Thermal Operations In Food Process Engineering: Theory And Applications

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Lecture - 49
Heat Exchangers (Contd.)

So, we are coming now to the Heat Exchangers more or less to the end part, it is not maybe some few more classes will be required because if you are able to handle heat exchangers and problems associated with that, then I hope you can handle heat transfer very efficiently, 'right'.

So, now we are coming to that lecture number 49, that is heat exchanger continuation, 'right'.

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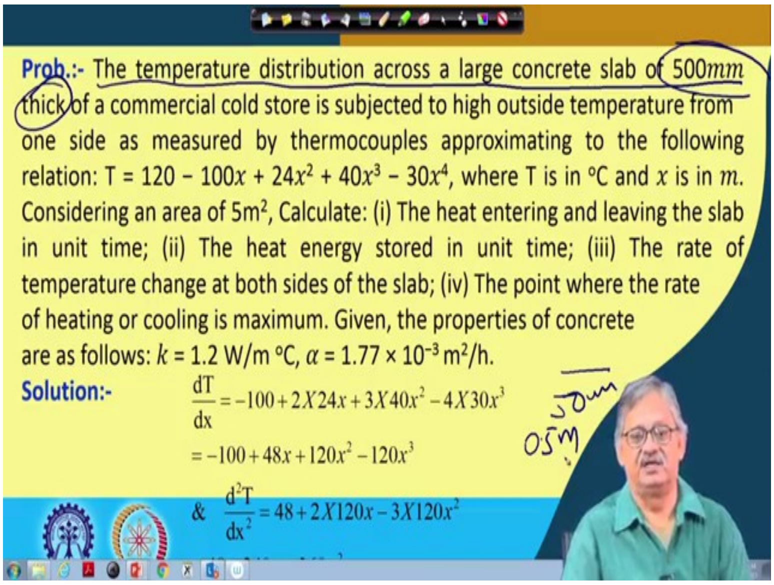

So, here also let us do a problem, this is a different problem than what you normally come across that is why I thought let me share with you also. The problem is like this. The temperature distribution across a large concrete slab just now we gave definition in the previous class. We gave that cold store or R22 things like that we had given.

So, that is why that if you see a cold store so, that is having a concrete, 'right' and then some insulation 'right' and then some barrier that is vapour barrier etcetera, 'right'. So, their concrete is also one of the vital thing. So, that is why I thought let it be also be taken care of or let it also be tried with. That the temperature distribution across a large concrete slab of 500 mm thick. So, that slab is 500 mm thick that is 50 cm, ‘right’, 500 mm means 50 cm means 0.5 m, ‘right’. So, half of a meter is thick. So, really it has to be because they are normally equal storages are 4-5 story.

So, if it is not well constructed then there may be a chance of collapse that is why they are well. I mean you know from the civil engineering point of view well done; however, our problem is not civil engineering our problem is heat transfer. So, let us look into that. The temperature distribution across a large concrete slab of 500 mm thick of a commercial cold store is subjected to high outside temperature from one side as measured by thermocouples approximating to the following relation. That is $T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$, ‘right’.

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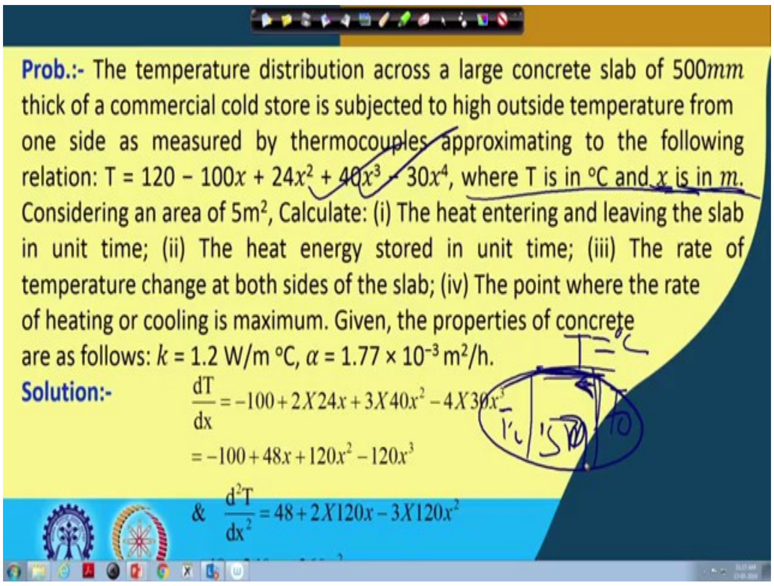

I repeat, at the end will repeat where T is in °C and x is in m, ‘right’. So, this was the concrete which was this length as thickness is 50 cm, ‘right’ or 0.5 m, ‘right’ and in this that the temperature distribution is like this, $120 - 100x + 24x^2 + 40x^3 - 30x^4$, where T is in centigrade and x that x is this as you are progressing this is having a temperature outside and this is a temperature inside. So, that this temperature distribution which we have said is a function of x where x is the distance from the end, ‘right’.

