# Thermal Operations in Food Process Engineering: Theory and Applications

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## Lecture - 26
## Heisler Chart (Contd.)

Good morning. So, we were dealing with that Heisler chart and we have shown for infinite long slab; infinitely long slab what is the Heisler chart, how it looks like; how we can determine the temperature distribution at the centre as well as any location. Given we have to find out that we have to know that thing. So, it is lecture number 26.

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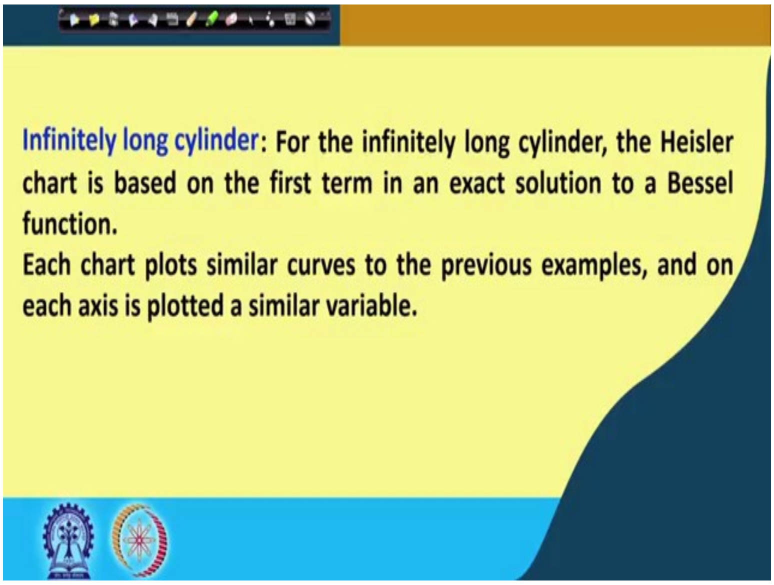


So, in lecture number 26 we now go to that Heisler chart this was done, ‘right’ for long cylinder. And now we are finding for infinitely long cylinder that was for long slab infinitely long slab, this is for infinitely long cylinder, ‘right’.

So, for infinitely long cylinder Heisler chart is based on the first term in an exact solution to a Bessel function. Each chart plots similar curves to the previous examples and on each axis is plotted a similar variable like this, ‘right’.

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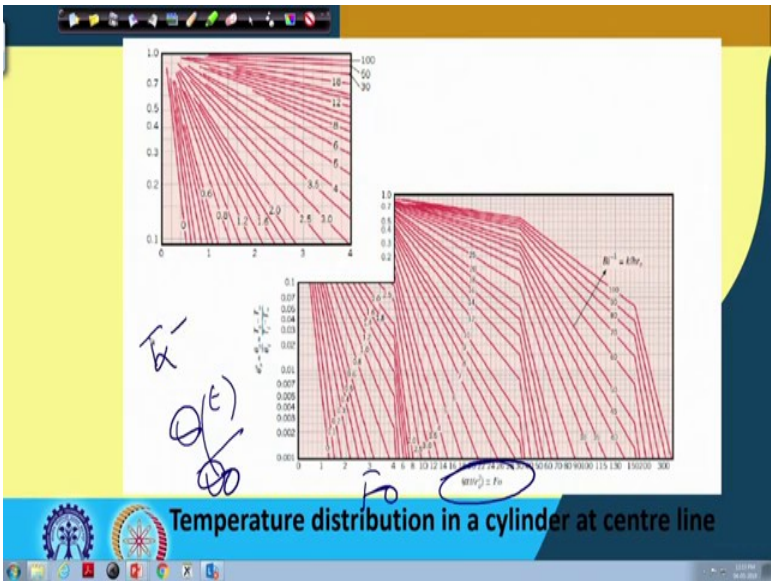


So, this is the Heisler chart for long cylinders, 'right' where $\alpha t$ $/L_{r0}^2$ square instead of L it is $r_o$, 'right', $\frac{\alpha t}{l^2}$ is the your x-axis. So, this is Fourier number and this is θ, i.e., $\theta_0$, $\theta_t/\theta_0$ is that which is T infinity minus rather T.

