Micro Irrigation Engineering

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Lecture-05
Tutorial-1
Numerical Examples on Fluid Mechanics and Soil Water

Hello everybody, welcome to lecture 5. In previous 4 lectures, we discussed about introduction of micro irrigation system. We discussed about basic fundamentals of fluid mechanics and soil properties. Now, in this lecture we will deal with various examples related to numerical problems, an application of fluid mechanic principles, numerical problem dealing with soil properties, and infiltration.

So, let us start doing exercise. These are simple exercises, this will make you to more understand more about these lectures which I have covered on theory part.

So, question 1, we are given 1 liter of liquid which weighs 2 Newton and we are required to determine a specific weight, density of this liquid, and a specific gravity of this liquid. So, basically, in this question we are given 1 liter of liquid and its weight and we will find out these 3 properties of this liquid and this is basically unit conversion.

So, if we see that 1 liter of a liquid, it has a weight of 2 Newton and 1 liter is 1 into 10 is to power minus 3 cubic meter volume. Now, if you want to find out the unit a specific weight of this liquid, so basically we will work out by

$$Specific\ Weight(\gamma) = \frac{weight\ of\ liquid}{Volume\ of\ liquid}$$

So, once we are writing this particular ratio, we are given 2 Newton is the weight and the volume is 1 into 10 is to power minus 3 cubic meter.

$$\gamma = \frac{2}{\left(\frac{1}{1000}\right)}$$

$$\gamma = 2000\ \frac{N}{m^3}$$

So, a specific weight it becomes 2000 Newton per cubic meter. Now, when this same liquid we are finding out what will be the density of this liquid. So, we have been given from the previous example, we got the specific weight as 2000 Newton per cubic meter. And we want to get density, so density unit is mass per cubic meter. So, already we have got 200 Newton per cubic meter and we can express same thing in terms of kilogram per cubic meter.

$$Density(\rho) = \frac{weight\ of\ liquid(w)}{Acceleration\ due\ to\ gravity(g)}$$

$$\rho = \frac{2000}{9.8066}$$

$$\rho = 203.94\ \frac{kg}{m^3}$$

So, the density of this liquid can be worked out as 203.94 kilogram per cubic meter. Now a specific gravity, when we want to find out, so a specific gravity is given by a weight density of a liquid divided by weight density of standard liquid, standard liquid is nothing but a water. And this water has got a density of 1000 kilogram per cubic meter and weight density of this liquid which we worked out as 203.94.

$$Specific\ gravity(G) = \frac{weight\ density\ of\ liquid}{weight\ density\ of\ standard\ liquid}$$

$$G = \frac{203.94}{(1000)}$$

$$G = 0.2039$$

So, when we are working out we get a specific gravity of this liquid as g is equal to 0.2039, this is the value. So, in this particular example, what we have found that there is a simple unit conversion, we are getting a specific weight and we are using the specific weight values to find out the density. And then having got the value of density we are getting a specific gravity of the same liquid. So, this is a simple unit conversion way how we are working out these values.

Now in question 2, we are given a main pipeline and this pipeline carries water and which has the velocity of flow as 2.4 meter per second. Now the diameter of main pipeline is 75 millimeter after a few meter of its length, this diameter is reduced to 50 millimeter. Now, we want to determine in the question, what is the velocity head at the inlet end of these 2 pipes, means 75 millimeter as well as 50 millimeter diameter pipeline.

And also we want to find out the discharge from this pipeline. Now in the tapered pipe that is 50 millimeter pipeline, 1 piezometer has been attached and the water level in the piezometer is 7 meter. So, it is asked in the question that what will be the water pressure, so 7 meter of water column is to be converted in equivalent unit. So, if you see here we are given 2 pipes one pipeline is 75 millimeter another pipeline is 50 millimeter.

And velocity through the main pipeline that is 75 millimeter pipeline is 2.4 meter per second. So, we are required to determine velocity head, we are required to determine rate of flow, and we are required to find out what will the pressure in the 50 millimeter diameter pipeline.

So, we are given these 2 diameters, from these diameter of the pipeline, main pipeline is 75 millimeter which is nothing but 0.075 meter. And for this diameter area of cross section is pi by 4 D 1 square. So, substituting the value of D 1, we get the area of cross section of the pipeline of 75 millimeter. Similarly, for second pipeline which is of 50 millimeter diameter A 2 is 1.96 into 10 is to power minus 3 meter square.

Now, we are given the velocity to the pipeline is 2.4 meter per second. So, one can work out what will be the velocity head. So, velocity head is given by V 1 square by 2g, so simply substituting the value of velocity and then g is 9.81.