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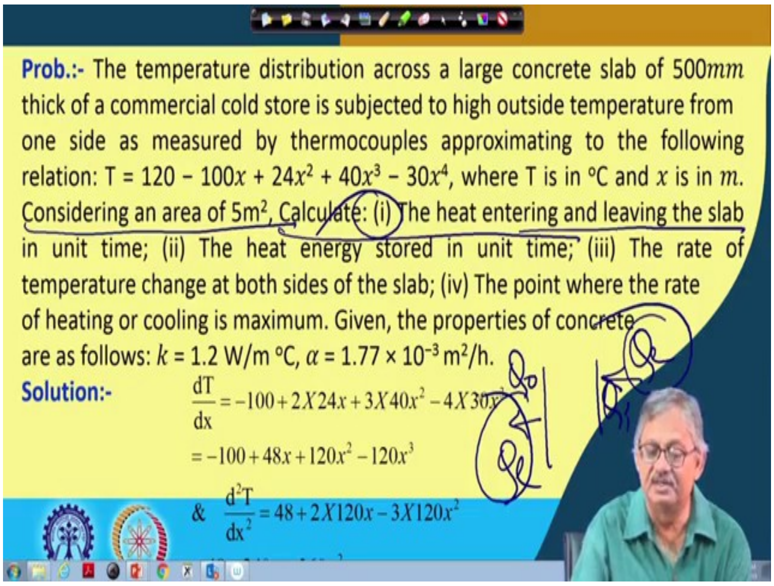

So, if these be true, then considering an area of 5 m^2, calculate the heat number i: the heat entering and leaving the slab, 'right'. So, this was the slab number i, find out the Q entering and Q leaving the slab 'right'. So, this is if it is entering and this is exiting or this can be simply written Q_i and this is Q_o that is much better, 'right' Q inlet and Q outlet, 'right', the heat entering and leaving the slab in unit time per unit time or second, 'right'.

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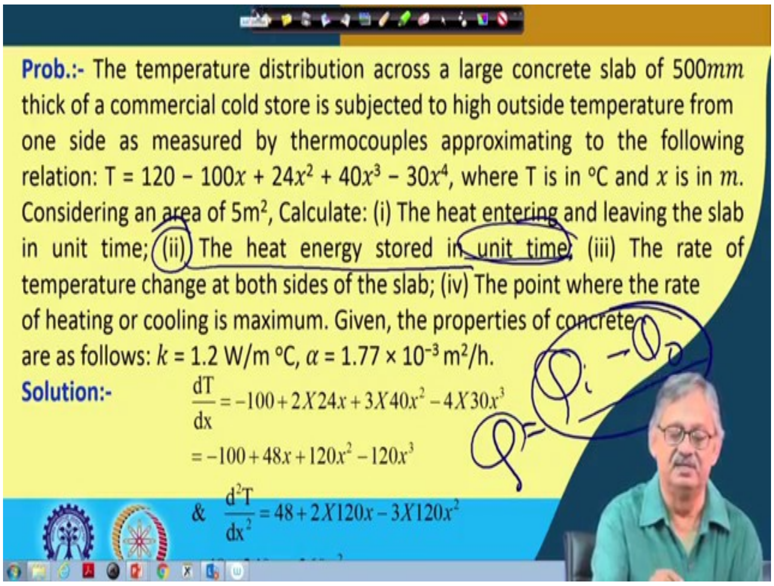

So, that is i. Number ii: the heat energy stored in unit time again what is the Q that has been stored that is Q_i - Q_o should be, 'right' how much Q has been stored should be whatever has come and whatever has left the difference should be the stored amount per unit time since the earlier one was also per unit time and this one is also per unit time. Third the rate of temperature change at both sides of the slab.

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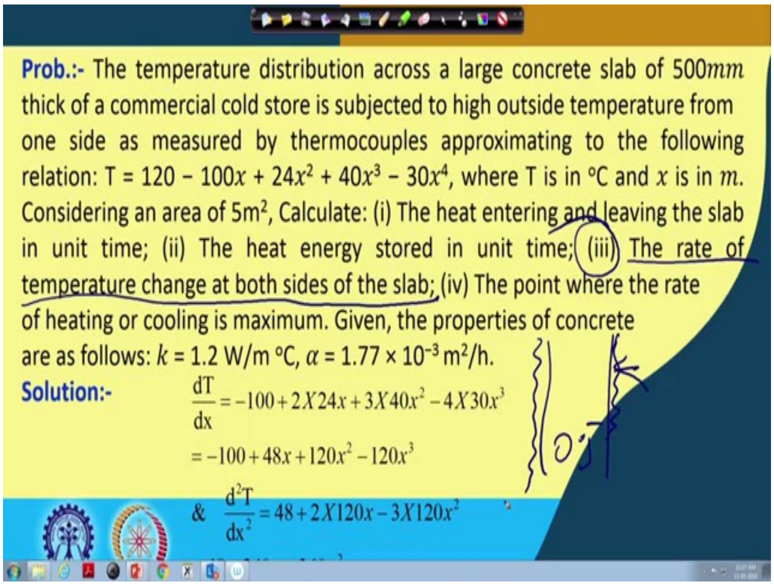

Again we have that 50 or 0.5 m slab, 'right', the rate of temperature change at both side of the slab. So, what is the rate of temperature change at this end and what is the rate of temperature change at this end of the slab that is also to be determined. And number iv: the point where the rate of heating or cooling is maximum, 'right'. So, this is both heating or cooling is maximum, 'right'. So that means, here also that maximum minimum concept will come into, 'right'.

So, some derivative is equal to 0 that is for the maximum or minimum. So, fourth one is that. Given the properties of the concrete like this k, conductivity of the concrete material is 1.2 W/m.°C, alpha that is thermal diffusivity is 1.77×10^{-3} m^2/h this is m^2/h, 'right'.