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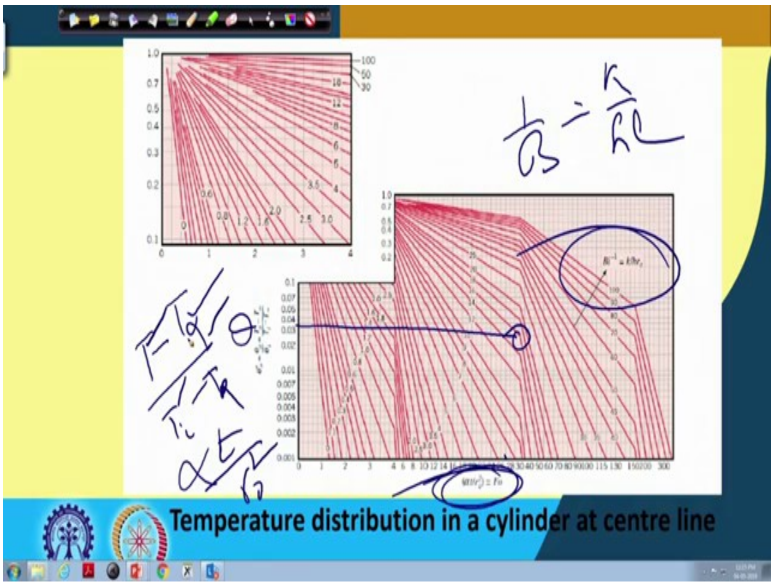

T at any place t at any place minus $T_\infty/T_i - T_\infty$, 'right'. So, this is for the long cylinder, 'right' and here also if we know Fourier number there is alpha T by instead of L it is L for slab it is $r_o$ square, 'right', that if we know r value of r then you know the value of Fourier number, then from that Fourier number here also you have inverse of Biot number there is 1/Bi that is hL/k. So, k/hL that value if you know, then you can point that where it is on the graph.

So, from there the value of theta you can find out and then you can find out this either this temperature or that temperature or this temperature accordingly if whatever is given, 'right'. So, this is for the temperature distribution at the centre.

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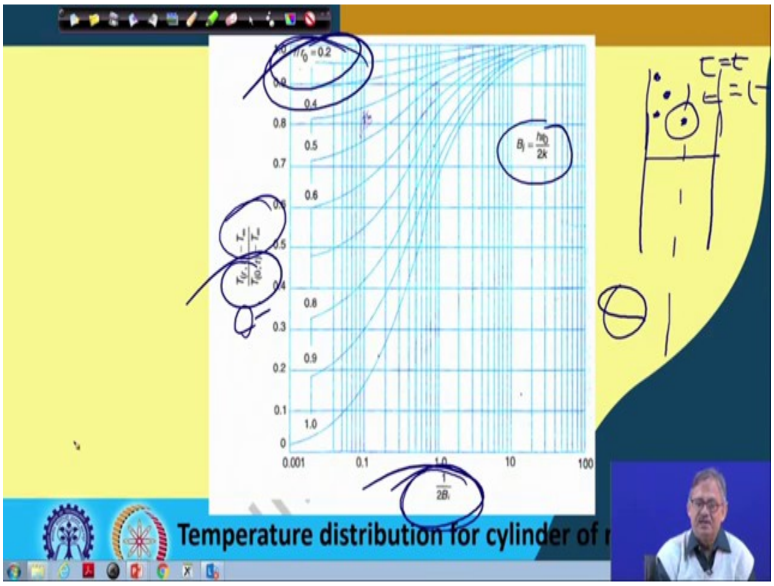


Then we get that temperature distribution and the special temperature distribution means in the cylinder body where we know Biot number is $h_{ro}/2k$, 'right' in earlier graph also it was $h_{ro}/2k$, 'right'. Here also it is $k/h_{ro}$, 'right' 1 by inverse of Biot number. So, here it is Biot number is $h_{ro}/2k$ and we can find out from the value of this is plotted 1/2Bi, 'right';that 1/2Bi is plotted against that θ, 'right' and knowing the value of Bi $r/r_o$ that is the position.

