Thermal Operations In Food Process Engineering: Theory And Applications

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Lecture - 47
Log Mean Temperature Difference

So, good morning. We were discussing about the Log Mean Temperature Difference. So, we were in the beginning of that derivation of how the log mean temperature difference for co-current, counter current are to be determined, how the expression is developed based on the heat transfer equations. So, that we are continuing here, 'right'. So, this is in lecture number 47, we are doing log mean temperature difference, 'right'.

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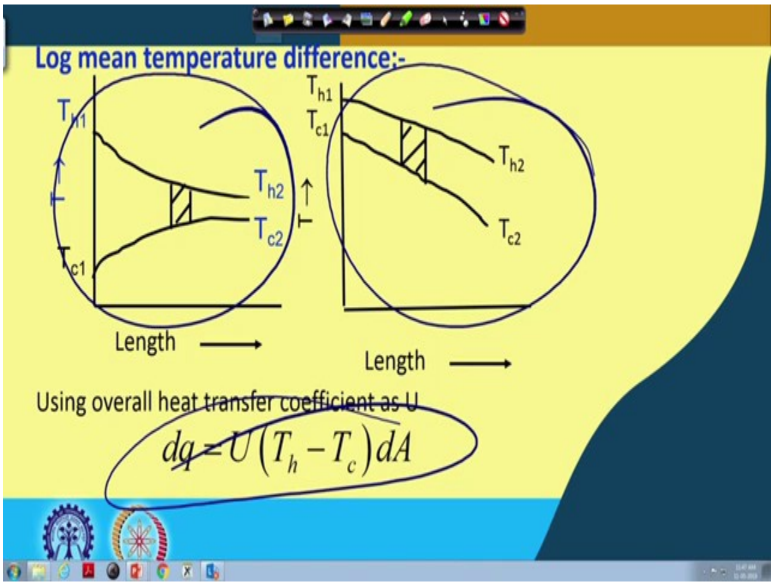

So, let us quickly look at where we were that is we started here that log mean temperature difference I am not going to emphasize again on this. This was for counter current and this was for co-current and this is the general equation; general equation for heat transfer, 'right'.

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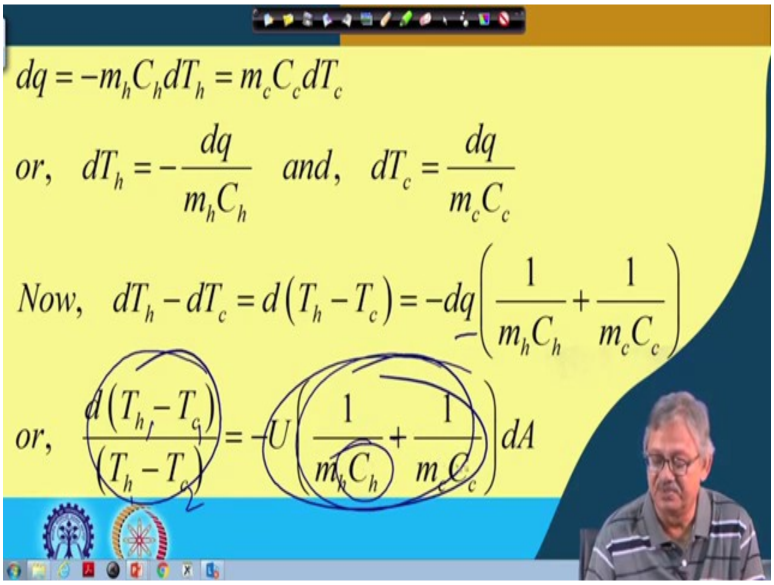

So, that we had taken to dq is m $-m_h$ C_h dT_h, from there we came to the point that dT_h of, we came to the point that dT_h of rather $d(T_h - T_c)$ / $dT_h - T_c$ we said that this cannot be; this cannot be taken of the reason being one is that is 1 and other is 2, that is why that either 1 and 2, depending on whether it is 1 and 2, this cannot be taken up. That was made equal to -U because that dq was in terms of U, $-U (1/m_hC_h + 1/m_cC_c)$ dA. This is where perhaps in the last class we had stopped, 'right'.

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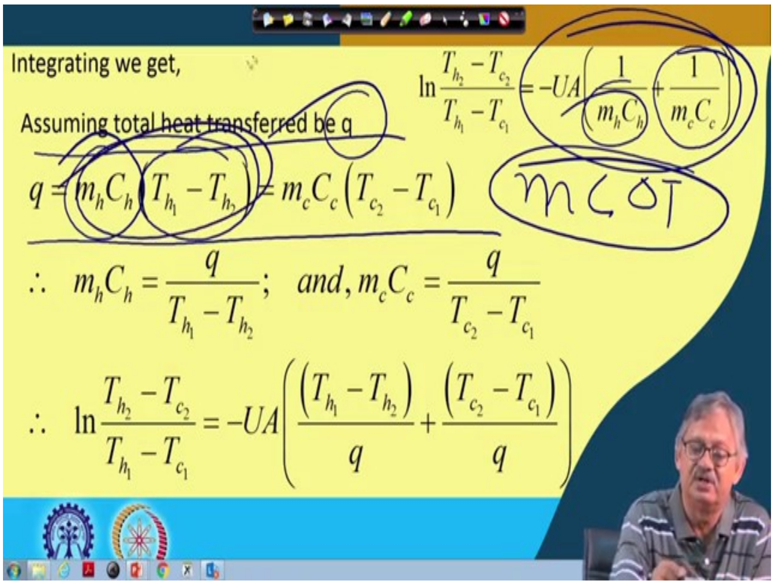

Now, if you look at this expression we can say that integrating this that $dT_h - dT_c - T_c$ over dT_h over $T_h - T_c$, 'right'. If we integrate and put the limits and limits where 1 and 2, that is inlet and outlet, 'right'. So, inlet and the outlet, if we do that, we can say that ln of that sorry ln of that that ln $((T_{h2} - T_{c2}) / (T_{h1} - T_{c1}))$ is that expression, 'right'.