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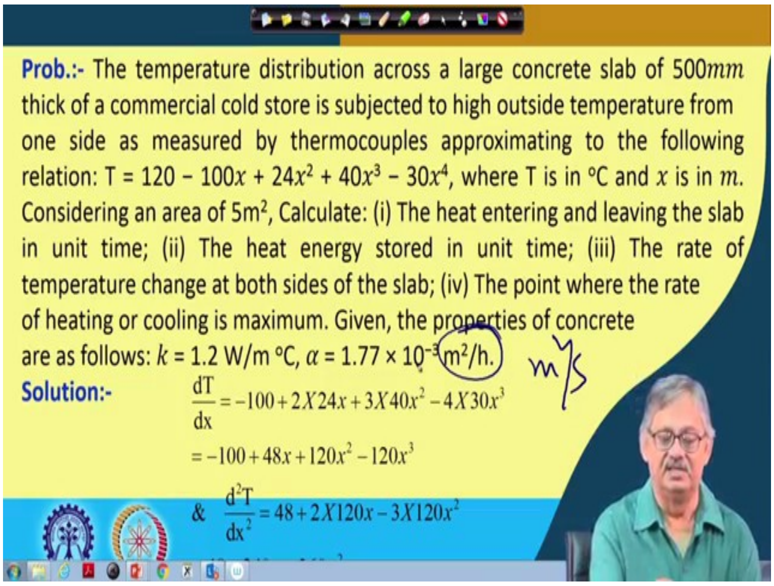

Normally α is in m^2/s or thermal diffusivity is m^2/s, when the moment it is like that the value will be somewhere high. So, that is why it is m^2/h; 1.77×10^{-3} m^2/h, 'right'. So, to do the problem as we normally do that we repeat the problem so that understanding becomes clear.

The temperature distribution across a large concrete slab of 500 mm thick of a commercial cold store is subjected to high outside temperature from one side as measured by thermo couples approximating to the following relation, $T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$, where T is in centigrade and x is in meter.

Considering an area of 5 m^2 area, calculate an area of 5 m^2 calculate number i, the heat entering and leaving the slab in unit time, number ii, the heat energy stored in unit time, number iii, the rate of temperature change at both sides of this slab and number iv, the point where the rate of heating or cooling is maximum given the properties of concrete as follows that k conductivity of the concrete material is 1.2 per m/°C and thermal diffusivity of the concrete material is 1.77×10^{-3} m^2/h, 'right'. So, this we have to do.

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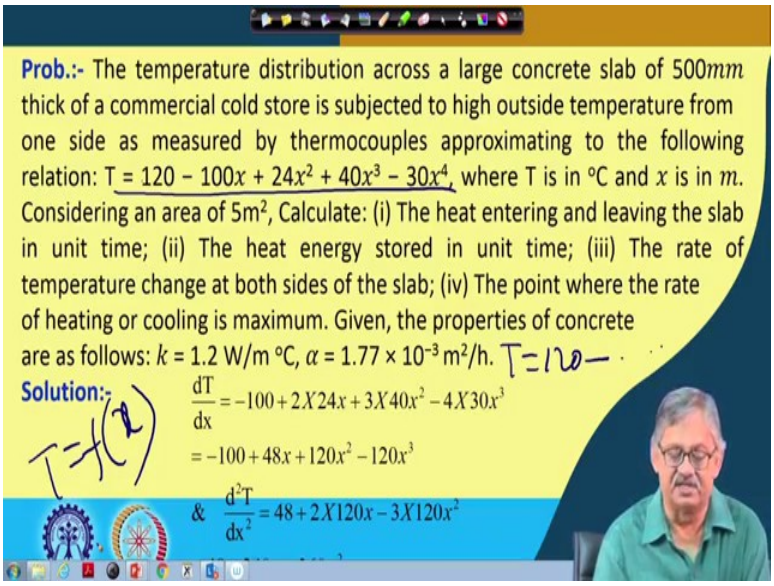

Now, to do that, first let us see we have been given this unit that T = 120 minus dot dot dot dot this thing, 'right'. So, write first this one. Now the moment you write it, so, T is we can write T is a function of x, 'right'.

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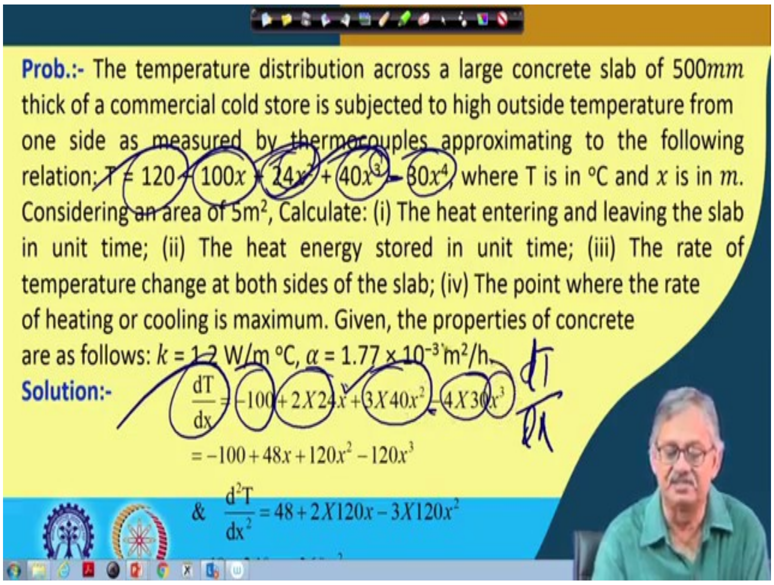

If T is a function of x then we can easily differentiate it as dT dx 'right'. So, that is what we have done. What is the dT/dx of this? dT/dx of this is this one is constant. So, not there, so, this is minus 100 x. So, minus 100, this is +24 x^2. So, 2 × 24x, this is on differentiation, this is $40x^3$ so, plus 3 x^3. So, 3 × 40 x^2, 'right' - 30 x^4; so, -4 ×30 × x^3.

