Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 09
One Dimensional Steady State Heat Conduction

So, good morning in the earlier class last class we had finished it up to that conduction heat transfer through a slab One Dimensional Steady State with say constant energy generation and constant conductivity. So, we came to the solution after integration of both the of the differential equation by integrating twice we got two constants C_1 and C_2 and we were supposed to solved for C_1 and C_2, 'right'.

But, we also said to solve the two bound, two constants, C_1 and C_2 we need to have 2 boundary conditions, which can be any one of prescribed heat flux boundary, prescribe temperature boundary, or prescribed convective boundary. Any one of these or in combination we need two boundaries to solve them, 'right'. So, now, let us do a problem and solve that ok. So, it is like that in this is one dimensional steady state heat conduction continuation in lecture number 9, we come to this that the problem is defined like this.

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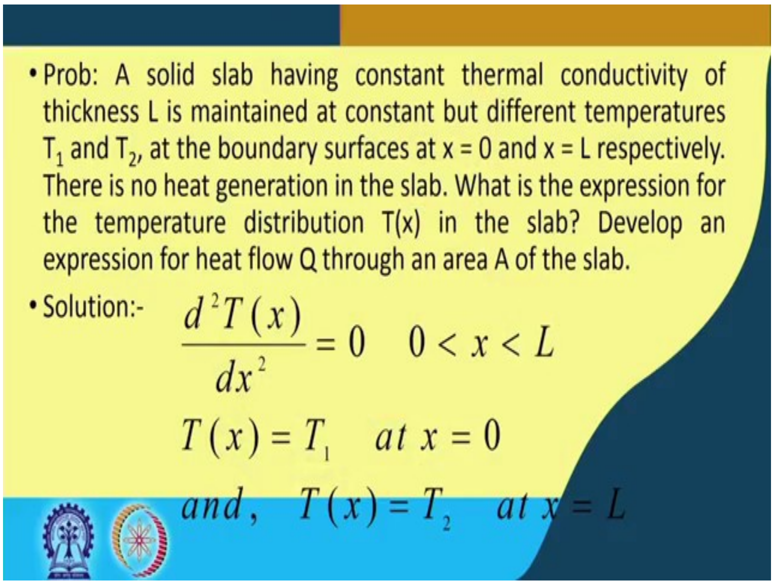

A solid slab having constant thermal conductivity of thickness L is maintained at constant, but different temperatures T_1 and T_2, at the boundary surfaces at x =0 and x=L respectively. There is no heat generation in the slab. What is the expression for the temperature distribution T(x) in the slab? And, develop an expression for heat flow Q through an area A of the slab.

I repeat, the problem is like this a solid slab having constant thermal conductivity of thickness L is maintained at constant, but different temperatures T_1 and T_2; that means, both the sides one side T_1 and another side T_2 at the boundary surfaces at x =0 and x= L respectively. There is no heat generation in the slab. What is the expression for the temperature distribution T(x)? You remember we came to the solution that T(x) is equal to this plus C_1 and C_2 this was our solution.

So, C_1 and C_2 was not evaluated, because boundaries were not given, but now our two boundaries are given one is prescribed temperature boundary and another is also prescribed temperature boundary, that is boundary surfaces is at T_1 at x =0 and T_L at x=L, 'right'. As, then we have we have to find out that temperature distribution T(x) in the slab as well we have to also find out the expression for the heat flow Q through an area A of the slab, 'right'. So, we start with that now here what do we have? We have the our situation is steady state; that means, the, 'right' side is 0, then we have no internal energy generation, because it is not specified unless it is specified, we can assume that no internal at energy generation. Because at there been internal energy generation or internal heat generation, then it would have been specified since it is not specified we can assume that there is no generation of heat number 2 and number 3 is that conductivity k is constant, 'right'.