So, this is equal to minus UA $(1/m_hC_h + 1/m_cC_c)$, 'right'. Now, assuming that total heat transferred be q, assuming total heat transferred be q, then $q = m_hC_h \times (T_{h1} - T_{h2})$ that is mass m_c ΔT, 'right'. From this, we can write this for the hot fluid and for the cold fluid we can write m_cC_c $(T_{c1} - T_{c2})$. Here also that at m_c Δc same thing we are writing, 'right'. So, in both the cases here you see you can say earlier we had written minus, but here it is there is no minus because we have taken care of the due minus T hot to 1.

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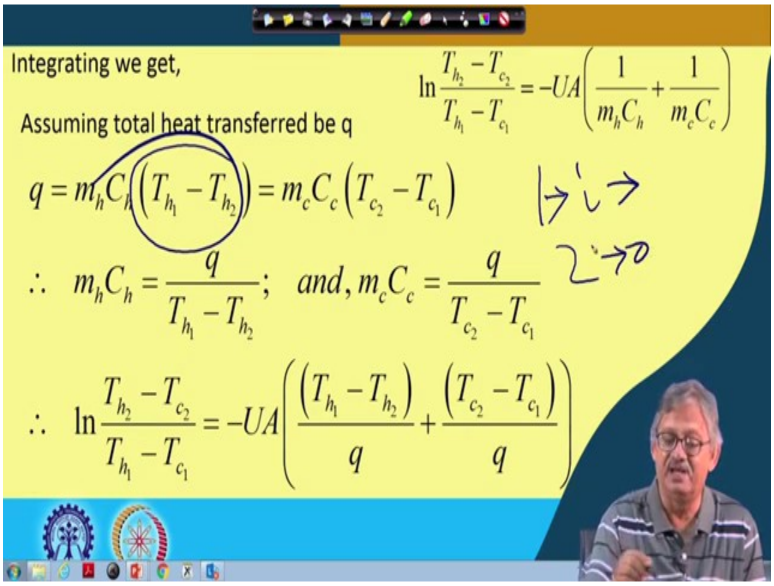

If 1 is inlet and 1 is inlet and 2 is outlet if that from is implemented here, then this $(T_{h1} - T_{h2}) = \Delta T$ and this is the + ΔT; that means, h_1 to h_2 it is decreasing. Whereas, this is $T_{c2} - T_{c1}$, cold fluid 1 which was entering is less than cold fluid 2 which is exiting is higher, 'right'. So, $T_{c2} - T_{c1}$ is also a positive one. That is why there is no negative sign. So, m_h C_h we can write this to equal to $q / (T_{h1} - T_{h2})$ and m_c C_c can be written as $q / (T_{c2} - T_{c1})$ from this relation, 'right'.

Therefore, if we take the log we can write this equation ln $(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})$ which was equal to minus UA that $1/m_cC_c$ and m_h Tm h C_h that we are now replacing, 'right',. So, that is equal to $(T_{h1} - T_{h2})/q$ + T_h $(T_{c2} - T_{c1})/q$, 'right'. So, this is that. So, we are replacing this with that $1/m_hC_h + 1/m_cC_c$ with this, 'right'.

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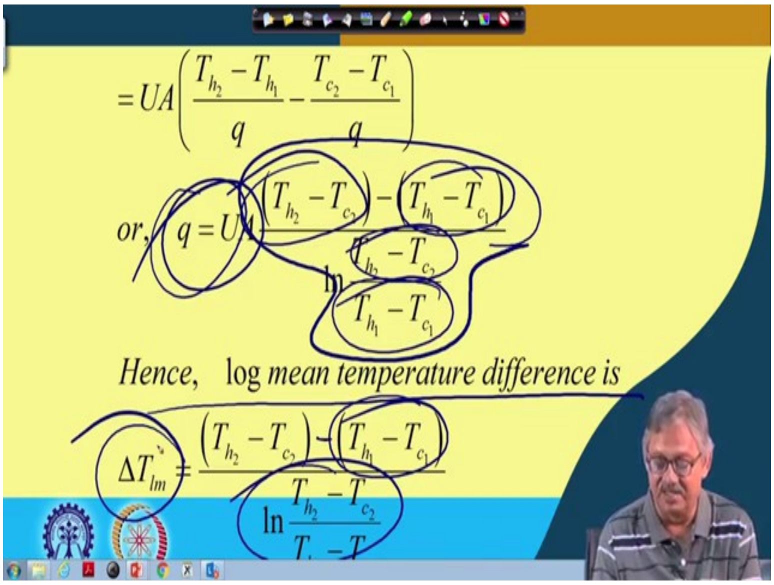

Then, we can further simplify it and already we have this side ln, so we can further simplify and write that is equal to UA sorry that is equal to UA $\{(T_{h2} - T_{c2})/q - (T_{c2} - T_{c1})/q\}$. The reason being you see that earlier in the earlier slide we had shown that this U was written negative, 'right' So, that negative we are taken care of by introducing it inside and this is now minus, 'right'. So, we can write $q = UA \{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})\}$, over all ln $\{(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})\}$, 'right'. So, q is equal to this we can write.

