

Thermal Operations in Food Process Engineering: Theory and Applications
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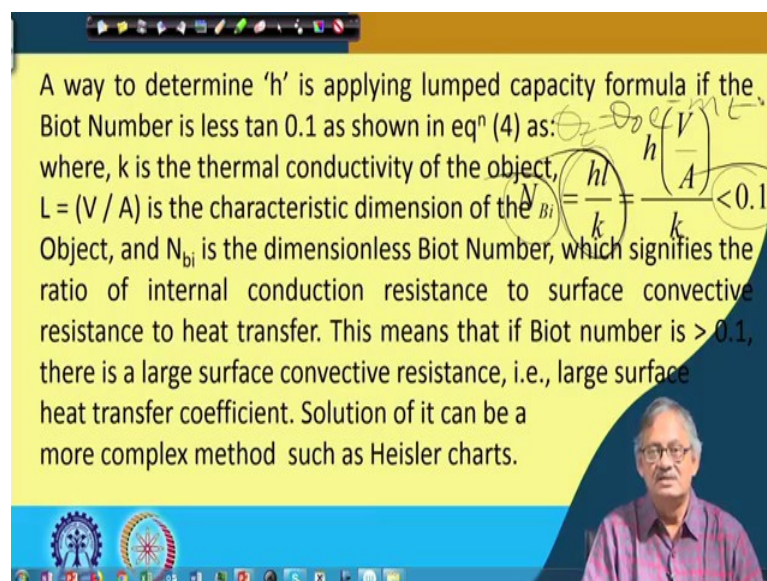
Lecture - 28
Heat Transfer by Convection (Contd.)

Good afternoon, my dear students. So, we were discussing about the Convective Heat Transfer, 'right'. So, when we are discussing about convective heat transfer we have seen in convection there are two types: one is that where it is by nature or by the buoyancy force and there it is due to the density difference of the medium of the fluid rather and fluid we have said that it is either liquid or gas, 'right'.

So, that density difference causes the natural convection or it can be by the application of some forced air or forcing that medium to move and that is called forced convection by explicitly by application of fan, 'right' and we were to the point where the convective heat transfer is meaningful and that we have to determine, 'right'.

So, now we are in the lecture number 28 that is Heat Transfer by Convection and this is continued, 'right'. These we have already done.

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A way to determine 'h' is applying lumped capacity formula if the Biot Number is less than 0.1 as shown in eqⁿ (4) as: $N_{Bi} = \frac{hL}{k} < 0.1$ where, k is the thermal conductivity of the object, L = (V / A) is the characteristic dimension of the Object, and N_{bi} is the dimensionless Biot Number, which signifies the ratio of internal conduction resistance to surface convective resistance to heat transfer. This means that if Biot number is > 0.1, there is a large surface convective resistance, i.e., large surface heat transfer coefficient. Solution of it can be a more complex method such as Heisler charts.

So, now we are coming to this point that a way to determine the heat transfer coefficient h, 'right', a way to determine the heat transfer coefficient h is by applying the lumped

capacity formula, 'right'. If the Biot number that is hl/k is less than 0.1, 'right' which we have already shown earlier also that if the Biot number; this is Bi or in cases it may be also written as N_{Bi} like many that dimensionless numbers are written as n of that number, 'right' Reynolds number you have come across by this time that is N_{RE} , 'right' similarly in Biot or N_{Bi} can be many numbers will come across afterwards, 'right'.

So, this is applicable when Biot number is less than 0.1 and the conductivity k of the material is or of the object is very high, 'right' and this L that is characteristic length is obtained as V/A or volume of the object over the surface area available of the object for heat transfer is the and this was this is what we have said earlier also as that characteristic dimension of the object, 'right', and N_{Bi} we have said to be the Biot number which signifies the ratio of internal conductance or in the internal conduction resistance to surface convective resistance, 'right'.