In that case if all these three are true, then we can write the generalized equation from that for the slab

$$\frac{d^2T(x)}{dx^2} = 0 \quad 0 < x < L$$

$$T(x) = T_1 \quad at\ x = 0$$

$$and, \quad T(x) = T_2 \quad at\ x = L$$

The reason being it is a steady state, there is no energy generation and the conductivity is constant, then we have written like this, 'right'. Now, we let us go into it is solution.

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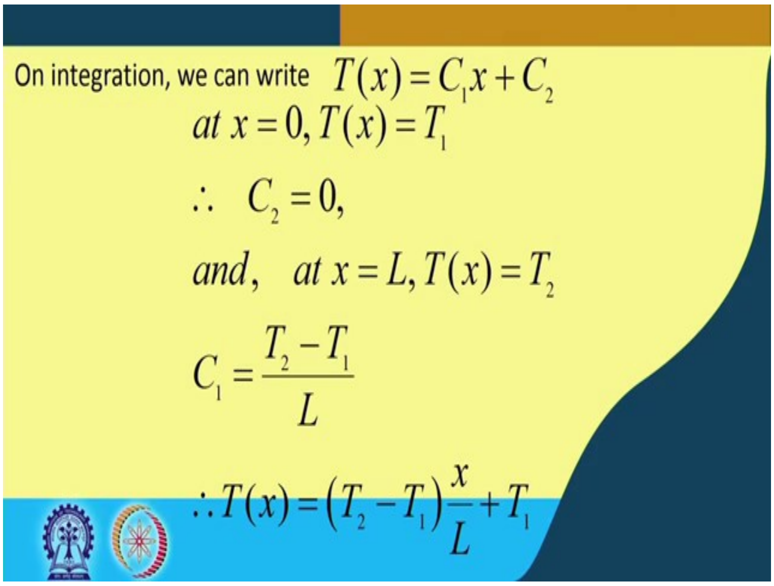

The solution of it can be written that on integration we can write, first integration T(x) is equal to something this is, 'right' and this is C_1x and then T(x) is equal to $C_1x + C_2$, 'right'. So, if that be true, then we can because first one came as the integration of $d^2T(x)/dx^2$, it became dt/dx is equal to constant C_1 and next integration it came T(x) is equal to C_1x plus C_2, 'right'.

And, we are given at x is equal to 0 that is the boundary at x =0, T(x) =T_1, 'right'. If, that be true at x =0 T(x) =T_1, then we can easily write that x =0, 'right' that you takes x =0, T_0 is T_1, 'right'. And, then we can write that this x is to 0 means this is 0 and this is T_1. So, C_2 is T_1 not 0, it should be C_2 is T_1, 'right'.

And, at x =L, T(x) =T(2), 'right'. So, we can write this is C_1 is we have now we are substituting. So, it is C_1 we have to find out, so T is T_x is T_2; T_2 is this equal to this one becomes C_1, C_1 L, 'right' and this is becoming T_1, 'right'. So, we can write C_1 is equal to this is $(T_2 - T_1)/L$ is C_1, 'right' because this is not C_2 is equal to 0 this was by mistake it was written. So, in many cases this may be purposefully made the mistake.

So, that you can identify, because it should it should click you that there is something wrong in it, 'right'. So, here I that is why I am explaining that on integration we got T(x) is $C_1x + C_2$ how you got it? We got it from here that $d^2T(x)/dx^2$ is 0, on first integration we got dT(x)/dx equal to C_1, 'right'. And, on second integration T(x) is equal to $C_1x + C_2$, which we wrote in once in one shot, 'right'.

So, this is on the integration and to solve this two C constant C_1 and C_2, we need 2 boundaries. 2 boundaries already given T(x) is T_1 at x =0 and T(x) is T_2 at x =L. So, if that be true in the first case at x equal to 0, T(x) is T_1, so; that means, here it is T_1 and x is 0. So, this goes off zone; that means, C_2 is T_1 not 0, 'right' this is T_1.