Hence, log mean temperature difference that can be written as ΔT_{lm} is nothing, but this part, 'right' and this part is $(T_{h2} - T_{c2}) – (T_{h1} - T_{c1})$ / ln $\{(T_{h2} - T_{c2}) / (T_{h1} - T_{c1})\}$ that is the log mean temperature difference, 'right'. Then, we can rewrite and say that this is the log mean temperature difference, where it is $T_{h2} - T_{c2}$ sorry, it is $T_{h2} - T_{c2}$; and $T_{h2} - T_{c2}$ ok.

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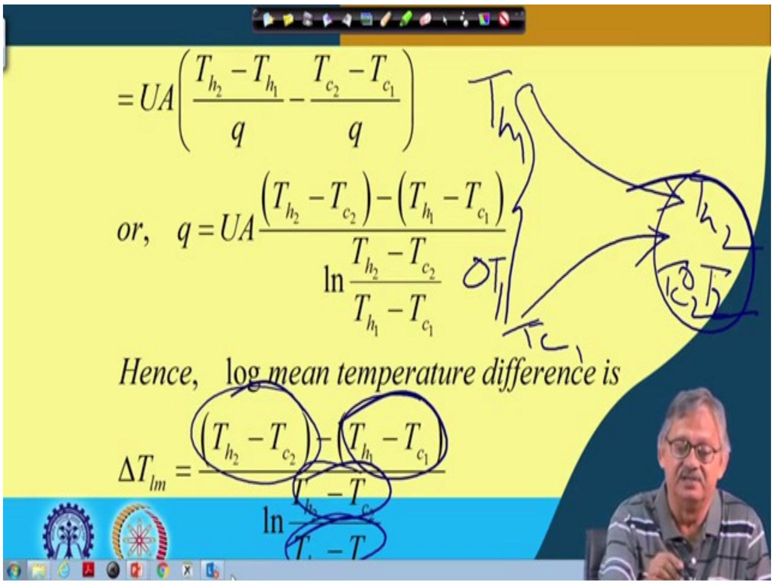

Let us look into really this is one, this is another, 'right', so T_{h1} and this is T_{h2} and this is T_{c2} and this is T_{c1}, 'right'. And this T_{h1} - T_{c1} is this one and T_{h2} - T_{c2} is this one, 'right'. So, this is what? Say we say ΔT_2 and say this is ΔT_1 ln (ΔT) it is 2 - T_{c2} / T_c, T_{h1} - T_{c1}.

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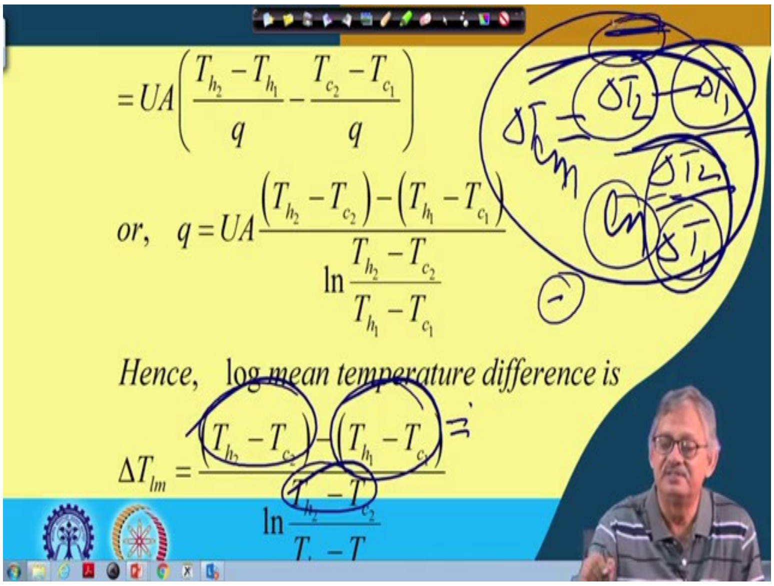

Now, this can be simplified and written as T, that ΔT_{lm} or let us write here ΔT_{lm} is equal to this we can write ΔT_2 and this we can write ΔT_1 over 1n of this we can write ΔT_2 and this we can write ΔT_1.

So, if properly they are written or the nomenclature is given, rightly then we can say that LMTD is (ΔT_2 - ΔT_1) / ln (ΔT_2/ΔT_1) or the vice versa that ΔT_{lm} can be written as (ΔT_1 -

ΔT_2) / (ΔT_1 - ΔT_2) sorry, over ln ($\Delta T_1/\Delta T_2$), 'right'. So, this can be properly written, 'right'.

You see, if ΔT_2 is low or less then ΔT_1, so there will be negative, but here that will also be taken care off. So, here also you have $\Delta T_2 < \Delta T_1$, the value which will come of ln which you will have all negative so this negative and that negative will go off, 'right'. So, this is how ΔT_{lm} we can write.

