Cooling Technology: Why and How utilized in Food Processing and allied Industries

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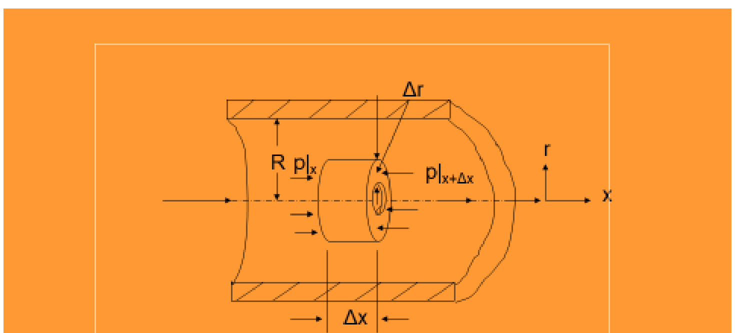
Module No 03

Lecture 11
Basics of Thermodynamics Contd.

Good afternoon my dear students. If you remember in the previous class we started with basics of thermodynamics and in that we saw that heat transfer, fluid flow, things are also coming. A little of heat transfer, I had already said and maybe at the end of this basics of thermodynamics may take some classes. So before that let us look at the flow through the conduit or flow through pipe.



Control volume for shell momentum balance on a fluid flowing in a circular tube

Now, this, I had given you yesterday I mean in the last class and I told that there are two types of force transfer, one is by bulk movement of the molecules and the other is by the molecular transport, that also I had explained to you, right. So assuming that we know molecular transport and we know bulk flow or bulk transfer then let us take this as a sample that is section of a pipe in which we are taking a volume element of radius delta r thickness rather and the length we are taking as delta x right and as you see this r and x this is the r and this is x right.

So I can show you that this is the r direction and this is the x direction right and the thickness we have taken as delta r, we have taken the length as delta x. So, this means there, this is two dimension and the third dimension is taken as unit right. So if we do the momentum balance, let me clear up, if we do the momentum balance then what we get, again we have to come back, if you do momentum balance, then what we get we said that, we will be ge sorry, we will be getting, if you are doing shell momentum balance which is valid for incompressible fluid, this also I said that incompressible fluid means that, the fluid which does not change its density with time or rather with pressure, right that is incompressible. Compressible fluids do change their density with pressure, but incompressible does not and the flow is one dimensional, it is a steady state laminar flow fully developed that also we said, and there is no end effects and velocity profile does not vary along the x axis or x direction, right.

So, the governing equation that we can write, as governing equation that we can write as momentum, rate of momentum in, rate of momentum in minus rate of momentum out plus sum of the forces acting on the volume element must be equal to rate of accumulation of momentum. This is the general equation for solving the flow through pipe equation. Now, we have already said that it is a steady state. So, momentum in by convection is equal to momentum out by convection. So, they are cancelling out each other, right.

So there is no then convective momentum left over because whatever was coming is also going out. So momentum in by convection equal to momentum out by convection that is why, because of steady state. Now momentum in by molecular transport, we can write that, it is equal to tau r x into 2 pi r into delta x at the position r. If we go back to the previous slide, where we had, this slide, you see how we are finding out the area. We are finding out that 2 pi r, right into delta x, right.

$$\tau_{rx}\, 2\pi r\, \Delta x\, |_{r}$$

This we are getting for the momentum flux ok. So, we are getting 2 pi r into delta x at the face r, right. So at the face r it is so. So what about at the face r plus del r? If you remember, I am drawing it here, it was our thing, right and this was our thing right, and we had taken delta r like this, and this was delta x right. So, 2 pi r delta x that is the area at the face r into the tau r x right.

$$\tau_{rx} .2\pi r\, \Delta x\, |_{r+\Delta r}$$

So, this of course, you know tau r x, what is that tau r x is? It is not going out, let it be. So, the other one is momentum out by molecular transport that is tau r x into 2 pi r into delta x at r is equal to r plus delta r. This is the momentum out by molecular transport right. This tau r x is nothing, but shear stress, right this is nothing, but shear stress and of course, it has a definition, and we define it after words, we will come it that momentum out and momentum in due to molecular transport, we have seen one is tau r x rather, into 2 pi r into delta x at r is equal to r and tau r x into 2 pi r into delta x at r plus delta r. So, this is the force for molecular transport.