And, at x equal to L, T(x) is T_2 that is the boundary given. So, we can write and substituting at x is equal to L, so this is C_1 L, C_2 is T_1 and this T(x) has become T_2, 'right'. So, we can write C_1 is equal to this $(T_2 - T_1)/L$. So, we can substitute C_1 and C_2 here we get T(x) is equal to $(T_2 - T_1)x/L + C_1$, 'right'.

So, this x by L came because this C_1 is $(T_2 - T_1)/L$. So, $(T_2 - T_1)x/L + T_1$ is T(x). So, as you were said in the problem find out the temperature distribution T(x) in the slab, 'right'. So, which we have found out that the C at T(x) that is temperature distribution in the slab is $(T_2 - T_1)x/L + T_1$, 'right'.

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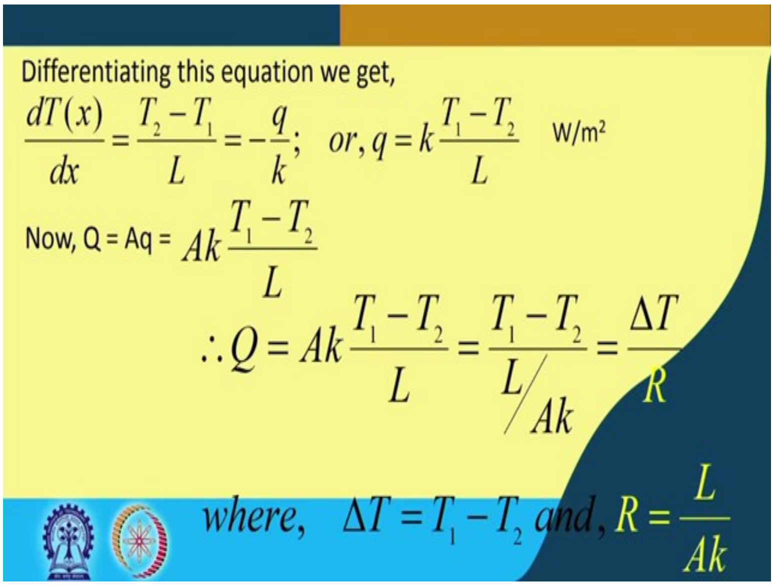
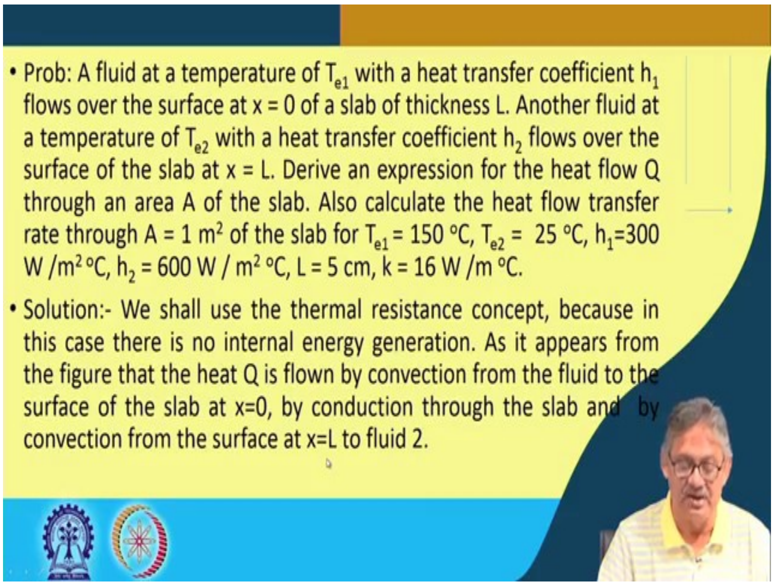