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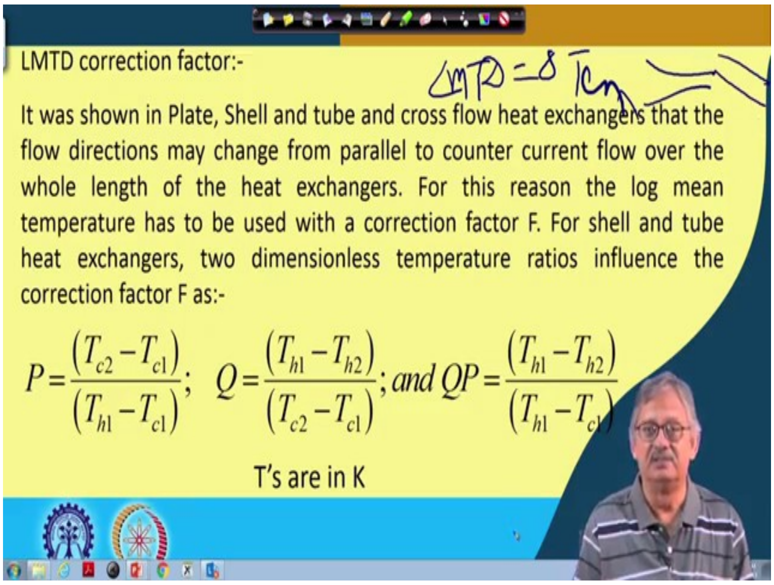

Now, again it comes another important thing is that LMTD correction factor. The earlier which we have shown that LMTD, that LMTD was ΔT_{lm} and whatever we had written that was true for counter current or co-current, 'right'. But if it is a cross flow like this, 'right', if it is a cross flow then it is not that is easy to find out for that there is a correction factor for LMTD which has to be introduced. And what is that? So, it was shown in plate shell and tube and cross flow heat exchangers that, the flow directions may change from parallel to counter flow over the whole length of the heat exchangers.

For this reason, the log mean temperature has to be used with the correction factor, for shell and tube heat exchangers two dimensionless temperatures ratios influence the correction factor F as P = (T_{c2} - T_{c1}) / (T_{h1} - T_{c1}) and Q = (T_{h1} -T_{h2}) / (T_{c2}/T_c - T_{c1}) and a product of QP is (T_{h1} -T_{h2}) / (T_{h1} - T_{c1}) where all these T's are in Kelvin, 'right' So, if you look at how it is proceeding that the correction factors F, how it is proceeding we can get it from solving problem, ‘right’.

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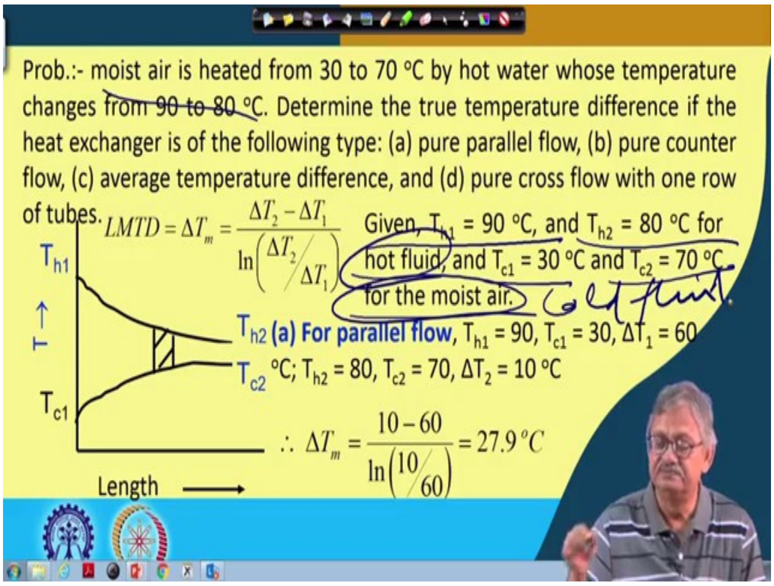

It is like this that, moist air is heated from 30 °C to 70 °C by hot water whose temperature changes from 90 to 80 °C. Determine the true temperature difference if the heat exchanger is of the following type. Number 1, pure parallel flow, number 2, pure counter flow, number 3, average temperature difference and number 4 pure cross flow with one row of tubes. So, there is no multiples, nothing, nothing single, ‘right’.

So, I repeat moist air is heated from 30 to 70 °C by hot water whose temperature changes from 90 to 80 °C. Determine the true temperature difference if the heat exchanger is of the following type, a pure parallel flow, or pure counter flow or average temperature difference and pure cross flow with one row, ‘right’. So, first let us try with the parallel flow, ‘right’.

So, if the parallel flow means as we have shown this is the axis, this is the length axis, this is the temperature, ‘right’. Then the flow direction is like this, ‘right’. And we had taken a volume element from where why extension we have done this equation, ‘right’. So, we got LMTD was equal to delta T m or normally mean or log mean it is written lm. So, it is $(\Delta T_2 - \Delta T_1) / \ln (\Delta T_2 / \Delta T_1)$. And in this given problem, we have been given T_{h1} is

90 °C, T_{h2} is 80 °C for hot fluid and T_{c1} is 30 °C and T_{c2} is 70 °C for moist air or we can write cold fluid, 'right'.