So, if this is the cylinder, if this is the axis then what is this if this is the initial temperature distribution uniform then after time t is equal to t, if it is heated then what is the temperature here that we have seen from the previous cylinder graph and what is the temperature after time t is equal to t at this location or at this location or at this location depending on what is the value of $r/r_o$, 'right'. Then you find out the value of θ and then you get either T(r t) or T(0 t) or $T_\infty$ minus T whatever value you want you can find out, 'right'. This is the second graph.

Third one as earlier we have seen is the heat which is being transferred, 'right'.

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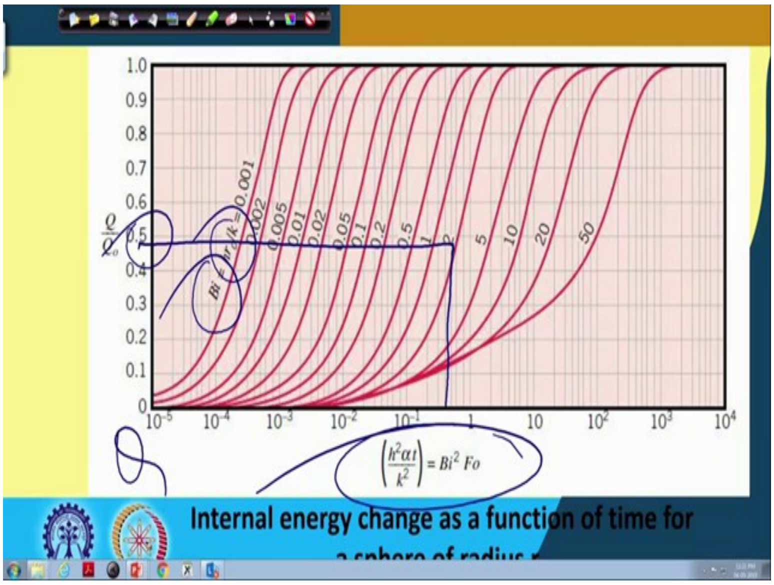


Heat which is being transferred this is again plotted Bi square was in times Fo versus Bi that is for different Bi $h_{ro}/k$ this is $Bi^2$ ,$Fo\times Bi^2$. So, that is how that for different Bi you get the value of $Q/Q_0$, 'right' and if $Q_0$ is known you can find out the value of Q, 'right'. So, this is another. So, quickly we are going through this and next one is this is for cylinder.

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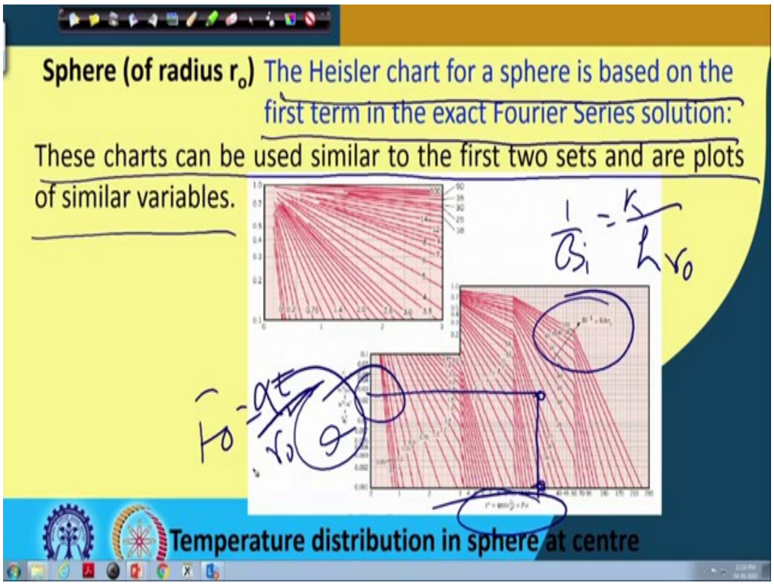


So, similarly for the sphere also we can get it, 'right'. For sphere of radius $r_o$, the Heisler chart for a sphere is based on the first term in the exact Fourier series solution, and these charts can be similar to the first two sets of the plots of similar variables. Like here it is Fo, here it is theta and here it is inverse of Bi that is h by or rather $k/h_{ro}$, 'right'. So, inverse of Bi is $k/h_{ro}$, 'right' and this is Fourier number that Fourier number is $\alpha t / r_o^2$, 'right' and this is for θ.

