Cooling Technology: Why and How utilized in Food Processing and allied Industries

Prof. Tridib Kumar Goswami
Department of Agriculture Engineering
Indian Institute of Technology, Kharagpur

Module No 08

Lecture 38

Gas as Refrigerant(Contd.)

Good afternoon, my dear boys and girls and friends. In the previous class, we had started with gas as the refrigerant, right and there, I asked you one thing, which, you, I have no idea, because, we are not corresponding directly. Whether, you could recognize that or not, that V 1 square plus p 2 by rho that is, the velocity head, plus pressure head, plus the gravitational head is constant. This equation, we had utilized, and I asked you to remember, or chance, I had taken that, whether, you can remember that or not. Hopefully, you have, but, for those, who could not have, that was the Bernoulli's equation, right, that velocity head, plus pressure head, plus gravitational head, plus any other forces, some of them is equal to 0 at steady state right. If it is not of course, steady state, then it will not be equal to 0.

It is under steady state, that is why, V 1 square by 2 and that side V 2 square by 2 you can write, at two points, right. So, that was Bernoulli's equation. I asked you of course, I cannot get back from you because we are not in direct contact. So, I hope, you could have figured it out and for those, who could not, it is Bernoulli's equation, ok.

Now, we had stopped in the previous class, the continuation of this, gas as refrigerant, up to the first step, 1 to 2 right. Usually, by this time, these steps are in your mind right. So, one like this 1 2 3 4 back to 1, this is what is our system, right. Again, say, from this end, I start 1 2 3 4 back to 1, this is in our system. So, we have done 1 to 2 and that was isentropic, right.

Process 2 -3 Isothermal heat rejection:

$$p_1 v_1 = p_2 v_2 = RT_2$$

$$-W_{2-3} = \int_2^3 v dp = -RT_2 \int_2^3 \frac{dp}{p} = RT_2 \ln\left(\frac{p_3}{p_2}\right)$$

Now, let us look at the other one, that is, that is, isothermal. To keep it in our mind, I always try to put back the system before your eye. So, it was T S, and this was the dome, this was point 1 to 2 to 3 to 4 back to point 1, right.

So, 1 2 3 4, so, 2 isentropic, and 2 isothermal, right. So, this was our system. Now, we are coming to the point. We come back to 2 to 3, and during this process, pressure increases, and hence, work has to be done on the system, and for the isothermal process, right. So, 1 2 3 4 back to 1, right 1 2 3 4 ok. So, from this was pressure Pe from this was pressure Pc.

So, Pc is much much greater than Pe. So the one which we are saying that during this process pressure increases. Hence, work has to be done on the system, and this is the system, which the process is isothermal. So, in that case, we can say, P1 V1 is equal to P2 V2 right. If it is adiabatic, then, we say, P V gamma is constant, but, if it is isothermal it is P1 V1 is equal to P2 V2 and that is, P V is constant, right. So, we can write that, P1 V1 is P2 V2 is equal to RT2, right RT1 or equal to RT2.

So, the flow of work for the open system, that we can write as, that we can write as, minus W 2 to 3 integral of 2 to 3, pdV, right. Since p is increasing so, we can write minus RT2 between 2 to 3, this V, we can replace with 1 by rho, or this is P 1 by rho, right. So, it is RT2 no, no, sorry, sorry, sorry, it is P rho. So, P1 V1 is P1 V1 is P2 V2 is equal to RT2. So, minus W 2 to 3, between 2 to 3, Vdp is minus RT2, 2 to 3, this is dP over P, because this PV is RT2 so, there it comes in, and we are writing dP over P right.

And this is RT2, since 1 by P, dP of 1 by P is ln P3 over P2 ok. So, so, minus W 2 to 3 is Vdp between 2 to 3 is equal to minus RT2 between 2 to 3 dP over P. So, ln P, that is, ln P3 over P2, that is fine. Only, I have a little hangover over this negative sign, this negative sign, where did it go? This is P3 over P2 ok, W 2 to 3 is minus RT2 ln P3 over P2, that is what. We had missed 1 minus, that was, by chance, right, Here also, it should have been minus RT2 ln P3 over P2. Otherwise, where this minus is going? So, I got stuck there ok.

So, we can now say that since, again, this was 2 to 3 right, this was 2 and this was 3. And both are under same temperature, that is T2. So, T2 and T3 are same, right, because, that is isothermal. So, we can write for T2 is equal to T3, and for a perfect gas, we can also write h2 is equal to h3. So, using that first law of thermodynamics, for steady flow system with delta KE and delta PE equal to 0.