This means that if Biot number is greater than 0.1, 'right', there is large surface convective resistance that is large surface heat transfer coefficient. Solution of it can be more complex method such as Heisler charts and analytical solutions.

So, by applying so, by applying that lumped system analysis that N_{Bi} is equal to hl/k if we know the Biot number we can find if it is less than 0.1 that solution if you remember θ_t is equal to $\theta_0 e^{-mt}$ that solution we can utilize and from there we can find out the heat transfer coefficient, 'right'. This is one and another could be from the Heisler chart which also we have done already, 'right'. So, heat transfer coefficient h can be found out.

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If the object is cylindrical, the dimensionless parameters used to determine unknown variables are as follows:

$$\theta = \frac{T(x,t) - t_{\infty}}{T_i - T_{\infty}}, \quad \frac{1}{Bi} = m = \frac{k}{hl}, \quad \text{and } n = \frac{x}{l}$$

$$Fo = \frac{\alpha t}{l^2}$$

Fo is plotted in the X-axis, and θ in the Y-axis., 'm' is used for determining the slope, and 'n' l used to determine increment of the slope. Variables introduced in these dimensionless numbers are α , the thermal diffusivity of the object, x is the one dimensional position in the object.

Now, if the object is cylindrical the dimensionless parameters used to determine unknown variables are as follows that Fo we have already said is Fourier number

dimensionless temperatures θ that is $\theta = \frac{T(x,t) - t_{\infty}}{T_i - T_{\infty}}$, Bi we have already said

$$Fo = \frac{\alpha t}{l^2}$$

inverse of Biot number is there and that that in some cases is written instead of 1/Bi in some cases it is written as m in some of the books some of the places in place of 1/Bi it is written as m and which is k/hl .

And, in another case which we had said earlier with capital X if you remember, so, that can be represented as n again that is x/l where x at any distance position and l is the characteristic length, 'right'.

So, these things are already said, but there may be though it is the recapitulation it is like that how at different books and different sources things are also presented. See, in some case if you find that instead of Biot number this is m and n that time you may feel cold that this was not told. So, this is like that ok.

So, Fourier number is plotted in the X-axis and theta in the Y-axis, 'right'; m is used for determining the slope, and n is used to determine the increment of the slope, 'right'. Variables introduced in the dimensionless numbers are alpha that is thermal diffusivity

of the object, X is the one dimensional position in the object and this way we can say that this is close to the plotting Heisler chart which we have done like this that you have in

the X-axis Fourier number there is $\frac{\alpha t}{l^2}$ and in the Y-axis you have theta that is $(T_t - T_e) /$

$(T_0 - T_e)$, 'right' and we had a plot of this m and n, 'right' these are for Biot number or 1 by Biot number equal to m and these are for different x or the for x by l, 'right'.

So, depending on whether it was for Fourier number or center temperature or inside temperature that will dictate how you are plotting, 'right'.

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Prob 1:- A 10 cm thick slab initially at 160 °C is immersed in liquid at 60 °C resulting in heat transfer coefficient of 1200 W/m² °C. Determine the temperature at the centerline and at the surface 100 sec after immersion. Properties for the slab are; $\alpha = 1.0 \times 10^{-5}$ m²/s, $k = 210$ W/m °C, $C_p = 0.9$ kJ/kg °C, $\rho = 1600$ kg/m³.

Prob 2:- A long steel cylinder having diameter of 10 cm and initially at 30 °C is placed into an oven at 200 °C having a local heat transfer coefficient of 150 W/m² °C. Calculate the time required for the axis temperature to reach 100 °C. Also, calculate the corresponding temperature at radius of 5 cm. Given, $\alpha = 5.0 \times 10^{-6}$ m²/s, $k = 30$ W/m °C.

So, if you solve a problem like 10 centimetre thick slab initially at 160°C is data is immersed in liquid at 60°C resulting in heat transfer coefficient of 1200 W/m² °C. Determine the temperature at this center line and at this surface 100 second after immersion. Properties for this slab are alpha is equal to point α is equal to 1.0×10^{-5} m²/s; k is 210 W/m °C; C_p is 0.9 kJ/kg °C; ρ is 1600 kg/m³, 'right'.

