Cooling Technology: Why and How utilized in Food Processing and allied Industries

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Lecture 44

Reciprocating Compressor (Contd.)

Good afternoon my dear boys and girls. We are continuing with the reciprocating compressor type and typically in the last class we have come with the ideal type of reciprocating compressor. Because, if we know the ideal, then, we can compare with the real one, right. So, we are continuing with reciprocating compressors, and that, today, we shall continue there. Work input to the ideal compressor, earlier, we have done with the equation number 4, if you remember, right. The total work input to the compressor in one cycle that can be given as W i d, right and this is equal to W DA plus W AB plus W BC, right.

$W_{id} = W_{D-A} + W_{A-B} + W_{B-C}$ … (5)

Obviously, where the individual terms, that W DA, is work done by the refrigerant on the piston during process DA, and this is equal to the area under the line DA on PV diagram, and that is equal to minus Pe into VA. W AB is work done by the piston on refrigerant during compression that is A to B and this is a under the area A to B on again PV diagram, and this is equal to integral of VA to VB PdV, right. W BC is work done by the piston on the refrigerant during discharge from B to C that is, area under the line B to C and that is equal to Pc into VB, right. $\int_{V_A}^{V_B} P.dV$

Then, we can say that, W ideal is equal to W id is W ideal is equal to Pe minus Pe into VA plus integral of VA to VB P into dV plus Pc into VB, that is, all equal to area under A, B, C, D, on the PV diagram. This PV diagram, in the previous class, we have shown.

$\therefore W_{id} = -P_e.V_A + \int_{V_A}^{V_B} P.dV + P_cV_B$ = Area A-B-C-D on P-V diagram = $\int_{Pe}^{Pc} V.dP$

Therefore, this can be written, equal to Pe to Pv integral Vdp, right, with the domain Pe to Pc. Now, the work input to the ideal compressor per cycle, that is equal to the area of the cycle, on the PV diagram. The specific work input, that is, W id in kilo joules per kg to the ideal compressor is given by W id equals to W id over mR that is equal to integral of Vdp within the domain Pe to Pc, where, of course, mR is the mass of the refrigerant compressed in one cycle, and V is the specific volume of the refrigerant. The power input to the compressor Wc is given as Wc equals to m dot W ideal is equal to V dot Sw over Ve integral of Vdp under Pe to Pc, right.

$$w_{id} = \frac{W_{id}}{M_r} = \int_{Pe}^{Pc} v.dP \quad \dots (6) \qquad W_c = \dot{m}\, w_{id} = \frac{\dot{V}_{SW}}{v_e}\int_{Pe}^{Pc} v.dP \quad \dots (7)$$

Then, we can say that the mean efficiency or effective pressure, MEP mean effective pressure, MEP, for the ideal compressor, that is given by MEP is equal to W id over W sw, sorry, W id over Vsw, and this is equal to 1 by Ve into integral of Vdp over Pe to Pc, right. The concept of mean effective pressure is used for real compressors, as the power input to the compressor is a product of the MEP and the swept volume rate. So, the power input to the compressor and its mean effective pressure, this can be obtained from this, I mean, relation, right, and this equation number 8 will give us that. So, if the relation between V and P during the compression process, AB is also known, we can say then, this equation which, we have just shown is valid for both isentropic and non isentropic cross compression processes.

$$mep = \frac{W_{id}}{\dot{V}_{SW}} = \frac{1}{v_e}\int_{Pe}^{Pc} v.dP \quad \dots (8)$$

However, the compression process must be reversible, as the path of the process should be known for the integration to be performed. Therefore, for the isentropic process, PV to the power k, may be gamma, or a constant k, is equal to constant. Hence, the specific work of the compression, that is, W id, that can be written or obtained by integrating and it can be shown to be equal to W id equals to integration of Vdp over Pe to Pc, that is equal to Pe into Ve into k by k minus 1 into Pc over Pe to the power k minus 1 by k minus 1, right. So, this we can say that k is the index of isentropic compression. If the refrigerant behaves as an ideal gas then k is equal to gamma.