Velocity head in 75 mm main pipe = $\frac{(V_1)^2}{2g} = \frac{(2.4)^2}{2\times 9.81} = 0.2935\ m$

So, once we are substituting we get velocity head as 0.2935 in the pipe of 75 millimeter diameter. Now, here we are using the concept of continuity equation which states that

$A_1\ V_1 = A_2\ V_2$.

Means flow from the pipeline in the 75 millimeter pipeline is equal to the flow in the 50 millimeter pipeline. So, from this we will be getting what will be the velocity. So, velocity in the second pipeline that is 50 millimeter pipeline can be obtained by putting A_1 V_1 by A_2 V_2 which when we are substituting area of cross section of the pipeline 1 and velocity and divided by A_2.

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{4.4178\times10^{-3}\times2.4}{1.9635\times10^{-3}} = 5.4 \text{ m/s}$$

So, we get the velocity in second pipeline is 5.4 meter per second. Now velocity head, once we have got the velocity. So, one can work out what is the velocity head and this velocity head.

Velocity head in 50 mm tapered pipe = $\frac{(V_2)^2}{2g} = \frac{(5.4)^2}{2\times9.81} = 1.4862\ m$

So equivalent velocity head after calculation, we get 1.4862 meter. Now, we have got velocity head that this was asked in the question that what is the velocity head. So, velocity head in 75 millimeter pipeline is 8.2935, velocity in 50 millimeter pipeline is 1.4862 meter.

Now, since using the continuity equation concept, one can find out what will be the discharge? So, discharge will be nothing but area of cross section multiplied by the velocity.

$$Q = A_1 V_1 = A_2 V_2$$

So, this will be velocity in pipe 1 or area of cross section of pipe 2 that is 50 millimeter diameter and velocity we have got 5.4 meter per second, so accordingly we can find out.

$$Q = 4.4178\times10^{-3}\times2.4 = 0.00106 = 10.6\ L/s$$

So, we got rate of flow as 10.6 liter per second. So, this is the 3rd part of the question which was asked. Now 4th part of the question it was asked, what is the pressure in the tapered pipe? That is a 50 millimeter pipeline. Now the water level in the piezometer is 7 meter.

$$p = \rho g Z$$

So, we can write 7 meter and the pressure equal to rho is the density of liquid or water which is 1000 kilogram per cubic meter, g is the acceleration due to gravity that is 9.81 meter per second square which when we are multiplying

$$p = 1000\times9.81\times7 = 68670\frac{N}{m^2} = 0.\ 7\frac{kg}{cm^2}$$

We get this 68670 Newton per meter square or which is equivalent to 0.7 kg per square centimeter. So, this is the 4th part of the equation which was asked in the question that what the pressure is. So, this is also simply unit conversion. Already we were given 7 meter of water column which is 0.7 kg per square centimeter but how to get 0.7 kg per square centimeter? Here, we are using appropriate unit and that is in Newton per square meter when we are then it can be obtained in 0.7 kg per square centimeter.

Now in third question, we will be again using the concept of fluid mechanics, how to obtain the component of total head? So, let us try to from this we have been given here in a sprinkler system water is flowing in a pipeline of 80 millimeter diameter and the pressure when it is at tapped in this pipeline it is having 30 Newton per square centimeter. And the velocity of flow in this pipeline is 2.4 meter per second.

We are given again this pipeline is laid in such a way that it is 4 meter above the reference plane or above the datum. So, means we are given diameter of the pipeline, we have got the pressure and then we have got the velocity of flow. So, here simply we will be using energy equation.

So, total head in the pipeline is given by pressure head, velocity head plus elevation head. Head due to elevation. Already I have told you that D is the diameter of the pipeline which is 0.08 meter. P is 30 Newton per centimeter square or I can say 30 into 10 is to power 4 Newton per square meter and elevation head is 4 meter. So, when we are writing, let us write total head, so total energy or total head it is given by

$$Total\ head\left(H_T\right) = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

So, all these components are available to us only simply we have to substitute the values. So, P is already given,

$$Total\ head\left(H_T\right) = \frac{(30\times10^4)}{(1000\times9.81)} + \frac{(2.4)^2}{(2\times9.81)} + 4$$

$$Total\ head\left(H_T\right) = 34.87\ \text{m}$$

So total head it becomes 34.87 meter. So, this was the question which was asked? So, total head in the pipeline is 34.87 meter.