So, dT/dx is this $-100 + 2 \times 24\,x + 3 \times 40\,x^2 - 4 \times 30\,x^3$ which on simplification can be written as $-100 + 48\,x + 120\,x^2 - 120\,x^3$ this is dT dx. So, dT dx is this first derivative, ‘right’. So, what will be the second derivative? That is d^2T/dx^2.

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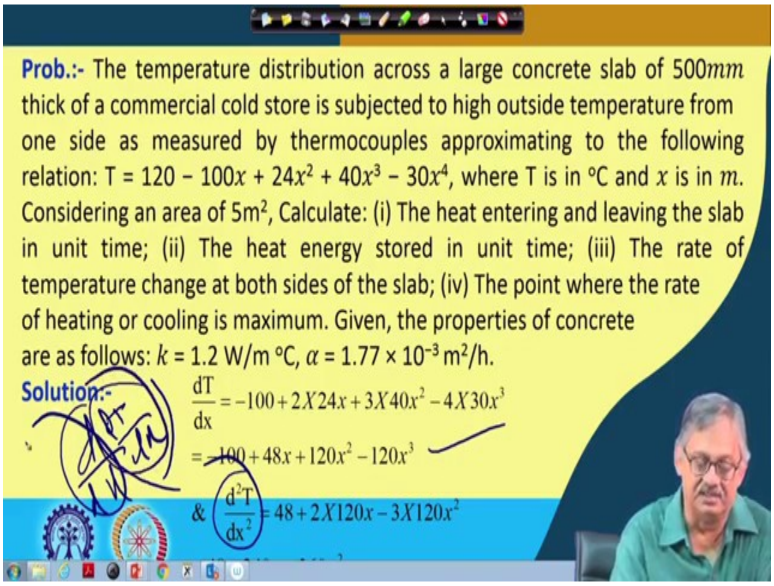

Second derivative will be again d dx(dT/dx), ‘right’ ddx of dT/dx is your d^2T/dx^2 or second derivative. So, if we again differentiate with respect to x the dT/dx, then we get that. So, this is to be again differentiated. Again if we differentiate first this term goes off, ‘right’. So, here it is 2×24 that comes directly, here it is 3×40 or 120×2 2×120 x, ‘right’ and this term it is 3×120 that is this.

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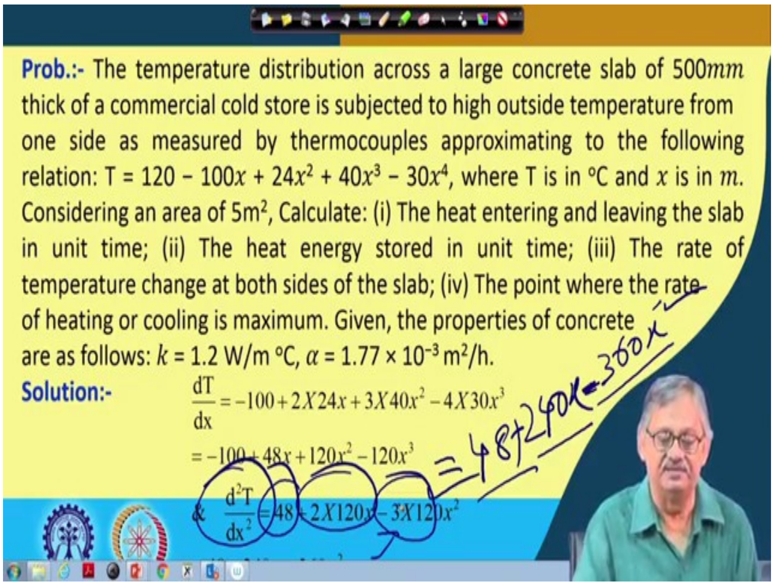

So, we can write d^2T/dx^2 is 48 + 2 × 120 x - 3 × 120 x^2 that can be written which is not visible here, but that can be written simply as 48 plus this is 120 × 2 that is 240 x, 'right' minus this is -3 ×120; that means, 360 x^2 is the simplification form of this 'right'. 48 + 240 x - 360 x which is written here, 'right' which is not visible for some reason beyond me, ok.