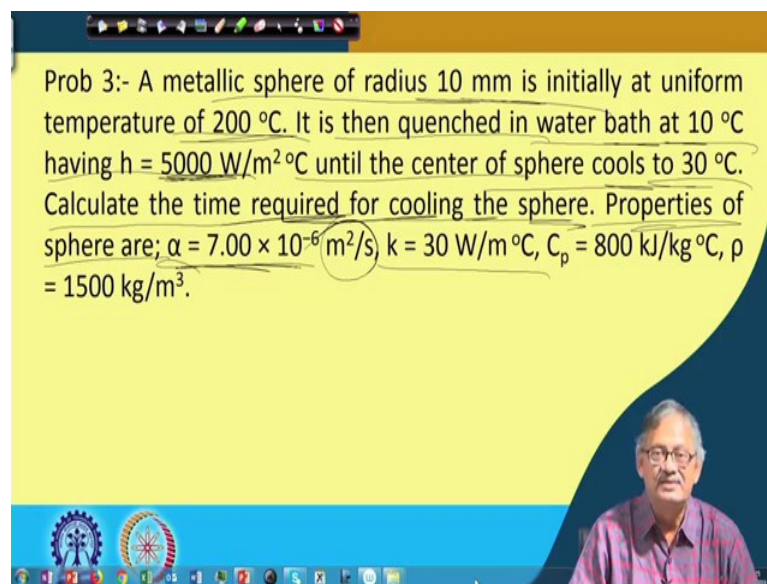
I repeat the problem a 10 centimeter thick slab initially at 160°C is immersed in liquid at 60°C resulting in heat transfer coefficient of 1200 W/m² °C 'right' determine the temperature at the center line and at the surface 100 second after immersion. Properties for slab are alpha is equal to 1.0×10^{-5} m²/s; k 210 210 W/m °C; C_p is 0.9 kJ/kg °C ρ is 1600 kg/m³ this is one problem.

Second problem you try a long steel cylinder having diameter of 10 centimeter and initially at 30°C is placed into an oven at 200°C having local heat transfer coefficient of 150 W/m² °C 'right'. So, it is 150 W/m² °C. Calculate the time required for the temperature to reach 100°C. Also calculate the corresponding temperature at radius 5 centimeter. Given $\alpha = 5.0 \times 10^{-6}$ m²/s; k 30 W/m °C, 'right'.

You repeat that a long steel cylinder having diameter of 10 centimeter and initially at 30°C is placed into an oven at 200°C having a local heat transfer coefficient of 150 W/m² °C. Calculate the time required for the temperature to reach 100°C.

Also calculate the corresponding temperature at radius 5 centimeter. Given, alpha thermal diffusivity thermal diffusivity to be $\alpha = 5.0 \times 10^{-6}$ m²/s and thermal conductivity equal to 30 W/m °C, 'right'.

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Prob 3:- A metallic sphere of radius 10 mm is initially at uniform temperature of 200 °C. It is then quenched in water bath at 10 °C having $h = 5000$ W/m²°C until the center of sphere cools to 30 °C. Calculate the time required for cooling the sphere. Properties of sphere are; $\alpha = 7.00 \times 10^{-6}$ m²/s, $k = 30$ W/m °C, $C_p = 800$ kJ/kg °C, $\rho = 1500$ kg/m³.

So, these are the two problems and let us and I say that you try a third problem is also given that a metallic sphere of 10 mm is initially at uniform temperature a metallic sphere; that means, we are giving that three types of problems one is slab, another is cylinder and the third is sphere, 'right'.

So, a metallic sphere of radius 10mm is initially at uniform temperature of 200°C. It is then quenched in water bath at 10°C having h of 5000 W/m²°C until the center of sphere cools to 30°C.