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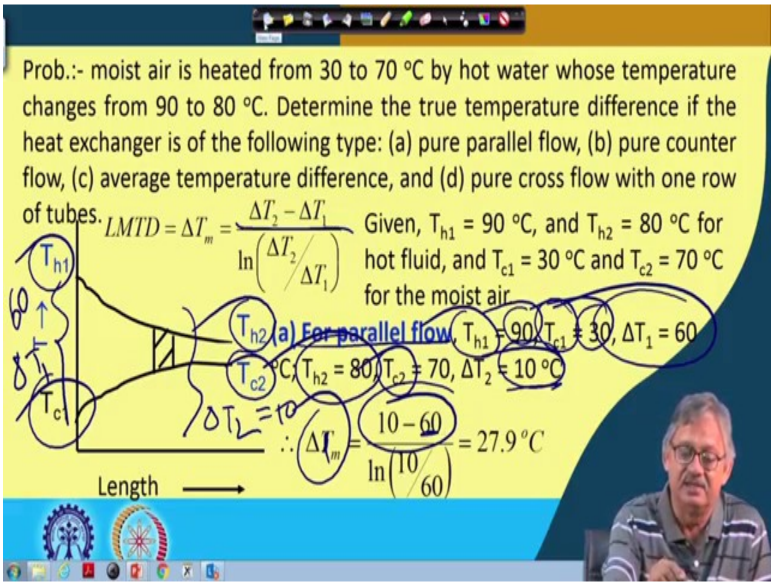

So, if this is true, then we can write for parallel flow T_{h1} = 90°, T_{c1} = 30°, so ΔT_1 = 60°. So, T_{h1} = 90°, T_{c1} = 30°, so this is the ΔT_1, 'right'. And T_{h2} is 80°, so this one is 80° and T_{c2} = 70°, so this one is 72°. So, their difference is ΔT_2 = 10 °C. So, that ΔT_1 came to be 60 and this ΔT_2 came to be 10°.

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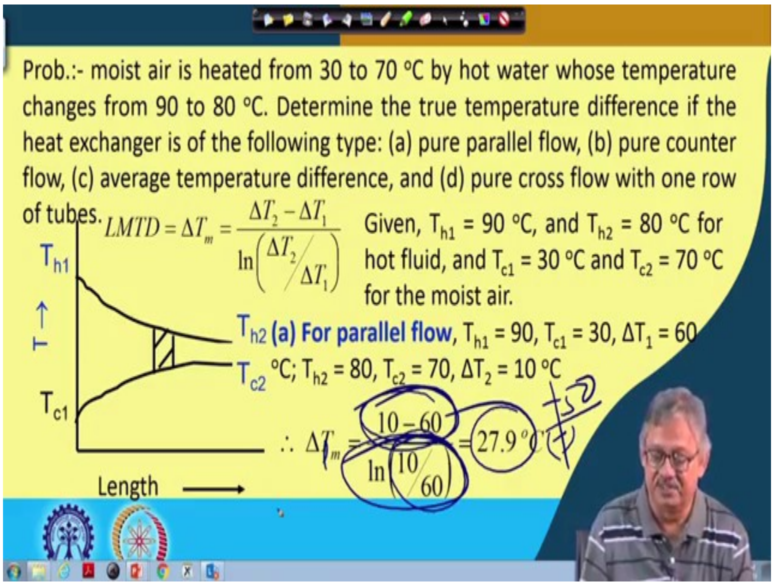

Then ΔT_{mean} or log mean we can write is 10 that T_2 $\Delta T_2 - \Delta T_1$. So, 10 - 60, 'right'. Let us erase. (10 – 60) / ln (10 / 60), 'right'. So, (10 – 60) is -50 and if you take 10 / 60 that is 1 / 6 ln (1/6) that will have some value with negative.

So, this two negative will go off and ΔT_{lm} as come up to be 27.9 °C. This you can also check with your calculator, 'right'. Since, we have cannot do it here with the calculator were perhaps we can also do, but yes let me check whether we have, yes, we have a calculator here.

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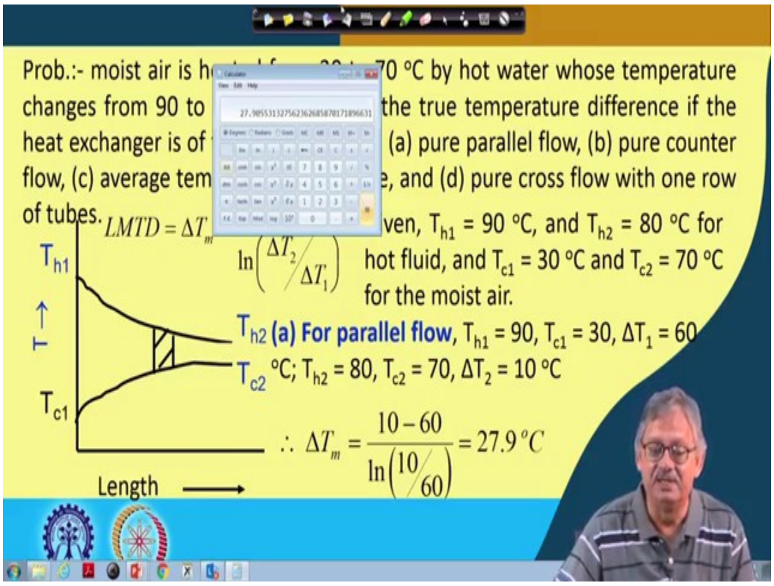

So, if we have a calculator here. So, we can write view scientific, yes. So, if that be true then if we see this, 'right', we can say that this was 10 minus this will go. So, 10 - 60 that was minus 50 over again 10 / 60. So, you see this has become 0.2166. And if we take its ln then it is -1.0. So, earlier we had -50 on the numerator, so dividing this becomes 27.9°, 'right'. So that means, we have, 'right'ly are correctly done which you also have seen it we have checked it, 'right'. So, 27.9 °C is the ΔT log mean for parallel flow.