So, there we have use energy equation. Now let us come to the 4th question, in this 4th question we are given the water is flowing through a pipeline having diameter 63 millimeter at section 1. So, there are 2 section one section which has a diameter of the pipeline at 63 millimeter. In the section 2, another pipeline which is of 50 millimeter diameter and the flow which is taking place in this pipeline is 5.5 liter per second.

Then elevation head at section 1 is given 4.5 meter above the datum and elevation head 3.5 meter above that deepened section 2 and pressure at section 1 is 20 Newton per centimeter square and we are required to determine the pressure at section 2. So, once again I am reading water is flowing through a pipeline having diameter 63 millimeter at section 1 and 50 millimeter diameter pipeline at section 2, rate of flow through pipe is 5.5 liter per second.

Section 1 is 4.5 meter above datum means it is at the inclined plane, means the pipeline is not in the same elevation. So, there is inclination that is 4.5 meter, the elevation head and then the pipeline at section 2; this is at 3.5 meters, so pipe is not in the same level. And then pressure head at section 1 is 20 Newton per centimeter square. So, we need to determine pressure head at section 2.

So, the values are given, so 63 millimeter is the diameter of the pipeline 1, then 50 millimeter at pipe 2. So, we are finding out area of cross Section A 1, that is pi by 4 D 1 square and this is the value when we are substituting 63 millimeter square, so that 63 is converted in meter. Similarly for the area of cross section of floor in pipe 2, we worked out that is 1.9635 into 10 is to power minus 3 meter square.

This value is given to us, we are given elevation head which we at section 1 is given by z 1 is equal to 4.5 meter, z 2 equal to 3.5 meter, flow is given we have converted this in liter per second to meter cube per second. So, here we will be using continuity equation, that already we know that

$$Q = A_1 V_1 = A_2 V_2$$

Means, discharge at section 1 equal to discharge at section 2. And then from this particular equation we are getting what is the velocity of flow.

$$V_1 = \frac{Q}{A_1} = \frac{5.5\times10^{-3}}{3.117\times10^{-3}} = 1.76 \text{ m/s}$$

We have got as 1.76 meter per second. Similarly we are substituting V_2, velocity at section 2 can be obtained by using the continuity equation.

$$V_2 = \frac{Q}{A_2} = \frac{5.5\times10^{-3}}{1.9635\times10^{-3}} = 2.80 \text{ m/s}$$

So, we get velocity in section 2 is 2.8 meter per second.

Now we are using here Bernoulli's equation. Bernoulli equation that pressure head plus velocity head plus elevation head equal to constant.

$$\frac{p_1}{\rho g} + \frac{(V_1)^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{(V_2)^2}{2g} + Z_2$$

Now when we are taking two section means velocity at section 1 velocity head, pressure head, and elevation head equal to pressure head plus velocity head plus elevation at section 2, these 2 at the 2 sections these are equal. So, having substituted the values of pressure head that is already given. And then we have got the velocity we have calculated, so velocity head we are substituting elevation head we are substituting except p 2**.** All the parameters in the right hand side of the equation is known to us.

$$\frac{20\times104}{1000\times9.81} + \frac{(1.76)^2}{2\times9.81} + 4.5 = \frac{p_2}{1000\times9.81} + \frac{(2.80)^2}{2\times9.81} + 3.5$$

So, we have worked out pressure head or intensity of pressure at section 2 is

$p_2 = 20.736 \times 10^4 \text{ N/m}^2$

Which can be written as 20.736 Newton per centimeter square or this can be further converted by dividing 20.736 Newton per centimeter square dividing with 9.81 then we get 2.1137 kg per square centimeter.

So, this is the value pressure at section 2 is this value. So, this is the answer, desired answer which was asked in the question.

Now, when we come to the 5th question. Now, 5th question deals with movement of water through soil by infiltration process. Now, in infiltration process we are given that relationship between accumulated depths of infiltration which is y, it is expressed in centimeter and this particular relationship was obtained when the infiltration test was conducted.

So, $y = 0.57t^{0.70} + 0.05$

And we are asked to determine the infiltration rate at 5 minute and at 30 minute elapsed time. So, the equation which was established it is the modified form of Kostiakov's equation. And the modified form of the Kostiakov's equation is given by $y = at^{\alpha} + b$

Now, in this equation which we have a is equal to 0.57, alpha equal to 0.7 and b equal to 0.05. Now this is a cumulative depth equation of infiltration or accumulated depth of infiltration equation. So, we are asked to determine infiltration rate at 5 minute elapsed time as well as for 30 minutes elapsed time.