Now, we need to find out q to find out q we need to first differentiate that was on first differentiation integration we get

$$\frac{dT(x)}{dx} = \frac{T_2 - T_1}{L} = -\frac{q}{k}; \quad or, q = k\frac{T_1 - T_2}{L} \qquad Ak\frac{T_1 - T_2}{L}$$

$$\therefore Q = Ak\frac{T_1 - T_2}{L} = \frac{T_1 - T_2}{L/Ak} = \frac{\Delta T}{R}$$

$$where, \quad \Delta T = T_1 - T_2 \; and, R = \frac{L}{Ak}$$



Now, let us see another problem, 'right'. A fluid at a temperature of T_{e1} with a heat transfer coefficient h_1 flows over the surface at x =0 of a slab of thickness L. Another fluid at a temperature of T_{e2} with a heat transfer coefficient h_2 flows over the surface of the slab at x =L. Derive an expression for the heat flow capital Q through an area of A of the slab.

And, also calculate the heat flow transfer rate through A is equal to 1 m^2 of the slab for T_{e1} = 150ºC, T_{e2}=25 ºC, h_1=300 W/m^2ºC, h_2=600 W/m^2ºC and L is 5 centimeter and k of the material is 16 W/mºC, 'right'.

So, this tells that we have a slab and one side is a convective boundary condition and another side of this slab is the another convective boundary condition where it is a T_1 T_{e1} or T_{e1} and this is T_{e2} with a transfer coefficient of h_1 and h_2, then we have to find the solution and the accordingly was the solution is over obtained the problem numeral can also be solved, 'right'.

So, a fluid at the temperature of T_{e1} with a heat transfer coefficient of h_1 close over the surface at x equal to 0 of a slab of thickness L. Another fluid at a temperature T_{e2} with the heat transfer coefficient h_{e2} flows over the surface of this slab at x equal to L. Derive and expression for the heat flow Q through an area A of this slab, also calculate the heat flow transfer rate through A is equal to 1 meter square of the slab for T_{e1} is equal to 150 degree centigrade, T_{e2} is equal to 25 degree centigrade, h_1= 300 W/m^{2o}C, h_2= 600 W/ m^{2o}C, L is equal to 5 centimeter, k is 16 W/m^{o}C.

So, if you remember in the previous class, we had shown that let me also draw this a little rate, that we had say this one and we also had say this one and we said that Q is flowing through this and a resistance over it was there, 'right', this was the resistance R, 'right'. So, and that resistance was equal to L/ Ak, 'right'. So, now, if we do this solution on the basis of thermal resistance concept, if we solve it on the basis of thermal resistance concept, then the solution becomes much easier it is not analytical solution earlier equation we have done analytical solution.

But, here we can use analytical definitely and that can be having the same answer, but much more easier because if there is no internal energy generation number 1, number 2 the conductivity is constant, number 3 it is a steady state. So, if all these three are prevailing, then the electrical resistance concept or thermal resistance concept we can do, 'right'.

So, the solution is like that, we shall we use thermal resistance concept, because in this case there is no internal energy generation. As it appears from the figure that the heat Q is flown by convection from the fluid to the surface of the slab at x=0, by conduction through the slab and by convection from the surface at x =L to the fluid 2 for example, this, 'right'.

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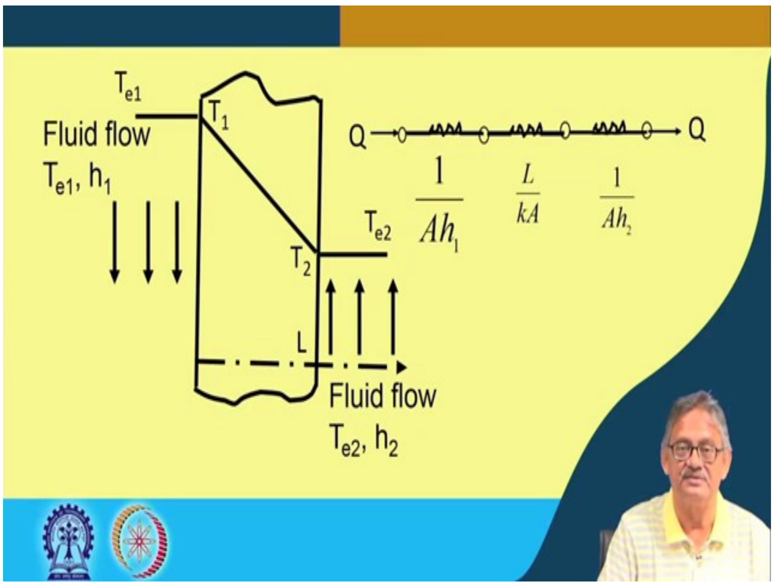