Calculate the time required for cooling this sphere properties of sphere are given $\alpha = 7.00 \times 10^{-6} \text{ m}^2/\text{s}$; k conductivity with $30 \text{ W/m } ^\circ\text{C}$; specific heat $C_p = 800 \text{ kJ/kg } ^\circ\text{C}$ and density of this sphere material is $\rho = 1500 \text{ kg/m}^3$, 'right'.

You repeat, a metallic sphere of radius 10 mm is initially at uniform temperature 200°C . It is then quenched to quenched means suddenly it is it is cooled, 'right', quenched in water bath at 10°C having h or heat transfer coefficient of $5000 \text{ W/m}^2^\circ\text{C}$ until the center temperature of the sphere equal to 30°C .

Calculate the time required for cooling this sphere properties of the sphere are $\alpha = 7.00 \times 10^{-6} \text{ m}^2/\text{s}$; k conductivity to $30 \text{ W/m } ^\circ\text{C}$; C_p specific heat to be $800 \text{ kJ/kg } ^\circ\text{C}$ and density is 1500 kg/m^3 , 'right'.

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Solution 1:

Given data:
 Slab thickness = 0.1 m ; Initial temperature ' T_i ' = 160°C ; Liquid temperature ' T_∞ ' = 60°C ; Heat transfer coefficient ' h ' = $1200 \text{ W/m}^2^\circ\text{C}$;
 Time of immersion ' t ' = 100 s ; Thermal diffusivity ' α ' = $1.46 \times 10^{-4} \text{ m}^2/\text{s}$;
 Thermal conductivity ' k ' = $210 \text{ W/m } ^\circ\text{C}$; Specific heat ' C_p ' = $0.9 \text{ kJ/kg } ^\circ\text{C}$;
 Density ' ρ ' = 1600 kg/m^3

To find:
 Centerline temperature ' T_0 ' = ?
 Surface temperature ' T_L ' = ?

So, this three problem I hope we can solve it here like this: that Slab thickness = 0.1 m ; Initial temperature ' T_i ' = 160°C ; Liquid temperature ' T_∞ ' = 60°C ; Heat transfer coefficient ' h ' = $1200 \text{ W/m}^2^\circ\text{C}$; Time of immersion ' t ' = 100 s ; Thermal diffusivity ' α ' = $1.46 \times 10^{-4} \text{ m}^2/\text{s}$; Thermal conductivity ' k ' = $210 \text{ W/m } ^\circ\text{C}$; Specific heat ' C_p ' = $0.9 \text{ kJ/kg } ^\circ\text{C}$; Density ' ρ ' = 1600 kg/m^3

So, what we have to do? We have to find out the centre temperature T_0 and the surface temperature T_L , 'right'. These two we have to find out, 'right'.

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Solution 1:

First, we have to find the Biot number, $Bi = \frac{hl}{k} = 0.286$ which implies that lumped system analysis not valid. $1/Bi = 3.5$.

Finding the Fourier number from Heisler chart for $Bi = 0.0286$, $1/Bi = 3.5$, and $Fo = 5.84$, the value of $\theta_0 = 0.27$.

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \quad \therefore T_0 = 0.27 \times (160 - 60) + 60 = 87^\circ\text{C}$$

$$Fo = \frac{\alpha t}{L^2} = \frac{1.46 \times 10^{-04} \times 100}{\left(\frac{0.1}{2}\right)^2} = 5.84$$

Again, from Heisler's chart for a plane wall, for the given Bi and $x/L = 1$, $\theta_0 = 0.83$.

$$\theta_0 = \frac{T_L - T_\infty}{T_i - T_\infty} = 0.83 \quad \therefore T_L = 0.83 \times (160 - 60) + 60 = 143^\circ\text{C}$$

To do that let us solve it that first we find out the Biot number, 'right'. Biot number is, hl/k from the given properties and given values we see that hl/k is 0.286. Why? Let us go back that how the properties were given that slab thickness is 0.1 meter, 'right' sorry slab thickness is 0.1 meter, 'right' h has been given 1200. So, $hl/1200$ this is 0.1. So, that means, 0.1 by 2 is the characteristic length.