So, if we are taking rate of infiltration, we have to obtain the first derivative of this equation. So, first derivative of this equation dy by dt which we can write as

$$\frac{dy}{dt} = a\alpha t^{\alpha-1}$$

a infiltration rate equal to a into alpha t alpha minus 1. So, a is 0.57 and alpha is 0.7, t is 5 minute and alpha minus 0.7 minus 0.1.

$$I = 0.57 \times 0.70(5)^{0.70-1}$$

$$I = 0.246\ cm/min$$

So, when we solve this equation rate of infiltration at 5 minute interval, it comes as 0.246 centimeter per minute, this can be converted in centimeter per hour.

$$I = 14.76\ cm/h$$

So, this can be written as I is equal to 14.76 centimeter per hour, so this is for 5 minute elapsed time.

Now, when we are interested to obtain a 30 minute, it is a very simple in the same way we did for 5 minute interval, we can do for 30 minute interval. Already we have the value of A that is 0.57, we have a value of alpha that is 0.7 and t is 30 minute, so we are substituting t 30 and alpha 0.7 minus 1.

$$I = 0.57 \times 0.70(30)^{0.70-1}$$

$$I = 0.1438\ cm/min$$

$$I = 8.63\ cm/h$$

So, this can be converted in centimeter per hour. So, I for 30 minutes elapsed time is 8.63 centimeter per hour. So, here we can see that at initial time the rate of infiltration is high and as we go for the longer time interval or longer elapsed time the rate of infiltration is decreasing with the time. And after some time you will obtain this rate of infiltration it becomes constant, and this rate of infiltration is known as basic rate of infiltration when it becomes constant.

Sometime when we do not have the value of hydraulic conductivity, so basic infiltration rate is used as the value of hydraulic conductivity of the soil.

Sixth question basically it deals with obtaining the physical properties of soil. These are all basic parameter which I explained in soil property class, those parameters are here. We are determining mathematically, numerically, you will find the value so that you get practice and you can have more experience and confidence in dealing with such problems. So, here a question is given an undisturbed soil sample of 500 gram was collected by using core sampler.

Core sampler, it is a device where the soil sample is collected at desired depth. So, this core sampler having a volume of 3 into 10 is to power minus 4 cubic meter and after oven drying at

105 degrees Celsius for 24 hours. The weight of soil sample was reduced to 400 gram, a specific gravity of this soil is 2.6 and depth of root zone of the crop is 60 centimeter. Now in this question, we are given total soil mass that is 500 gram.

Then, that sample was kept for drying for 24 hours, then its weight is reduced that is dry weight of the dry soil is 400 gram and the specific gravity of the soil is 2.6, depth for which we will be finding out the values in the soil sample that is for 60 centimeter. And the core volume, core cutter, volume or core sampler volume is 3 into 10 is to power minus 4 cubic meter. Now from these given data, we will work out moisture content by weight basis.

We will find out bulk density, we will determine dry density of the soil, we will find out what is the equivalent depth of water in 60 centimeter root zone of the crop. And we will work out what is the void ratio of this soil? What is the porosity of the soil and degree of saturation?

So, let us try to do this particular solve this problem. So, mass of wet soil is 0.5 kg, it is given we are given after drying that is 0.4 kg. And then volume is 3 into 10 is to power minus 4 cubic meter, a specific gravity of the soil is 2.6 depth of root zone is 0.6 meter. So, moisture content on weight basis or we can say gravimetric soil moisture content is means the mass of water in the soil sample, because we have been given the total soil mass, the weight of the dry soil mass.

a) Moisture content (θ_m) of soil on weight basis

Mass of water in soil sample (M_w) = M_s - M_d = 0.5 – 0.4 = 0.1 kg

Therefore, Moisture content (θ_m) = $\frac{M_w}{M_d} = \frac{0.1}{0.4} = 0.25$

Moisture content (%) = 25

So, weight of water is 0.1 kg or we can say 100 gram. Now, when we want to find out moisture content by this weight basis, so this can be given mass of water is 0.1 gram, mass of dry soil is 0.1 kg and then mass of dry soil is 0.4 kg, so this is equal to 0.25. So, moisture content by gravimetric method is 25%, so this is the answer one of this question.