So, one side this fluid is flowing with a heat transfer coefficient of T_{e1} and at heat transfer coefficient of h_1, with a temperature T_{e1} of the fluid coming to this surface. Surface temperature is T_1 and then it is getting conducted through the material to the temperature T_2 and then getting dissipated through a fluid with heat transfer coefficient of h_2 at a temperature of T_{e2}, 'right'. Which if we draw according to the thermal resistance, then Q quantity of heat is coming to this place that is here, 'right' up to this it is coming outside.

And then we have a heat resistance according to the fluid properties that is $1/Ah_1$ coming to the point T_1 where it is having a metal thermal or material thermal resistance there is L/kA. So, like this coming to the point this, where another fluid with their resistance according to the temperature and environment condition that is that is your h that heat transfer coefficient, it is $1/Ah_2$. So, this resistance coming to this outer side and the same Q is going out, 'right'.

So, this is called thermal resistance concept. One resistance is here, 'right', one resistance is here, another resistance is through this, another resistance is through this these are the three resistances corresponding to $1/h_1$, L /kA and $1/Ah_2$, 'right'.

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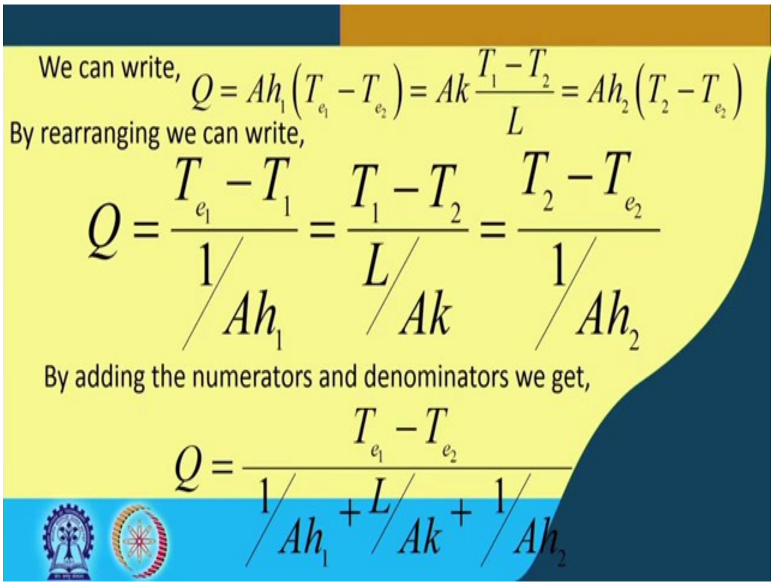

Then, we can solve we can write $Q = Ah_1\left(T_{e_1} - T_{e_2}\right) = Ak\frac{T_1 - T_2}{L} = Ah_2\left(T_2 - T_{e_2}\right)$

These are the three resistances flowing through the whole system where Q quantity of heat is flowing, 'right'.

$$Q = \frac{T_{e_1} - T_1}{1/Ah_1} = \frac{T_1 - T_2}{L/Ak} = \frac{T_2 - T_{e_2}}{1/Ah_2}$$

Now, these three terms if we add by adding the numerators and denominators we get Q is

$$Q = \frac{T_{e_1} - T_{e_2}}{1/Ah_1 + L/Ak + 1/Ah_2}$$

So, we can get capital Q where, but here we are not finding temperature, because all the temperatures are already given. That ambient temperature or outside temperature or environmental temperature is given, then we are also given that ambient temperature both side and the surface temperatures are also given.

