight of 2400 kg with a weight distribution of 35 percent and 65 percent at the front and rear axles, respectively. The track width 1.2 meter and it is cg is at a height of 0.8 meter from ground and this tractor is moving across a sloppy land, which is 20^o with the horizontal. The front tire has a cornering stiffness, camber stiffness. The rear tire has a cornering stiffness of this much. The camber angle at which the tire is fitted with the front axle is given. If both the cornering force and camber thrust are acting in the same direction and they are linearly related to slip and camber angles with a slip angle of 4^o for the front tires and 3^o for the rear tires. Verify whether the tractor is stable during operation against overturning and sliding. So, for overturning we have to find out $tan\beta$ value.

So, $tan\beta$ is nothing, but 20^o, beta is given as 20^o. Now you have to find out

$$\frac{\text{track width}}{\text{height of c g from the ground}}$$

If $tan\beta$ value is greater than this, then there is a possibility of overturning. So, in our condition track width is given as 1.2×0.8/2. So, if you take tan inverse of this value, then this will give around 36^o. Whereas, $tan\beta$ will give you a value around 0.36 and and this value is around 0.5 something. So, that means $tan\beta$ is lesser than this expression. So, the vehicle is stable, it is not going to overturn. So, this is one condition.

The second condition is; we have to verify the stability against sliding. We have verified that the vehicle is stable against overturning. Now for sliding, we know that there is a vehicle or tractor is moving. c g is here. So, it will have a component $W \sin\beta$ and this is lateral force, which is developed at the rear wheels. Similarly, lateral force will be developed at the front wheels. Now $W \sin\beta$ has to be supported by this lateral force. So, now the second component we have to find out, what is the total lateral force acting on the tractor or the vehicle. So, to find out total. There will be two front wheels, there will be two rear wheels, assuming that each of the front wheels will be developing the same lateral force. So, for the front wheel, we have to calculate lateral force for the front wheel. We have to calculate lateral force due to slip angle. This has to be added with lateral force due to camber angle and they are linearly related, the thrust is a linearly related.

Total lateral force for front wheel = lateral force due to slip angle + lateral force due to camber angle.

$$=1000 \times 133.2 \times 4 \times \frac{\pi}{180} + 229 \times 4 \times \frac{\pi}{180}$$

This is the lateral force, which is developed at the front wheel. This has to be multiplied with 2 to find out the total lateral force developed by the front wheels.

Similarly, you have to find out the lateral force which is developed at the rear wheel since there is no camber angle in case of a tractor at the rear wheels. So, we are only considering lateral force due to slip angle only. So, it is given rear wheels cornering stiffness is given as 90.

$$= 1000 \times 90 \times 3 \times \frac{\pi}{180}$$

So, this has to be multiplied with 2 to find out the total lateral force developed by the 2 rear wheels. Now, since total lateral force you have to find out for the tractor. So, that will be summation of this force plus this force. So, that will be equal to, at least equal to $W \sin\beta$. W is given as $2400 \times 9.81 \times \sin\beta$.

So, this should be equal to at least then only the vehicle is not going to slide. If it is higher than these two summations, then the vehicle is going to overturn. If it is lesser than these two then the vehicle is stable. So, let me repeat this one. The first case is we have to find out regarding the overturning.

For overturning we have to find out $tan\beta$ value should be greater than T by 2 H, ok. So, this condition has to be satisfied. If $tan\beta$ is less than this value, then there is no overturning. Now, the second condition of instability is your sliding. So, that will come, this $W \sin\beta$ component should be lesser than this side force which is developed by the tractor. Since the tractor has 4 tires and we assume that each of the front wheel will develop the same lateral force and each of the rear wheels will be developing the same lateral force. So, we try to find out what is the lateral force developed by the front each of the front wheel. So, since front wheel has both slip angle and camber angle so you have to find out lateral force due to slip angle and you have to find out lateral force due to camber angle. And since the cornering stiffness and the camber

stiffness are given so by multiplying this. We can find out, what is the total lateral force acting on each of the wheel. Then we multiply with 2 to find out what is the total force from the front wheels.

And in case of rear wheel, since there is no camber. So, only you will calculate the lateral force which is developed due to slip and we follow the same procedure because the slip angle is given and the cornering stiffness for rear tires are given. So, whatever value is obtained that has to be multiplied with 2 and the summation of these 2 forces should be greater than this $W \sin \beta$ then only there will be no sliding otherwise there will be sliding. So, in brief I have discussed how to calculate lateral force, then how to calculate the steering characteristics that means whether the vehicle is understeer, oversteer or neutral steer. And basically this is calculated taking into consideration the understeer coefficient which is a function of weight distribution and is a function of the cornering stiffness. Then we try to calculate the side slope stability for 2 conditions, one is your overturning the other one is sliding.

I hope these 3 problems is, they are going to help you in understanding these concepts, concepts of over steering, under steering, side slope stability. Thank you. You can refer to theory of ground vehicles by Wong J Y.