So, 1200 into 1/2 is the hl and k conductivity here was is given 210 $\text{W/m}^\circ\text{C}$. So, 2×210 . If you solve it will come to that point to something like that, 'right'. So, 210 one 0 goes out. So, it is 120 by 42 hl/k , 'right'. Oh this 1 is half 0.01 slab thickness of 0.1 this is centimetre, not meter, 'right' that C has gone out; so, 0.1 centimeter. So, that is this into 10 to the power minus 2, 'right'.

So, that is how it will come that 120×10^{-2} ; that means, it will become 1.2 and this 2 into 21 is 42. So, 1.2 by 42, this will come to somewhere the value which we have said, 'right'. So, that means, it is coming somewhere closer to 0.2, 'right' or 0.3 rather.

So, that if we see that how much we have obtained we have obtained 0.286, 'right' that this implies that when it is 0.286 that is the Biot number being high. So, lumped system analysis cannot be done, 'right'. So, that the lumped system analysis is not valid and $1/Bi = 3.5$ inverse of 0.286 is 3.5.

And, finding the Fourier number from the Heisler chart we if we find out the Fourier number then from Bi is equal to 0.286 and $1/Bi$ is 3.5. If we see that and Fourier number

if that is $\frac{\alpha t}{l^2}$ alpha is given, t is also given, l is also known. So, we know the Fourier

number and Fourier number comes to equal to 5.84, 'right' and the value of theta is from the plot of Fourier number versus theta for $1/Bi$ equal to 3.5 then from that graph we can find out that what is the value of theta and this value of theta came to be 0.27, 'right'

this came up to be 0.27, 'right'. Here we have shown Fourier number is $\frac{\alpha t}{l^2}$ that is

. So, only the thing which is asking me that here we have

$$F_o = \frac{\alpha t}{l^2} = \frac{1.46 \times 10^{-04} \times 100}{\left(\frac{0.1}{2}\right)^2} = 5.84$$

taken 0.1 by 2 that is it is 0.1 meter, but here what we have done we said that was to be 0.1 centimeter, but it is not true it is 0.1.

So, Bi which we have done to be equal to hl/k ; h was 1200, 'right', l is 0.1 if it is and k is 210 'right'. So, h is 1200.1. So, $120/210$. Yes, it is 0.1 centimeter yes it is 0.1 centimeter and if it is 0.1 centimeter then only it is coming this. So, what we said earlier it was 0.1 meter, 'right'.

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Solution 1:

First, we have to find the Biot number, $Bi = \frac{hl}{k} = 0.286$ which implies that lumped system analysis not valid. $1/Bi = 3.5$.

Finding the Fourier number from Heisler chart for $Bi = 0.286$, $1/Bi = 3.5$, and $Fo = 5.84$, the value of $\theta_0 = 0.27$.

$$Fo = \frac{\alpha t}{L^2} = \frac{1.46 \times 10^{-04} \times 100}{\left(\frac{0.1}{2}\right)^2} = 5.84$$

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \quad \therefore T_0 = 0.27 \times (160 - 60) + 60 = 87^\circ C$$

Again, from Heisler's chart for a plane wall, for the given Bi and $x/L = 1$, $\theta_0 = 0.83$.

$$\theta_0 = \frac{T_L - T_\infty}{T_i - T_\infty} = 0.83 \quad \therefore T_L = 0.83 \times (160 - 60) + 60 = 143^\circ C$$

So, that is what that hl/k was 1200 into 0.1/2 into 210, 'right'. So, that 0.1 became this in went to up. So, to a $120/210 \times 2$. So, 420. So, 120 by 420 or this point goes out, so, 12 by 42 somewhere it is 0.286, 'right'. So, 0.1 meter true. So, Fourier number came to be 5.84 and if Fourier number is 5.24 if $1/Bi$ is 3.5 then from the Heisler chart we can find out that the value of theta which is 0.27, 'right'. If the value of theta is 0.27 then T_0 from here of course, this is that 0 means that center temperature, 'right'; T_0 is the center temperature.