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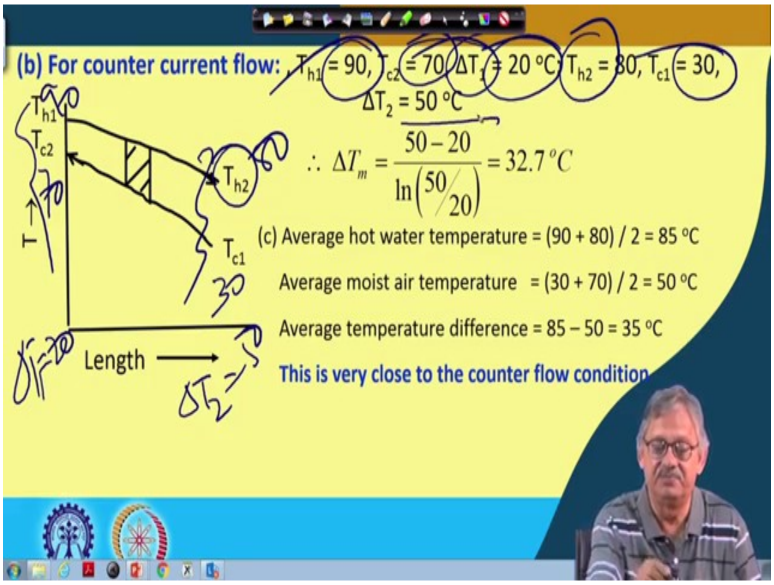

For counter flow we can write here. So, this is the flow direction, this is the flow direction, 'right' and this is T_{h1}, this is T_{h2}, this is T_{c1}, this is T_{c2}, a T vs L plot we have, 'right'. So, in that since T_{h1} is 90, T_{c2} is 70. So, T_{h1} is 90, T_{c2} is 70 and ΔT_1 we can write to be 90 - 70. So, this was 90 and outlet of c told was 70, so ΔT if we say one that ΔT_1 is 20, 'right'. Similarly, T_{h2} is 80, that is T_{h2} is 80 and T_{c1} is 30.

So, T_{c1} is 30, so ΔT_2 that is this one is 80 - 30 that is 50, 'right' that is 50. So, 50 and 20 is correct. Now, we can write ΔT log mean ΔT_{lm} = (50 – 20) / ln (50/20). That is, if we again see that with the help of a calculator, if we again see then we can see that 50 - 20 is equal to 30, 'right' divided by, so we have 50/20, 'right'. This will be taken ln, so that is this one. So, if we divide, so that is 32.7, 'right'.

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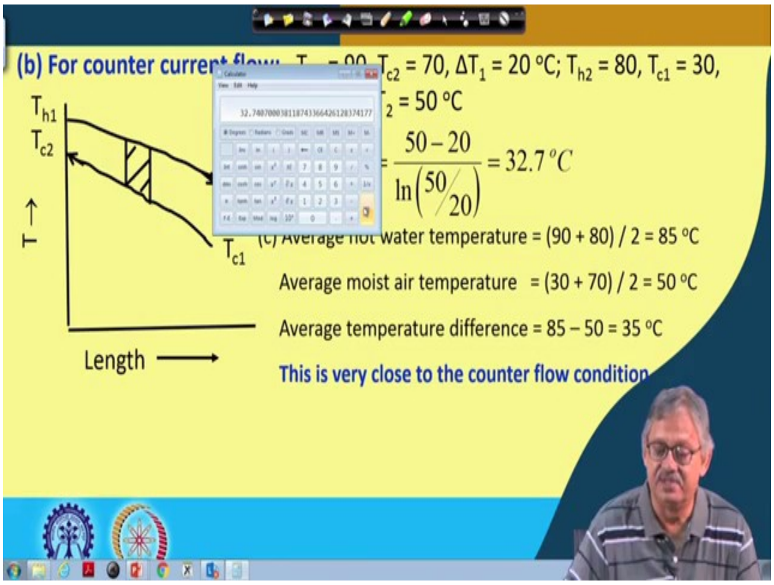

So, we got 32.7 as the ΔT log mean for counter current, 'right'. So, for counter count we got 32.7 °C as the log mean temperature difference, 'right'. Now, third one was available temperature. So, average temperature of the hot fluid is what? So, this was T_{h1} and this was T_{h2}. So, T_{h1} was 90 and T_{h2} was 80. So, average of that is 85 °C. So, average temperature of the inlet or fluid is 85 °C.

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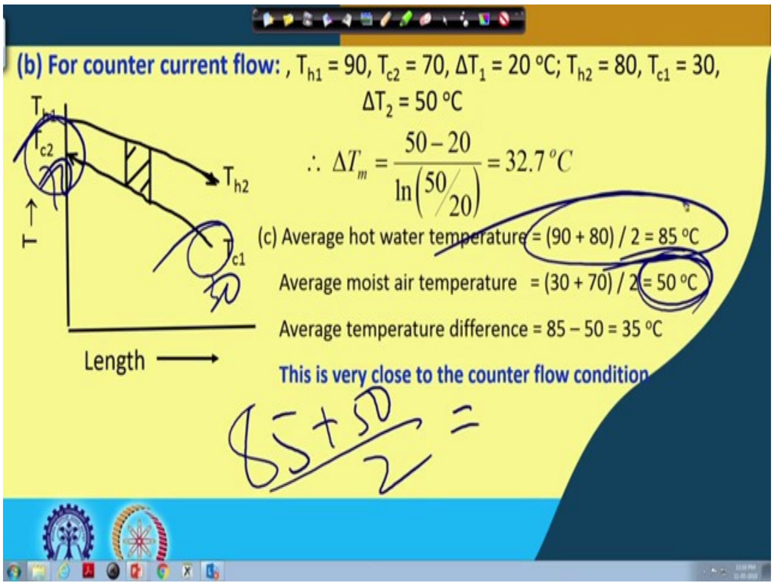

Similarly, average of the cold fluid is this was exit 30° inlet was sorry inlet was 30, exit was 70. So, 70 + 30, 100 / 2 is 50 °C, 'right'. So, we have inlet 85 exit 50. So, (85 - +50)/ 2 that is or we can say simply already we have done that average.