So, we get Q 2 to 3 is W 2 to 3, right Q 2 to 3 is W 2 to 3 because no internal energy is there. So, we can say now that Q 2 to 3 is equal to W 2 to 3 is equal to minus RT2 ln P3 over P2. So, here, there was a negative, which was omitted, right, because of cut and paste, ok. So, I was searching it, where it is, how could it be? So, this is minus RT2 ln

P3 over P2, right, that negative sign indicates that, the work is done on the system and heat is rejected by the system.

$\Delta KE = 0$ and $\Delta PE = 0$, we get $q_{2\text{-}3} = W_{2\text{-}3}$

$$\therefore \quad q_{2-3} = W_{2-3} = -RT_2 \ln\left(\frac{p_3}{p_2}\right)$$

Work is done on the system, and heat is rejected by the system, this was 2 to 3. So, from where QC was rejected, or QH, rather, was rejected at constant temperature TH, right. This you remember. So, we can now say that for, similar to 1 to 2 process, we can write that S3 is equal to S4, and Q3-4 is equal to 0, right. S3 is equal to S4, and Q3-4 is equal to 0, which one is that, this was 3 and this was 4, and this is isentropic in TS diagram, right. So, this being isentropic in TS diagram, we can write that S2 is equal to, no, S3 is equal to S4, and Q3-4 is equal to 0. Now, again, here, applying the first law of thermodynamics, we can write that. Thus, for a steady flow, we can write, minus W3 to 4 is equal to h4 minus h3, right or h3 minus h4 is equal to Cp T3 minus T4, right.

process 1 – 2, we can write, $s_3 = s_4$ and $q_{3\text{-}4} = 0$

$$-W_{3-4} = h_4 - h_3 \quad or, \quad h_3 - h_4 = C_p\left(T_3 - T_4\right)$$

$$p_3 v_3^{\gamma} = p_4 v_4^{\gamma}$$

$$-W_{3-4} = \int_3^4 v\,dp = \frac{\gamma}{\gamma - 1}\left(p_4 v_4 - p_3 v_3\right) = \frac{\gamma}{\gamma - 1} R\left(T_4 - T_3\right)$$

So, for open system, the same result can be found out, that, the flow of work for isentropic process is PV gamma or P3 V3 gamma is equal to P4 V4 gamma right. So, this is true. Therefore, we can write this, minus W3-4 is integral of 3 to 4 V dp, is gamma by gamma minus 1 into P4 V4 minus P3 V3, right, is equal to gamma by gamma minus 1 into R T4 minus T3. Therefore, we can write W3-4 is equal to gamma by gamma minus 1 into R T4 into P3 by P4 to the power gamma minus 1 by gamma minus 1, right. This is gamma by gamma minus 1 R T4 P3 is to P4 to the power gamma minus 1 by gamma minus 1. Now, can you identify, there is a mistake again, that sign mistake is there. Can you identify? Here we said gamma by gamma, sorry, here we said gamma by gamma minus 1 V dp, 3 to 4 and P4 V4 that is upper limit and this is 3, P3 V3, fine, and this is related to gamma by gamma minus 1 R T4 minus T3, right and T4 and T3 are different.

Now, converting it into P. From this relation, we write W3 to 4 is gamma by gamma minus 1, is R T4 into, R T4 is taken out, into P3 over P4. It was P4 over P3, right. It was P4 over P3. Here also, you see that, this is P4 over P3, that is P4 V4 minus P3 V3. So, where from suddenly, this, no, this has come, because, this gamma by gamma minus 1, here it is gamma minus 1 by gamma minus 1, right. So, this sign has been taken in with P3 over P4. So that you please check. I will also check right, you please check. I will also check that whether this is, this is correct, this is the final, but, intermediate, this one with the symbol negative. We will check up ok. So it is observed that W1 to 2 is equal to and opposite to W3 to 4 right. It is, that is, W1 to 2 is equal, but opposite to W3 to 4 right. So, if W1 to 2 is working like this, then, W3 to 4 is working like this, that is opposite, because of the sign right.

$$W_{3-4} = \frac{\gamma}{\gamma - 1} RT_4 \left[\left(\frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

So, we can now say that, we can now say that, the last process, that is process 4 to 1 being isothermal, heat rejection, or heat absorption, in this case isothermal heat absorption. There, the process pressure decreases and hence, work is done by the system. We had pressure Pc, now, we have come to pressure Pe. So, Pc is much greater than Pe, so, the pressure has come down. So, the work has been done by the system, right, for the isothermal process, and in that case, we can say that P4 V4 is equal to P1 V1 is equal to RT1 because, it is isothermal right. So, it being isothermal, P4 V4 is P1 V1 is equal to RT1. Therefore, flow work for the open system is, that can be said that, minus W4 to 1 is equal to integral of 4 to 1 V dp, and that is equal to minus RT1, 4 to 1 dp over P, and this is minus RT1 ln of P1 over P4. So, here we have written the sign correctly right.
It is observed that W_{1-2} is equal and opposite to W_{3-4}