So, once we know that again if we know the value of Fourier number, if you know value of 1 by Bi then you can know the value of theta and from there you can find out the temperature, 'right'. This is second way of this sphere and similarly internal distribution of temperature you can find out. This is 1/3Bi instead of 1/2Bi for sphere it is 1/3Bi and this is Bi $=h_{ro}/3k$ Bi is equal to each other by 3k.

Once we plot that then for a given value of 1/3 Bi versus this $r_o/r_i$ also rather $r/r_o$ that is for different values of that we can get this and from there we get the value of θ as it is given here $T(r\ t)-T_\infty/T(0\ t)-T_\infty$. So, that from there you find out the value of theta, 'right'.

Similarly, the other one is for the heat flow that is Q. So, the value of Q you have here similarly $Bi^2Fo$ for the sphere and for different Bi value is equivalent to $h_{ro}/k$ you get the value; get the Bi value and get the value of $Q/Q_0$ and $Q_0$ if you know earlier so, you can find out the value of Q, 'right'. This is how the different charts are to be read, 'right'.

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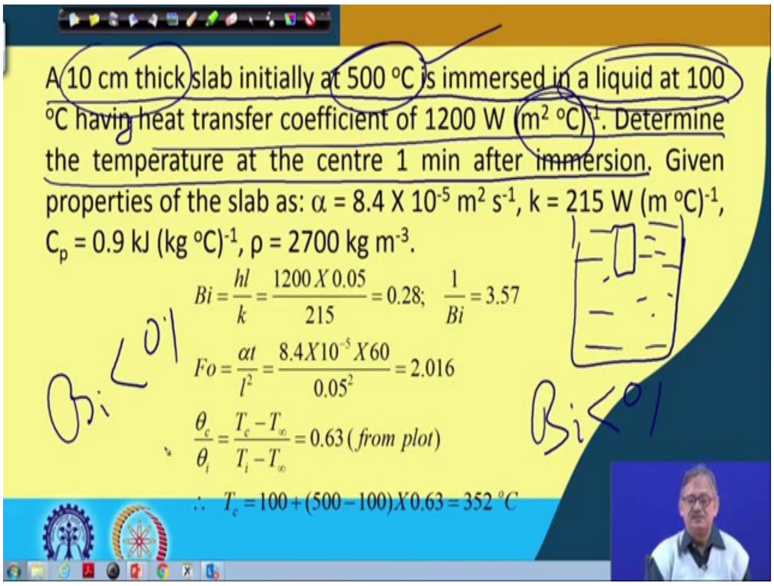


Now, we have to utilize these charts. So, for that I think it is the best thing to know by solving problems, 'right'. So, we solved the problems maybe 1, 2 or 3 or as many you can subsequently you can you do yourself we are giving some example. So, based on that example you can do subsequently and may do very; I mean, you can experience as well you can learn.

So, problem is like this 10 centimetre thick slab centimetre initially at 500ºC is immersed in a liquid at 100ºC having heat transfer coefficient of 1200 W $(m^2\ ºC)^{-1}$, 'right'. This we have written in this work.

Determine the temperature at the centre one minute after immersion, 'right'. Here also you see what do you have a 10 centimetre thick slab initially at 500ºC is the immersed in a liquid at 100ºC. So, you have a liquid you have immersed a slab in that liquid. Initially it was at uniform temperature of 500ºC, suddenly it is immersed. So, it appears that you can use the lumped system, but again for lumped system we have said that Bi has to be less than 0.1. So, if Bi is less than 0.1 then only you can apply lumped system.

So, the first thing you have to check whether Bi is less than 1.1 or not, 'right'. So, from the given properties that given properties are like this the slab has. $\alpha$ = 8.4 X $10^{-5}$ $m^2\ s^{-1}$; conductivity of 215 W $(m\ ºC)^{-1}$; specific heat $C_p$ = 0.9 kJ $(kg\ ºC)^{-1}$, and density of $\rho$ = 2700 kg $m^{-3}$, 'right'.