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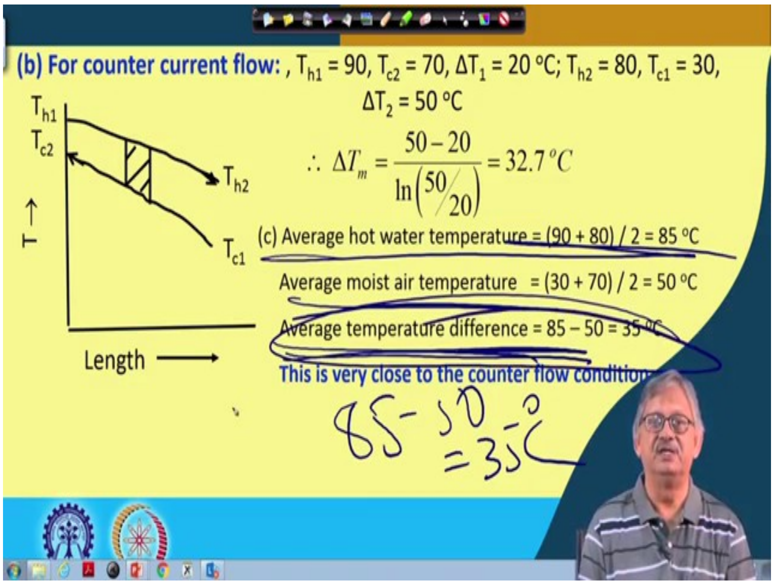

So, this can be the average temperature is average delta T, average temperature difference is ΔT 85 - 50 that is 35 °C, 'right'. 35 °C is the average temperature difference because we have taken the average of the in hot fluid, we also have taken the average of cold fluid. So, the difference is the average of the hot and cold fluid, 'right'. So, that is what average temperature we have seen.

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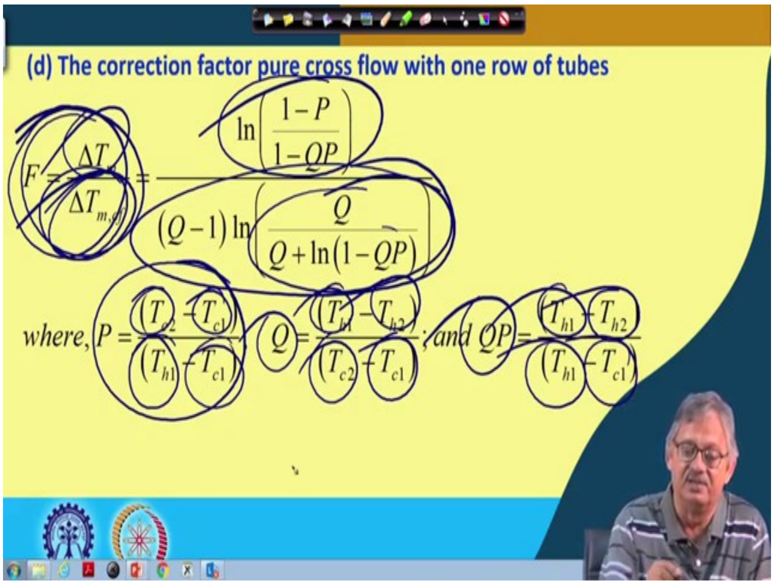

Then it is remaining that is 4th. Fourth one was that what is the temperature for LMTD. For the cross flow cross now for flow we have F = ΔT_{lm} or mean over ΔT_m cross flow or mean temperature log mean temperature for cross flow that is the F.

So, if that be true we can write ln ((1 – P)/(1-QP)) for this and ΔT_m cf is (Q – 1) ln (Q / (Q + ln (1 – QP))), 'right', where P = (T_{c2} - T_{c1}) / (T_{h1} - T_{c1}) and Q = (T_{h1} – T_{h2}) / (T_{c2} - T_{c1}), 'right' and QP is (T_{h1} – T_{h2}) / (T_{h1} - T_{c1}).