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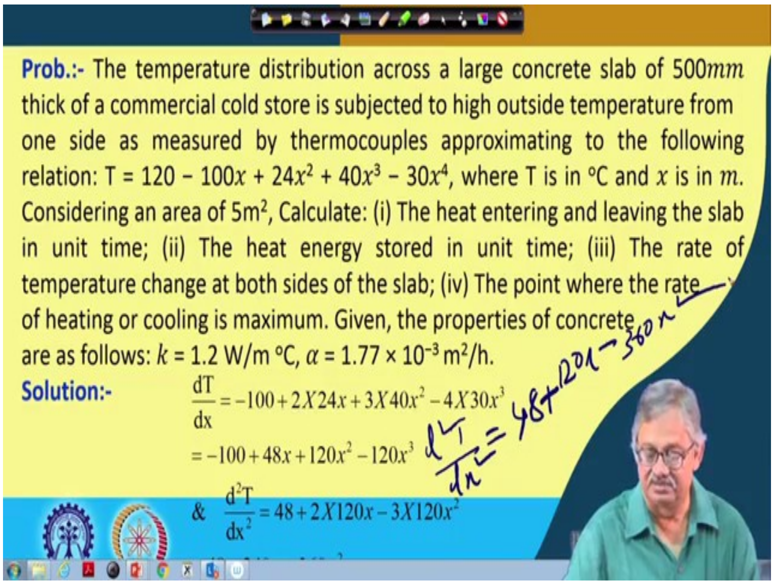

So, I repeat that we have d^2T/dx^2 on simplification is equal to 48 + 120 x - 360 x^2, 'right' that is the second derivative, 'right'.

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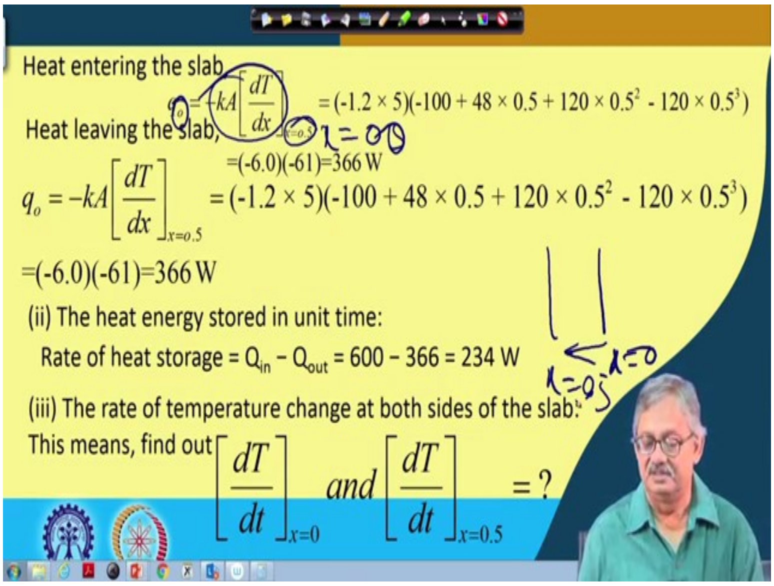

Now, if we have this, then we can write heating heat entering the slab, ‘right’ heat entering the slab that was our first question. So, heat entering the slab that can be written.

So, q_o if it is -kA dT/dx at x = 0 instead of 0.5, it should be written x is equal to because both the sense, ‘right’. So, one end x = 0 and other end x = 0.5, ‘right’ that is logical, ‘right’.

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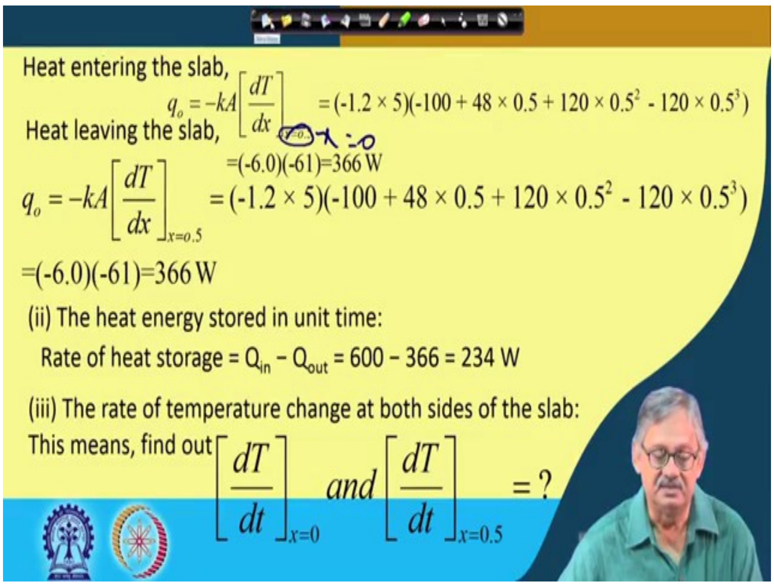

So, at x = 0 they should be x = 0 we write our expression was -1.2, ‘right’ into the again we again we made that mistake, that mistake is here you see it is α is one point not 2, 1.77 oh k is 1.2, 'right' k is 1.2 and α is 1.77 10 to the minus 3. So, when you are doing it with your calculator, please look into the corrections that this is x = 0 and our dT/dx -kA. So, we have -1.2 that is k, A was 5, 'right' and dT/dx first thing which came up is this which came up was minus 100 + 2 × 24 that is plus 48 x + and 120 x^2 - 120 x^3, 'right'.

So, this was our this thing dT/dx.