Process 4 – 1 Isothermal heat absorption:

$$p_4 v_4 = p_1 v_1 = RT_1$$

$$-W_{4-1} = \int_4^1 v dp = -RT_1 \int_4^1 \frac{dp}{p} = -RT_1 \ln\left(\frac{p_1}{p_4}\right)$$

$$or, W_{4-1} = RT_1 \ln\left(\frac{p_4}{p_1}\right)$$

Therefore, W4 to 1, that can be written as, this negative, and this negative goes out, RT1 ln of P4 is to P1, no, sorry, this negative sign is, and this negative sign are different, and that is, that is why this minus RT1 ln of P1 over P4. This negative sign is taken inside, and that becomes RT1 of ln P4 is to P1, right. So, instead of here, P1 is to P4, it is ln P4 is to P1. This negative sign has been taken, and W4 to 1 also, for it, T1 is equal to T4, there is no ambiguity, this was T4, this was T1, right. So, T1 is equal to T4, and hence for

a perfect gas also we can write h1 is equal to h4. So, using the first law of thermodynamics for a steady flow system, where delta KE and delta PE are 0, we get Q4 to 1 is equal to W4 to 1, right. So, Q4 to 1 is equal to W4 to 1.
also, for it, $T_1 = T_4$, and hence, for a perfect gas, $h_1 = h_4$. Using first law of thermodynamics for a steady flow system with $\Delta KE = 0$ and $\Delta PE = 0$, we get $q_{4\text{-}1} = W_{4\text{-}1}$

$$\therefore \quad q_{4-1} = W_{4-1} = -RT_1 \ln\left(\frac{p_1}{p_4}\right) = RT_1 \ln\left(\frac{p_4}{p_1}\right)$$

So, there are two ways to find the net work, and COP, the first thing, first method, can be like this. Since T2 over T1 is P2 over P1 to the power gamma minus 1 by gamma is equal to T3 over T4 is equal to P3 over P4 to the power gamma minus 1 by gamma. So, applying the first law of thermodynamics to the whole system and hence we can write that P2 over, we can write P2 over P1 is P3 over P4. Therefore, W net can be written as Q2 to 3 minus Q4 to 1, which, we have shown as RT2 ln P3 over P2 minus RT1 ln P4 over P1. So, we can write mod of W net is equal to RT2 minus T1 ln P4 over P1 right. Therefore, COP can be written as Q4 to 1 over W net and that is RT1 ln of P4 over P1 by RT2 minus T1 ln of P4 over P1 and this is nothing, but, T1. This T1 over T2 minus T1, this T2 minus T1, all other are cancelling out.

$$|W_{net}| = |q_{2-3}| - |q_{4-1}| = RT_2 \ln\left(\frac{p_3}{p_2}\right) - RT_1 \ln\left(\frac{p_4}{p_1}\right)$$

$$\therefore \text{we can write} : |W_{net}| = R(T_2 - T_1)\ln\left(\frac{p_4}{p_1}\right)$$

$$\frac{p_3}{p_2} = \frac{p_4}{p_1} \qquad \therefore \quad COP = \frac{q_{4-1}}{|W_{net}|} = \frac{RT_1 \ln\left(\frac{p_4}{p_1}\right)}{R(T_2 - T_1)\ln\left(\frac{p_4}{p_1}\right)} = \frac{T_1}{T_2 - T_1}$$

That ln of P4 over P1 and ln of P4 over P1, we have already shown that here P2 is to P1 is P3 is to P4 right. That means, P4 over P1 is P2 over P3, right, and that we have used here. So, we can say, P4 is to P1 is P3 is to P2. P4 is to P1 is P3 is to P2, right and that is how we have written here, this and that, and ultimately, we got T1 by T2 minus T1 right. So, COP, we got equal to T1 over T2 minus T1, right. Then we can conclude it before saying thank you, we can conclude it that for a gas as refrigerant, COP is also coming, the same as T lower over T higher minus T lower, right and this we have proved with the help of first law, second law and normal thermodynamic equations including, as we said earlier that, we started with the equation which I said in the beginning that pressure rate plus velocity head plus the gravitational rate plus any frictional loss is equal to 0, which was Bernoulli's equation right.

We had done with that and in very very few classes we could come to the thank you slide because either the class got continued to the next class or the thank you for which may be the thank you slide was at the end. But fortunately, this class we could have finished where it was the last slide and that is thank you and this is the one which our NPTEL system in all the classes, everywhere, you will find, if you come across with the thank you slide. So, thank you all for listening and I wish you that you study as much as you can ok. Thank you so much.