Second part, it is asked what is the bulk density of the soil? So, bulk density is defined as

b) Bulk density of soil (ρ_b)

$$\rho_b = \frac{Mass\ of\ soil}{Total\ volume\ of\ soil} = \frac{M_s}{V_T} = \frac{0.5}{3\times10^{-4}} = 1666.66\ \frac{kg}{m^3}$$

Dry density, just simply here we will be using mass of dry soil divided by total volume of soil. So, mass of dry soil is given as

c) Dry density (ρ_d)

$$\rho_d = \frac{Mass\ of\ dry\ soil}{Total\ volume\ of\ soil} = \frac{M_d}{V_T} = \frac{0.4}{3\times10^{-4}} = 1333.33\ \frac{kg}{m^3}$$

Now, when we want to find out equivalent depth of water in the soil root zone or we can say when we are considering 60 centimeter of the soil column. So, what is the volume of water which is available? So, volume of water in 60 centimeter soil column. So first let us see that we have been given theta m we have calculated that is 0.25 and then density of the soil that we have got the dry density of the soil that is rho d by and then we know what is the density of water that is 1000 kg per cubic meter.

So,

d) Equivalent depth of water in the soil root zone (d)

Volumetric moisture content can be determine as

$$\theta_v = \theta_m \times \frac{\rho_d}{\rho_w} = 0.25\times\frac{1333.33}{1000} = 0.333$$

Therefore, Equivalent depth (d) = θ_v × depth of root zone

$$d = 0.3333 \times 60 = 20\ cm$$

This water is 20 centimeter in 60 centimeter of soil column.

Void ratio is another important parameter of this soil. And in this question, we are given certain parameter we will use to compute the void ratio. So, void ratio is given by

e) Void ratio (e)

$$e = \frac{V_v}{V_s}$$

Volume of solid (V_s) = $\frac{Mass\ of\ solid}{specific\ gravity\ of\ soil \times density\ of\ water} = \frac{0.4}{2.6 \times 1000} = 1.538 \times 10^{-4}\ m^3$

Volume of void (V_v) = $V_T - V_s = (3 \times 10^{-4}) - (1.538 \times 10^{-4}) = 1.462 \times 10^{-4}\ m^3$

Therefore, $e = \frac{1.462 \times 10^{-4}}{1.538 \times 10^{-4}} = 0.95$

So when we are dividing the volume of void with the volume of solid then we get the void ratio as 0.95. So, this is the answer for of void ratio in the given question.

Porosity is another parameter, it is important parameter which is it matters when we deal with the water movement in the soil, when we supply water through drip irrigation system. So, porosity plays an important role, and so when we want to calculate porosity, this porosity is given by volume of void divided by total volume of the soil sample. So, this total volume of the soil sample is already given to us in the question that is the volume of core sampler and volume of void, we have calculated this value.

f) Porosity (n)

$$n = \frac{V_v}{V_T} = \frac{1.462 \times 10^{-4}}{3 \times 10^{-4}} = 0.487$$

So, the porosity of the soil in the given question is 0.487.

Degree of saturation, degree of saturation is defined by the ratio of volume of water divided by volume of void. So, volume of water can be computed by taking the mass of water, this divided by the density of water and then volume of void already we have calculated. So, mass of water we have already got that is 0.1 kg and density of water we know 1000 kg per cubic meter.

g) Degree of saturation (S_r)

$$S_r = \frac{V_w}{V_v} = \frac{\left(\frac{M_w}{\rho_w}\right)}{V_v} = \frac{\left(\frac{0.1}{1000}\right)}{1.462 \times 10^{-4}} = 0.683$$

When we work out numerical the substitution of these values, we get degree of saturation is 0.683. Now 0.683, it is the value which we say that 68.3% is the saturation level in the given soil sample. So, this is a important value when we are dealing with the various soil moisture constant or when we are dealing with the saturated soil sample, when we deal with the field capacity, when we deal with the wilting point or available water content.

So, in this particular tutorial, we have worked out the numerical problems related with simply unit conversion. We have dealt application of basic concept of fluid mechanics where we use continuity equation, where we use the energy equation that is the Bernoulli equation, when we deal with the physical properties of soil like infiltration capacity of the soil, when we deal with the porosity, bulk density, particle density, void ratio or degree of saturation.

So, such are what is the depth of water? So, these conversion from the gravimetric moisture content into volumetric water content such type of concept we dealt in these examples. So, for your practice I hope you have understood these things. Now in forthcoming lecture, we will discuss concept of evapotranspiration that will be dealt in the coming class.

Now, to these problems you can refer books, these books can be used as your textbook for the subject particularly for these topics. So irrigation and theory by Professor A.M. Michael, Land and water management engineering by V.V.N Murty, and there are quite good amount of material available in a net. So, that you can refer these net also. So, by this I am just closing my lecture and thank you very much.