So, center temperature we have to find out that is what we have been asked to do. So,

. So, we found out the center

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \quad \therefore T_0 = 0.27 \times (160 - 60) + 60 = 87^\circ C$$

temperature to be 87.

Now, we are asked that find out the surface temperature, 'right'. From Heisler chart again we can do for a plane wall for the given Bi and given X/L is equal to 1. Why X/L is equal to 1? Because that if it is a cylinder it was for cylinder I hope it was 0.1, 'right' if we hl/k ; l was 0.1 if we remember that. So, oh the thickness was 0.1

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Solution 1: $Bi = \frac{hl}{k} = 0.286$
 First, we have to find the Biot number, $Bi = \frac{hl}{k} = 0.286$ which implies that lumped system analysis not valid. $1/Bi = 3.5$.
 Finding the Fourier number from Heisler chart for $Bi = 0.286$, $1/Bi = 3.5$, and $Fo = 5.84$, the value of $\theta_0 = 0.27$. $Fo = \frac{\alpha t}{L^2} = \frac{1.46 \times 10^{-04} \times 100}{\left(\frac{0.1}{2}\right)^2} = 5.84$
 $\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \therefore T_0 = 0.27 \times (160 - 60) + 60 = 87^\circ C$
 Again, from Heisler's chart for a plane wall, for the given Bi and $x/L = 1$, $\theta_0 = 0.83$.
 $\theta_0 = \frac{T_L - T_\infty}{T_i - T_\infty} = 0.83 \therefore T_L = 0.83 \times (160 - 60) + 60 = 143^\circ C$

Then you can say that for X/L 'right' for X/L is equal to 1. When L a X is on the surface, 'right' when this is if this is the material if this is the center then when X is the surface that time we can say that this X is nothing, but equal to L . So, it is XL/L that is equal to 1, 'right'. That is why we have taken X/L is equal to 1 and from the Heisler chart we can find out the θ_0 is 0.83 corresponding to X/L is point X/L is equal to 1, 'right'. This is from the second chart where we had done it earlier, 'right'.

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Solution 1: $Bi = \frac{hl}{k} = 0.286$
 First, we have to find the Biot number, $Bi = \frac{hl}{k} = 0.286$ which implies that lumped system analysis not valid. $1/Bi = 3.5$.
 Finding the Fourier number from Heisler chart for $Bi = 0.286$, $1/Bi = 3.5$, and $Fo = 5.84$, the value of $\theta_0 = 0.27$. $Fo = \frac{\alpha t}{L^2} = \frac{1.46 \times 10^{-04} \times 100}{\left(\frac{0.1}{2}\right)^2} = 5.84$
 $\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.27 \therefore T_0 = 0.27 \times (160 - 60) + 60 = 87^\circ C$
 Again, from Heisler's chart for a plane wall, for the given Bi and $x/L = 1$, $\theta_0 = 0.83$.
 $\theta_0 = \frac{T_L - T_\infty}{T_i - T_\infty} = 0.83 \therefore T_L = 0.83 \times (160 - 60) + 60 = 143^\circ C$

So, we can say that

$$\theta_0 = \frac{T_L - T_\infty}{T_i - T_\infty} = 0.83 \quad \therefore T_L = 0.83X(160 - 60) + 60$$
$$= 143^\circ C$$

So, this way from Heisler chart we can also find out the temperatures at the center as well as at the surface, 'right'. So, today we have come to the end of this class. We have given you three problems out of which one we have already done here and others two you try. Maybe next class we will solve those two problems and you will also be able to check up, 'right' and I will also be able to counter check whether we have done it correctly or not, 'right', ok.

Thank you.