So, if the temperatures are given then we have to find out the quantity of heat Q flowing through them, 'right'. And, we have found out an expression based on the thermal resistance concept Q has become $Q = \frac{T_{e_1} - T_{e_2}}{1/Ah_1 + L/Ak + 1/Ah_2}$

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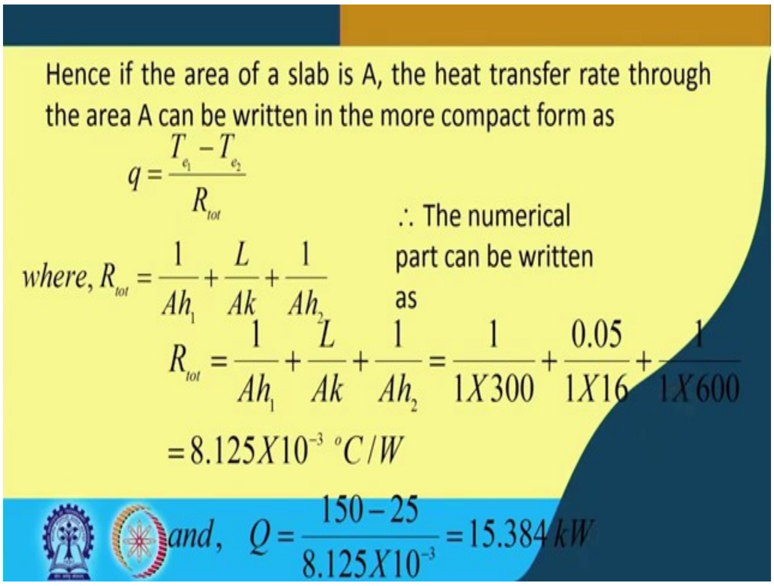

So, from and this quantification which we have seen that quantification can be written as, if area of the slab is A, the heat transfer rate through the A can be written as the Q is

$$q = \frac{T_{e_1} - T_{e_2}}{R_{tot}}$$

$$where,\ R_{tot} = \frac{1}{Ah_1} + \frac{L}{Ak} + \frac{1}{Ah_2}$$

So, which if we substitute the numeral values where this A was given as 1, h_1 was given as 300, h_2 was given as 600, and k was given 16 and length was given 0.05, 'right'. So, 5 centimeter so, 0.05 meter, so we can rewrite that as 1 by 1 into 300 plus 0.05 by 1 into 16 plus 1 by 1 into 600, 'right'.

So, these on simplification or calculation comes to be 8.125 into 10 to the power minus 3 watt per degree centigrade per watt, it is not watt per degree centigrade it is degree centigrade per watt since it is the resistance, 'right' degree centigrade per watt. You remember we had said that this resistance equal to that q is equal to delta T by R, 'right', delta T is in centigrade q is watt and this resistance is the delta T is degree centigrade. So, resistance becomes equal to delta T that is degree centigrade by q that is watt. So, degree centigrade by watt is the resistance; so, which we have come across as 8.125, 10 to the power minus 3 degree centigrade per watt.

$$R_{tot} = \frac{1}{Ah_1} + \frac{L}{Ak} + \frac{1}{Ah_2} = \frac{1}{1X300} + \frac{0.05}{1X16} + \frac{1}{1X600}$$
$$= 8.125X10^{-3}\ {}^{o}C/W$$
$$and, \quad Q = \frac{150-25}{8.125X10^{-3}} = 15.384\,kW$$

you will also you should also make yourself this solution or calculations, so, that you can check whether the number is correct or not because I may also do some calculative mistake, but through calculation.

So, you do also yourself this will type your problem and see it is solution, 'right'. With this we conclude to this class, next we will go may be a continuation of this slab and another solution some problem solution also to be done. So, that if you solve more and more problems, you are much better off or you really understand the principles lying over it, 'right'.

Thank you.