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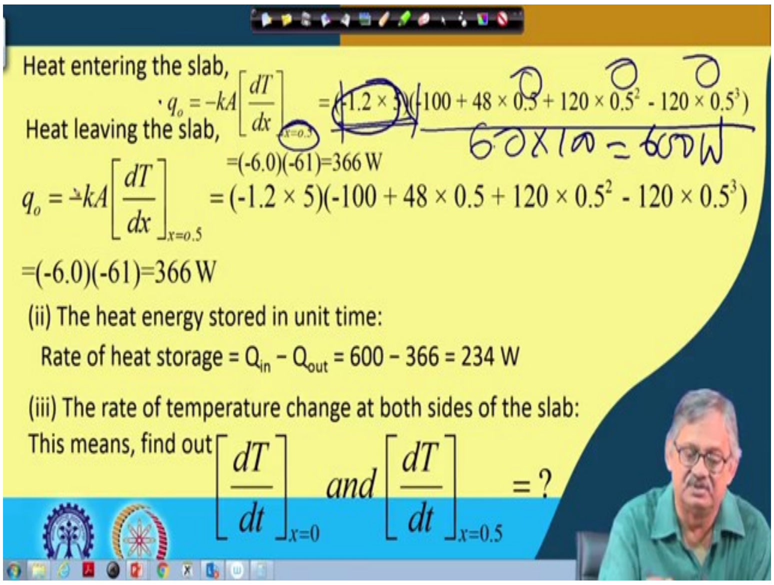

So, dT/dx at x = 0, this was that this is –kA, 'right'. So, if we take this should have been this actually should have been -100 into this is 0 this is 0 this is 0, 'right'. So, 1.2 × 5, 'right' 1.2 × 5 is how much? 60, 'right', 1.2 × 5 is 6, 'right' and this minus and this minus made it 100. So, it is 600. So, 600 watt actually q at x = 0; the same both the things are this and that are identical, somehow it got over overwritten, 'right'.

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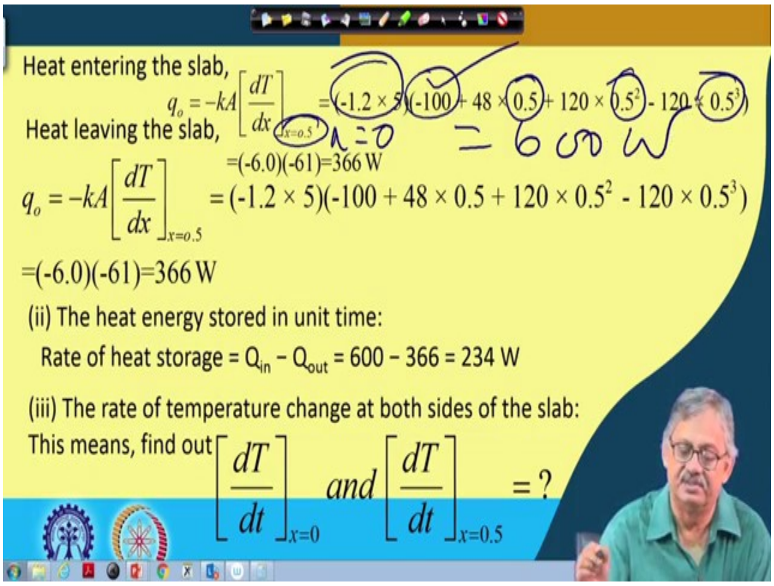

So, q at x = 0 you put all in this expression everywhere 0 x is 0 x is 0 then only minus 100 remains.

So, -1.2 × 5 is 6 times 100 = 600 W, 'right'. So, q is 0 is 600 W 'right' similarly q_o at x = -0.5, x equal 0.5 is -kA dT dx. So, now, here we are writing that here we are writing -1.2 × 5, 'right' into -100 + 48 × 0.5 + 120×0.5^2 -120×0.5^3, 'right'. So, this if we calculate then it comes this side is - 6 and this side after simplification becomes - 61.

So, total is this minus and this minus goes off. So, 6 × 61 is 366, what happen? When we have I have we have done I have done it. So, perhaps these also got super imposed unfortunately, 'right'. So, please correct it. This is x = 0 you put in all the x as 0, 'right'.

Then you have only 100 ×6. So, it becomes 600 W at x = 0 and this should be $q_{0.5}$, 'right', $q_{0.5}$ and -k A x = 0.5 that comes 366 W, 'right'. So, second part is over, third part is the heat energy stored in unit time, 'right' heat energy stored in unit time.

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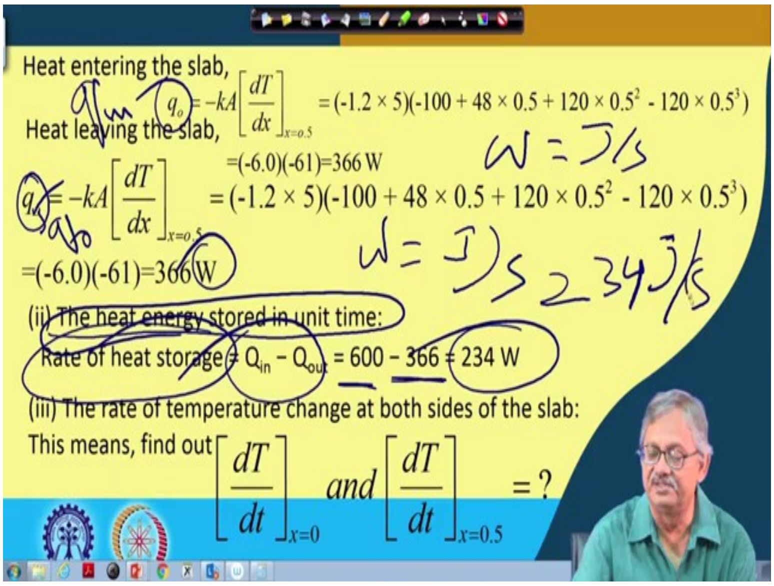

So, all where this watt means Joules per second ‘right’ this watt means J/s, ‘right’. So, if we have $q_o – q_{0.5}$ then that is what is or this could have been written which I said q in and this could have been written q out, ‘right’. So, q_{in} - q_{out} is the total heat which is going out. So, it is 600 was in 366 was out. So, it is 234 W, ‘right’ so; that means, 234 J/s per unit time, ‘right’.