Now, there are other forces like pressure force. So, pressure force is also similarly working at p at x is equal to x and that is equal to p times the area, what is the area? Area is 2 pi r into delta r at the face x right, that is the pressure force. Similarly, we can say pressure force at the outlet that is p at x plus delta x. so that we write p times the area that is 2 pi r delta r at x plus delta x. So, if we know this then pressure forces are known, molecular transport forces are known and there is also gravity force, but since we have taken one horizontal tube right.

$$\text{Pressure force in} = P|_x = p\, 2\pi r\, \Delta r\, |_x$$

$$\text{Pressure force out} = P|_{x+\Delta x} = p\, 2\pi r\, \Delta r|_{x+\Delta x}$$

So, we can neglect the gravitational force, gravity. So, neglecting gravity because it is not there, we can then write in the governing equation that tau r x into 2 pi r into delta x at r is equal to r minus tau pi r x into 2 pi r delta x at r is equal to r plus delta r plus p into 2 pi r into delta r at x minus p 2 pi r into delta r at x plus delta x, right this is equal to 0 right. So, this is equal to 0. So, if we rearrange, it you know by the definition of derivative, derivative is, how it is made? It is out, sorry, it is out minus in over the distance, right that is, what is, any derivative, out minus in over the distance. Now, if we say that, dividing both side by delta x into delta r, if we divide both sides by delta x and delta r, then we can write this r tau r x at r is equal to r plus delta r minus r tau r x at r is equal to r over delta r.

$$\therefore\ \tau_{rx}\, 2\pi r\, \Delta x\, |_r - \tau_{rx}\, 2\pi r\, \Delta x\, |_{r+\Delta r} + p\, 2\pi r\, \Delta r\, |_x - p\, 2\pi r\, \Delta r|_{x+\Delta x} = 0$$

$$\text{or, } (r\tau_{rx}\, |_{r+\Delta r} - r\tau_{rx}\, |_r) / \Delta r = r\, (p|_x - p|_{x+\Delta x}) / \Delta x$$

$$\text{or, } \partial(r\tau_{rx}) / \partial r = - r\, \partial p / \partial x = (\Delta p/L)\, r$$

$$\text{or, } r\tau_{rx} = (\Delta p/L)\, r^2/2 + c$$

$$\text{or, } \tau_{rx} = (\Delta p/L)\, r/2 + c/r$$

This is equal to p at x minus p at x plus delta x over delta x right. So, we can write, from the definition of derivative, that del del r of r tau r x equal to minus r del p del x, equal to delta p over L, that p at x plus delta x, right, and p at x, this difference, we can write, to be delta p, right. So, we can write that del del r of r tau r x is equal to minus r del p del x and that is equal to del p over L into r, right. So, this on rearrangement, we can write, r tau r x is equal to delta p by L into r square by 2 plus integration constant, as c right. So we have to find out what is the value of c that is the integration constant.

We have not done. It is not a definite integration. It is indefinite integration. So we need a constant c to be evaluated. So, one unknown so we need one equation, as boundary condition, right.

So this on rearrangement we can write tau r x is equals to delta p over L into r by 2 plus c by r this is the tau r x that we are looking for. Now the boundary, we need to know the boundary, and that boundary is at r is equal to 0 since tau r x is not equal to infinity, therefore, c is 0. We go back to the previous slide, here we had said that tau r x, right is equal to delta p by L into r by 2 plus c by r and we need to find out the constant c and that is only possible by knowing the boundary condition and boundary condition is what? Boundary condition is at r is equal to 0 since tau r x that is the tangential force or what we call it to be the shear force, which we are coming across with layer by layer or molecules by molecules. So, that is the tau r x and since it is not infinity, therefore c has to be 0 at r is equal to 0, otherwise, if it is infinite, then obviously it would be very very difficult to solve, right, it becomes a mathematical problem. So, to avoid that the best boundary is taken that at r is equal to 0, since tau r x is not infinity therefore c is 0.

$$\text{at } r = 0, \text{ since } \tau_{rx} \neq \infty, \; c = 0$$

$$\therefore \quad \tau_{rx} = (\Delta p/L)\, r/2,$$

$$\therefore \quad \text{and since } \tau_{rx} = -\mu\, dv_x / dr$$

$$\mu\, dv_x/dr = -(\Delta p/L)\, r/2$$

$$\text{or, } dv_x/dr = -(\Delta p/2\mu L).r$$

$$\text{or, } V_x = -(\Delta p/2\mu L)\, r^2/2 + c_1$$

Then we can write tau r x we can write tau r x is equal to delta p by L into r by 2. This we remember. In future, not only here, but in any other, maybe, whenever you are doing any other fluid flow, this in many cases, you start with this, right, if the other conditions remain similar. So, tau r x is equal to delta p by L into r by 2 right. Now we define tau r x, right.

Here you see tau is associated with 2, one is r another is x, right, and first one is r with tau second one is x, right. So, when it is defined, tau is defined, it is said that it is equal to minus mu, that is viscosity, minus mu into dv x dr right. So the second one is the velocity and the first one is the direction, second one is dv x, right and this is the direction, dr, sorry dr. So, dv x dr is the tau r x. So if we substitute in place of tau r x, if we substitute this, minus mu dv x dr then we can write mu dv x dr is equal to this minus we have taken to this side.