$$w_{id} = \int_{Pe}^{Pc} v.dP = P_e v_e \left(\frac{k}{k-1}\right)\left[\left(\frac{Pc}{Pe}\right)^{\frac{k-1}{k}} - 1\right] \quad \dots (9)$$

In general, the value of k, for refrigerants, varies from point to point and if its value is not known then an approximate value of it can be obtained from the values of pressure and specific volume at the suction and discharge conditions, which is k equal to ln of Pc over Pe over ln of Ve over Vc, right. The work of compression for the ideal compressor can also be obtained by applying energy balance across the compressor, right. And, this figure says that the energy balance across a steady flow compressor, what it is? It is saying that, Wc quantity of work, that is done by the compressor, and Qc quantity of heat is expelled, right and inlet is m Pe Te he hc, outlet is m or m dot, whatever we call, Pc Td hd and Asd. So, all discharge h, enthalpy discharge, As entropy discharge, T, that is temperature discharge, P compressor that condenser, that is discharge, m also at that. So, Se Ar also evaporator that is the entropy h, enthalpy evaporator, T, temperature evaporator, P, again the pressure at the evaporator, right.

$$k \approx \frac{\ln(P_c / P_e)}{\ln(v_e / v_c)}$$

So, we can now say that since, the process is assumed to be reversible, and adiabatic, and if we assume, changes in potential and kinetic energy to be negligible, which we have also seen in earlier cycles, then, from energy balance across the compressor, we can write W id is equal to Wc over m dot, and that is equal to hc minus he. This equation can also be obtained from the thermodynamic relation, that is, Tds is equal to dh minus Vdp, is equal to dh, and that is equal to Vdp, since ds is 0, for isentropic process. Therefore, W id can be written as integral of Vdp over Pe to Pc and that is equal to integral of Pe to Pc dh, and that is equal to hd minus he, right, that is h discharge minus h evaporator, right. Now, the above expression is valid only for reversible adiabatic compression, right. So, the ideal compressor with clearance, we had earlier, ideal compressor with no clearance.

$$w_{id} = \frac{W_c}{\dot{m}} = \left(h_c - h_e\right) \qquad \ldots (10)$$

Now, we introduce that clearance, ideal compressor, but, with the clearance. So, in actual compressors, a small clearance is left between the cylinder head and the piston to

$$Tds = dh - vdP \Rightarrow dh = vdP \quad (\because ds = 0 \quad for \quad isentropic \quad process)$$

$$\therefore w_{id} = \int_{Pe}^{Pc} vdP = \int_{Pe}^{Pc} dh = (h_d - h_e)$$

accommodate the valves and to take care of thermal expansion, and machining tolerances. As a thumb rule, the clearance, C, in millimeters, that can be written as C equal to 0.005 L plus 0.5 millimeter. This is a thumb rule, where L is the stroke length in millimeter. I hope, stroke length, you understood, this was our piston, this was here, and right, and the discharge was here. So, this can be taken as the L, right. So, if we make it, then, C is 0.005 L plus 0.5 millimeter, where, L is the stroke length in millimeter. So, this space along with all other spaces between the closed valves and the piston at the inner dead center, that is, IDC, inner dead center, is called as ance volume or VC right. The ratio of the clearance volume to the swept volume, that is called clearance ratio and this is written as epsilon, right. This is written as epsilon, and epsilon is nothing clear, but VC that is V clearance over V SW, that is the swept volume right.

$$C = (0.005L + 0.5)\ \text{mm} \qquad \ldots (12)$$

Then, we can say that the clearance ratio, epsilon, depends on the arrangement of the valves in the cylinder, and the mean piston velocity. Normally, epsilon is less than 5 percent for well designed compressors with moderate piston velocities, that is 3 meter per second. However, it can be higher for higher piston speeds. Due to the presence of the clearance volume, at the end of the discharge stroke, some amount of refrigerant at the discharge pressure, PC will be left in the clearance volume. Now, we can say, as a result, suction does not begin as soon as the piston starts moving away from the IDC.