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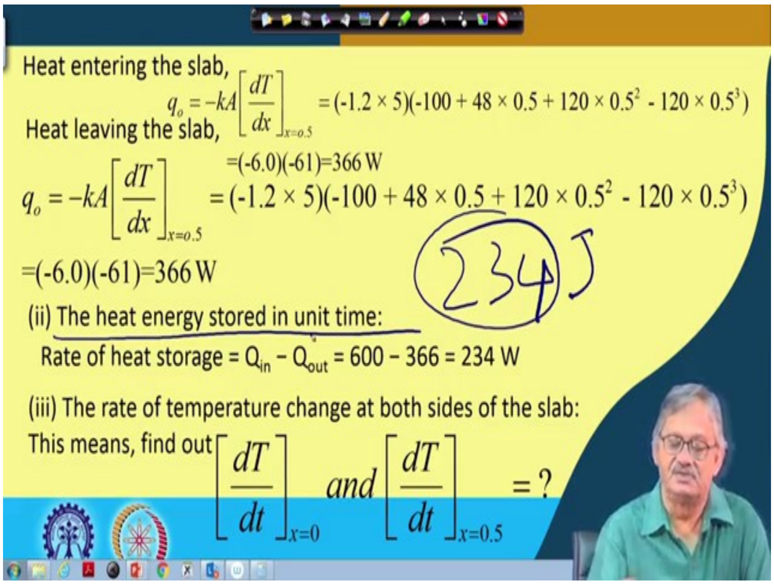

Actually it should have been written 234 J because we are already asked per unit time.

So, 234 J, 'right' per unit time. So, that is the net or the energy heat energy stored in unit time, 'right'. Now the fourth one which is left this was the second, now the third one that is the rate of temperature change at both sides of this slab, 'right'.

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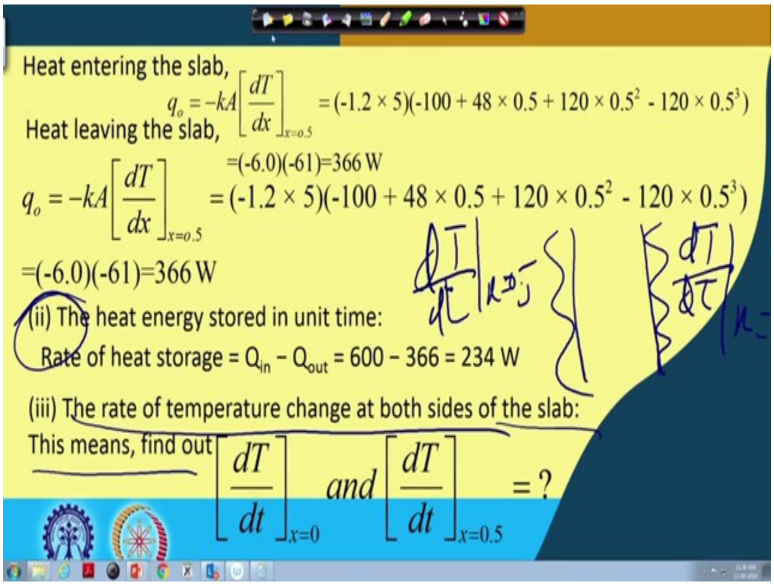

So, rate of temperature change at both sides of this slab, this was our slab, what is the rate of temperature change T that is d rate; that means, with respect to time.

So, dT/dt what is the value of dT/dt at x = 0 and dT/dt at x = 0.5, 'right'.

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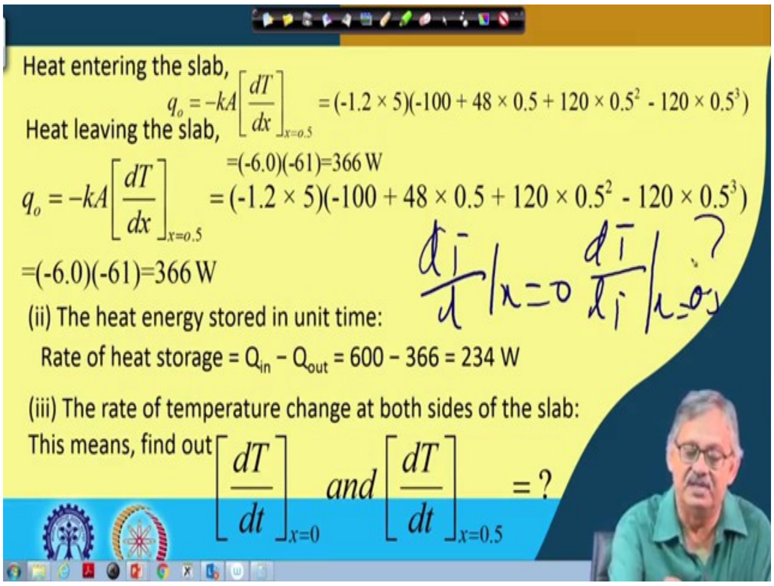

So, if this is to be done then we are finding out dT/dt at x = 0 and dT/dt at x = 0.5. So, how much is the value that we have to find out.