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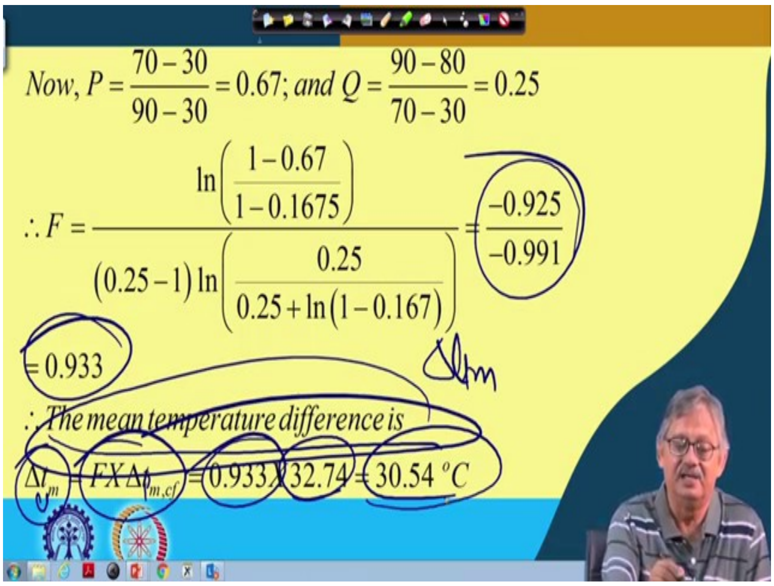

So, with all this parameters let us substitute the values and we get that P = (70 – 30)/(90 – 30) that is 0.67. Q = (90 – 80) / (70 – 30) that is 0.25. And then F = ln ((1- 0.67)/(1 - 0.1675)) / (0.25 - 0.1) × ln (0.25 / (0.25 + ln(1-0.167))) which comes on simplification as minus on the numerator -0.925 and on the denominator -0.991. So, this ratio comes to be 0.933, 'right'.

So, the mean temperature difference that can be written as ΔT_{lm}, 'right' equal to F× $\Delta T_{m,cf}$ = ΔT_{lm} cross flow. So, ΔT_{lm} cross flow that is then it is 32.7 4, 'right', 32.7 4 × 0.933 that is 30.54 °C, 'right'. So, ΔT_m has become 30.54, 'right'.

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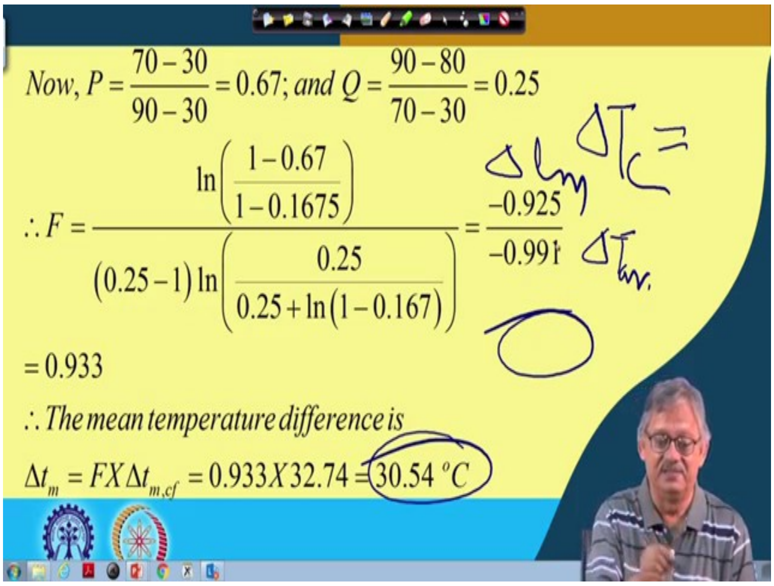

So, out of this 4 we see that ΔT counter current, 'right', that gave a value which is very very good, but that was very close to the average, ΔT average. Since, the temperature differences were small, so ΔT_{lm} values were not that significantly different. But if the ΔT_m is or rather if the difference of temperatures are very high then definitely the on the ΔT_{lm} or LMTD that will be highly appreciated.

So, that is why when the difference is high ΔT_{lm} is taken and if the difference is less, then the average temperature $(T_1 + T_2) / 2$ or ΔT of hot - ΔT of cold is taken as the average temperature, 'right'.

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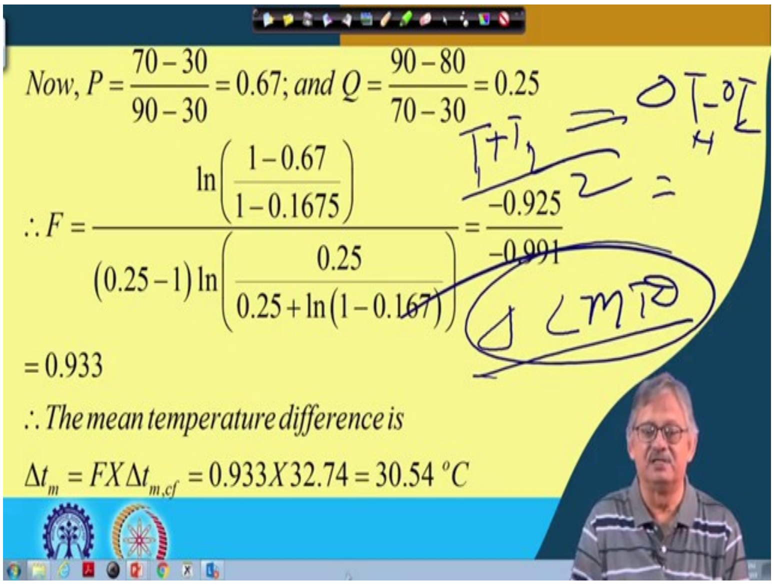

So, with this for the log mean temperature difference finding out, since the time is over. So, let us call it a day that LMTD for counter current, co-current and for the cross flow we have found out with a numerical value, ‘right’. And we have also shown with the calculator on the screen that the things which we have collected corrected or which we have done were correct, ‘right’. So, with this let us end this class. Stop this class here.

Thank you.