So minus delta P by L into r by 2 right. So we get by rearranging, dv x dr is equal to minus delta P by 2 mu L into r right. So on integration of this, we get v x is equal to minus delta p by 2 mu L. Now this r becomes r square and divided by 2 plus an integration constant called C 1, other was C and this one is C 1, right. So, we need another boundary to find out this integration constant C 1, we need another boundary.

Now, at $r = R$, $V_x = 0$

$$\therefore C_1 = (\Delta p/4\mu L)\,R^2$$

$$V_x = (\Delta p/4\mu L)\,(R^2-r^2)$$

So, what could be the boundary our pipe was like this, right and fluid was flowing like this, right, and it is laminar, etc. all things were said. So, the velocity here, it is v x. So in the direction of x, velocity is v x. So this velocity which is in the pipe wall, so that is not moving because it is clinging to the surface, right, this is called clinging to the surface.

So, the fluid which is clinging to the surface is not moving and therefore, the velocity at r is equal to say, capital R is equal to 0, sorry, v x is 0 at r is equal to capital R, v x is 0 right. So, that is what we have taken and by solving we get the value of C 1 equal to delta P by 4 mu L into R square, right. So, delta P by 4 mu L into R square, we get and substituting the value of C 1 to the value of v x, if we substitute, then we get v x is equal to delta P by 4 mu L into R square, capital R square minus small r square right. Here again we have taken that volume element small, here, and the radius we have taken to be equal to r right this is r and if it is the axis right.

$$v_x = \frac{p_{in} - p_{out}}{4\mu L} R^2 \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

So, we have gotten the velocity in this, is called instantaneous velocity at any position because x we have not said, x can be anything, like x can be anything, like here.

So that is v x 1 or here that is v x 2 or here v x 3, right. We have not said. So, that is why it is called instantaneous velocity and the velocity is then determined as the maximum velocity or average velocity. So that instantaneous velocity v x we have gotten delta P by 4 mu L into R square minus r square capital R square minus small r square, right. Since, we have gotten it, we can find out the average velocity.

The velocity which normally we talk about is the average velocity, that is, you have this pipe here you have one velocity as we said v x 1 here we have another velocity v x 2 here we have another velocity v x 3 dot dot dot as many points we get right. So we can say that the average velocity is the velocity which we talk about. So how can, we find out average velocity. So v average is 1 by A area into integration of the area, right. So, that is 0 to 2 pi and the other one that is 0 to L, that is the x, right.

So if this two we take then it is v x into d A. Now this d A is nothing but r d theta and d v x, right, r d theta d r into v x. So, this we substitute as 1 by r 1 by A integral of, double integration of v x d A and this v x d A, we are substituting as v x 1 by pi R square right and v x into r d theta, r d theta is that area right, that this is r d theta is that angle between this and this. So this is the d theta. So r d theta into d r right into v x is the expression.

Now, on simplification, on integration, we can write that delta P is P in minus P out obviously, that delta P this P 1 is much more than p P right, that is P in is higher than p out right. So that we can take as P in minus P out over 4 mu L times R square into 1 minus small y capital R whole square right. This we are getting and we get that this v average that becomes equal to P in minus P out into capital D square by 32 mu L right. This is known as Hagen-Poiseuille equation. Most of the cases, you are asked, how can you find out the pressure drop in a pipe.

$$V_{av} = 1/A.\iint v_x dA = 1/\pi R^2 \cdot \iint v_x\, r\, d\theta\, dr$$

$$= 1/\pi R^2 \int 2\pi r\, dr\, v_x$$

$$= (2\pi/\pi R^2) \int r\, (\Delta p/4\mu L)(R^2 - r^2)\, dr$$

$$= (P_0 - P_L)/2\mu L\, (R^2/2 - R^2/4)$$

$$= (P_0-P_L)/8\mu L\ R^2 = (P_0-P_L)/32\mu L\ D^2$$

So that you can say through the Hagen-Poiseuille's equation. If you know pressure drop, or if you do not know pressure drop then, if you know average velocity, if you know the viscosity, length, and diameter, then you can find out what is the pressure drop in the pipe, right. This is called theoretical and from there we get v max, which is how much v is max at, this was the pipe, v is max at r is equal to 0. So that r is equal to 0 v is equal to v max right. So by substituting that we get P in minus P out into R square by 4 mu L this is the v max and from the relation we get v average is equal to v max by 2.

$$v_{av} = \frac{\left(P_{in} - P_{out}\right) D^2}{32\ \mu\ L}$$

$$V_{max} = (P_0-P_L)\ R^2\ /4\mu L, \text{ at } r = 0$$

$$\therefore V_{av} = V_{max}\ /2$$

v average equal to v max by 2 ok. Hope you have understood how the pressure, how the pressure drop in the pipe can be calculated with the help of Hagen-Poiseuille's equation ok. Thank you.