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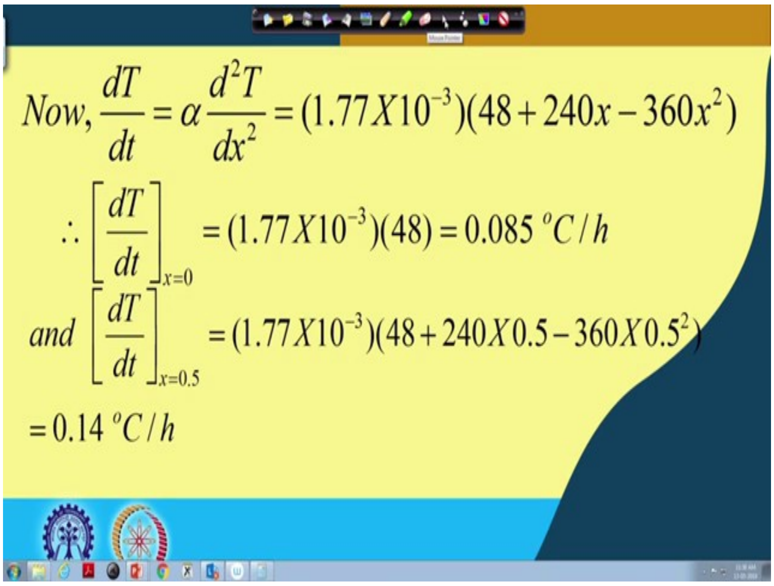

So, let us look into that and we see that dT dt is nothing but α d^2T/dx^2, 'right' dT/dt that is d temperature with α time is α d^2T/dx^2.

So, that you can write α 1.77×10^{-3}, already we have found out d^2T dx^2 = 48 + 240 x – 360 x^2 this is the general expression. So, we can write dT/ dt that is d with of temperature with time that is the differential of temperature with time at x = 0 is 1.77×10^{-3} and since x = 0 here you have done so, correctly and x = 0. So, it comes only 48.

So, 48 × 177 1.77×10^{-3} is 0.085 °C/h. So, that is dT/dt at x = 0. Similarly dT/dt at x = 0.5, we can write it is 1.77×10^{-3} times now instead of x you write 0.5; so, 48 + 240× 0.5 - 360 × 0.5^2. So, this on simplification comes to 0.14 °C/h, 'right'.

So, it was 0.085 °C/h and this is 0.14 °C/h.

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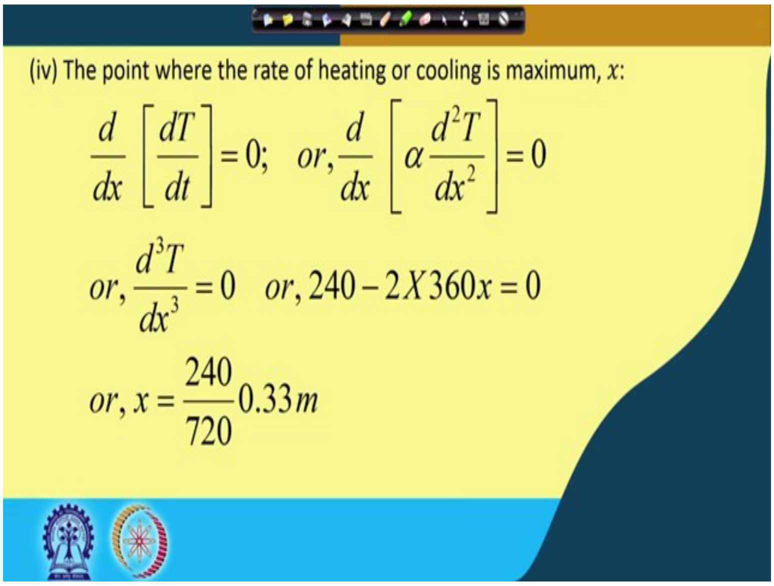

Now, we have still one more to do and that is the point where the rate of heating or cooling is maximum and that is what is the value of x you have to find out which means that ddx of dT/dt that should be 0, where it is either that it is maximum to become maximum the change of dT/dt that is the change of rate of change of temperature with respect to x that must be equal to 0, then only this can become maximum, 'right'.

So, we can write that ddx of α d^2T/dx^2 because dT/dt we have seen it is nothing but α d^2T/ dx^2. So, that is equal to 0, 'right'. So, this we can rewrite, 'right', we can rewrite that d^3T this is, 'right' this we can write d^3T/dx^3 because α goes out, 'right'.

d^3T dx^3 is 0, 'right' and this value we have already seen that was 240 - 2 × 360 x that was equal to 0, 'right'. So, we can write x = 240 / 720 that is 0.33 m. I hope you have understood that how we have done, 'right' because we are asked that where is the value where it will be maximum to become maximum, the change of temperature with respect to time if it is further differentiated with respect to x, then it becomes d dx / dT dt and when it becomes ddx of dT/dt, dT/dt value we know is α a d^2T dx^2, 'right' since we are again making it with respect to x.

So, it becomes d^2T/d^3T d^3T/dx^3 because α goes out because we are differentiating that is why and then d^3T/dx^3 is 0 and now we are substituting that value that is 240 - 2 × 360 x that from that earlier expression is 0 and that is how we got x = 240/ 720 or 0.33, 'right'. This is how this problem can be done and it is a problem which is not normally we come across that is why I like to put this problem before you. So, that a new type of thing which obviously, this is based on mathematics nothing more and mathematics which we have done, ‘right’.

Thank you.