When, it was earlier, when there was no clearance volume, we saw, we told that the moment the discharge is over, then, suction starts, right, as the piston is moving. But here it is not so, since, there is a clearance, or clearance volume, rather, right. So the piston starts moving away from the IDC, it is not so. So, since the pressure inside the cylinder is higher than the suction pressure, PC, because, PC is greater than PE. So, as it is shown in the next figure that suction starts only when the pressure inside the cylinder falls to the suction pressure, in an ideal compressor with clearance.

This implies that, even though, the compressor swept volume V sw being equal to V a minus V c, the actual volume of the refrigerant, that entered the cylinder during suction stroke, is V a minus V d. As a result, the volumetric efficiency of the compressor with clearance, that is, yeeta V c, that is, less than 100 percent, right. That is, this is the figure, which, we are, we were referring, but, let us see the equation, which we arrived at yeeta V clearance is equal to actual volume of the refrigerant compressed over swept volume of the compressor, right. This is equal to V a minus V d over V a minus V c right. So, this is, ‘d’ this is ‘a’ this is ‘b’ this is ‘c’ back to ‘d’, right.

This is a P theta diagram, and in PV diagram, this is a to b, b to c, c to d back to a right. And the length, as we said, is this, where, the piston is moving, right, and these are the suction and discharge pressures valve, rather, ok. So, for ideal compressing pressure with clearance it looks like this. Therefore, we can say now, that the clearance volume efficiency or clearance volumetric efficiency can be written as yeeta clearance equal to V a minus V d over V a minus V c, that is, V a minus V c plus V c minus V d over V a minus V c, and this is equal to 1 plus V c minus V d over V a by V a minus V c, rather. Therefore, the clearance ratio epsilon the which was defined as V c over V sw is now, coming to be V c over V a minus V c, which is, roughly equal to, V a minus V c, that is, V c over epsilon, right.

$$\varepsilon = \frac{Vc}{V_{SW}} \quad \ldots (13)$$

Substituting this equation in the expression for clearance volume efficiency, that is, this, we get clearance volume efficiency is 1 plus V, which was here 1 plus V c minus V d over V a minus V c, that is equal to 1 plus epsilon into V c minus V d over V c, that is equal to 1 plus epsilon minus epsilon into V d over V c, right. Then, we can say that, the mass of refrigerant in the cylinder being at point C and D and therefore, point C and D, the mass are same and we can express the ratio of cylinder volume at points D and C in terms of ratio of specific volumes of refrigerant at point D and C, that is, V d over V c is equal to small v d over V c, right. Hence the clearance volume efficiency, that can be written as yeeta V clearance is 1 plus epsilon minus epsilon into V d over V c, equal to 1 plus epsilon minus epsilon into V d over V c, right. Therefore, we can say that, if we assume re-expansion process also, to be followed in the equation as p V to the power k is equal to constant. Then, we can say that V d over V c is equal to p c over p d to the power 1 by k and that is p c over p e to the power 1 by k.

$$\eta_{V,cl} = \frac{Actual \quad volume \quad of \quad refrigerant \quad compressed}{Swept \quad volume \quad of \quad the \quad compressor} = \left(\frac{V_A - V_D}{V_A - V_C}\right) \quad \dots (14)$$

Since, the clearance volumetric efficiency is given as yeeta volume clearance equal to 1 plus epsilon minus epsilon into p c over p e to the power 1 by k. Therefore, we can say this is equal to 1 minus epsilon into r p to the power 1 by k minus 1 where, r p is the pressure ratio, that is, p c over p e, right. Now, we can write that, the equation, which we have found out for the ideal clearance and that is efficiency, rather, holds good for any reversible compression process with clearance, right. If the process is not reversible, adiabatic, that is, non isentropic, but a reversible polytrophic process with an index of compression and expansion equal to n, then k in this equation, which, we have just derived that, in that place, k can be replaced by n. That is, in general, for any reversible compression process, yeeta volumetric clearance is equal to 1 plus epsilon minus epsilon into p c over p e to the power 1 by n and that is 1 minus epsilon r p to the power 1 by n minus 1, right.

$$\eta_{V,cl} = \left(\frac{V_A - V_D}{V_A - V_C}\right) = \frac{(V_A - V_C) + (V_C - V_D)}{(V_A - V_C)} = 1 + \left(\frac{(V_C - V_D)}{(V_A - V_C)}\right) \quad \dots (15)$$

This expression shows that, yeeta volumetric clearance, this we can say that, it is going down, as r p is going up, right, yeeta volumetric clearance is going down as r p is going up, and epsilon is going up, and this is, this is evident in this, sorry, this is evident in this figure, where, we are seeing that yeeta volumetric clearance is rather, is varying with r p, or rather, r p is in the x axis, that is, independent, and yeeta volumetric clearance is dependent, and n is increasing in this order, right. So, this is n 1 less than n 2 less than n 3 like that. So, effect of pressure ratio, this is r p, and index of compression, n on clearance volume efficiency, yeeta volumetric clearance is like that. So, for a given r p, and n, yeeta volumetric clearance, we can find out from there, right, and for the same, for the second one, as n is increasing, yeeta volumetric efficiency is also increasing, same is true for the third case, for the same r p

$$\varepsilon = \frac{Vc}{V_{SW}} = \frac{V_C}{V_A - V_C} \Rightarrow (V_A - V_C) = \frac{V_C}{\varepsilon} \quad \dots (16$$

$$\eta_{V,cl} = 1+\left(\frac{(V_C-V_D)}{(V_A-V_C)}\right) = 1+\frac{\varepsilon(V_C-V_D)}{V_C} = 1+\varepsilon-\varepsilon\left(\frac{V_D}{V_C}\right) \quad \ldots (17)$$

right. This limiting pressure ratio is obtained from the equation yeeta volumetric clearance is equal to 1 minus epsilon into r p to the power 1 by n minus 1, that is equal to 0, and this corresponds to r p maximum, and that is 1 plus epsilon by epsilon to the power n. The mass flow rate of the refrigerant compressed with clearance, that can be given as, m dot clearance is yeeta V cl, the V dot sw over V e, right and we thus, mass flow rate and hence, the refrigeration, both mass flow rate and the refrigeration capacity of the system decreases as the volumetric clear efficiency reduces in the order, in other words, rather, the required size of the compressor increases, as the volumetric efficiency increases, right. Today, our time is up. So, in the next class, perhaps, we will conclude about the compressor, because it will complete the 9th week ok. Thank you so much.

$$\left(\frac{V_D}{V_C}\right) = \left(\frac{v_D}{v_C}\right) \quad \ldots (18)$$

$$\eta_{V,cl} = 1+\varepsilon-\varepsilon\left(\frac{V_D}{V_C}\right) = 1+\varepsilon-\varepsilon\left(\frac{v_D}{v_C}\right) \quad \ldots (19)$$

$$\left(\frac{v_D}{v_C}\right) = \left(\frac{P_C}{P_D}\right)^{1/k} = \left(\frac{P_c}{P_e}\right)^{1/k} \quad \ldots (20)$$

$$\eta_{V,cl} = 1+\varepsilon-\varepsilon\left(\frac{P_c}{P_e}\right)^{1/k} = 1-\varepsilon\left[r_p^{\,1/k}-1\right] \quad \ldots (22)$$

$$\eta_{V,cl} = 1-\varepsilon\left[r_p^{\,1/n}-1\right] = 0$$

$$\Rightarrow r_{p,\max} = \left[\frac{1+\varepsilon}{\varepsilon}\right]^n \quad \ldots (23)$$

$$\dot{m}_{cl} = \eta_{V,cl}\frac{\dot{V}_{SW}}{v_e} \quad \ldots (24)$$