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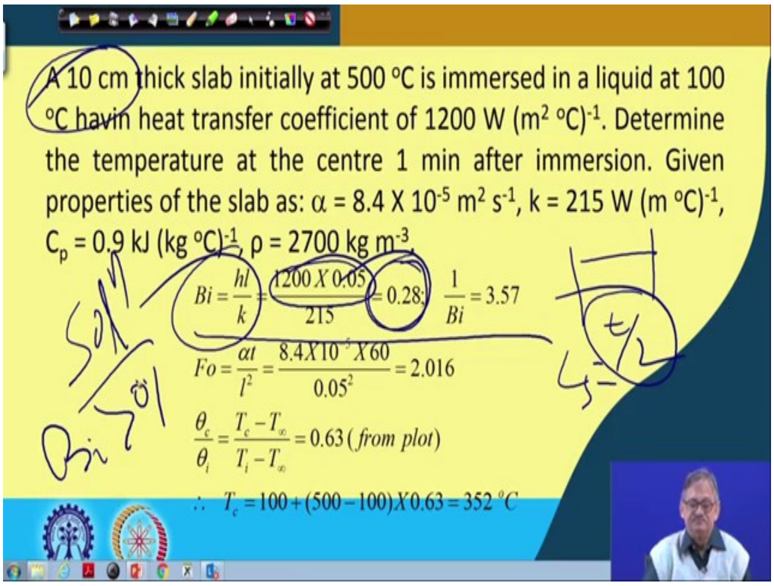


So, for the solution what do you need? First you look into that what is the value of Bi. So, Bi is hl/k; this is for slab. So, it is given 10 centimetre. So, 10 centimetre is the thickness. So, t/2 is the characteristic Length; so, $L_s$ being t/2. So, it is 10 centimetre thick. So, it becomes 10/2 that is 5 centimetre.

$$Bi = \frac{hl}{k} = \frac{1200 X 0.05}{215} = 0.28; \quad \frac{1}{Bi} = 3.57$$

which means Bi is greater than 0.1. So, you cannot you cannot employ this lumped system.

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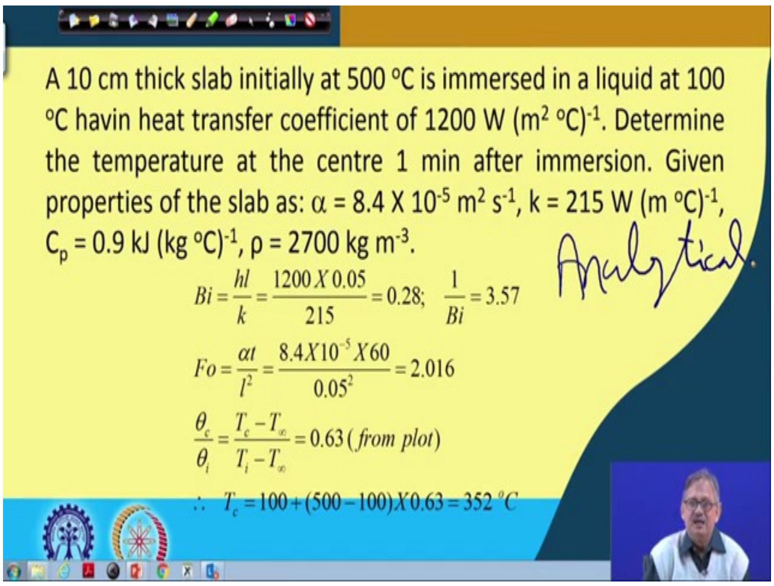

So, if you cannot employ lumped system what other it could have been? It could have been an analytical solution, ‘right’, but analytical solution you have seen, we have shown you also earlier that it needs lot of calculation or analysis.

Now, is to avoid that just we have said that the Heisler charts they are very handy, ‘right’. So, we can utilize the Heisler chart here also. So, Bi you have found out 1 by Bi is 3.57. Now, from $Fo = \frac{\alpha t}{l^2}$; alpha is given, t has been given, t is after 1 minute, ‘right’; so, 60 second t is given. Alpha t by l square; l already 0.05 we have seen. So, $Fo = \frac{\alpha t}{l^2} = \frac{8.4X10^{-5}X60}{0.05^2} = 2.016$ So, Fourier number is 2.016.

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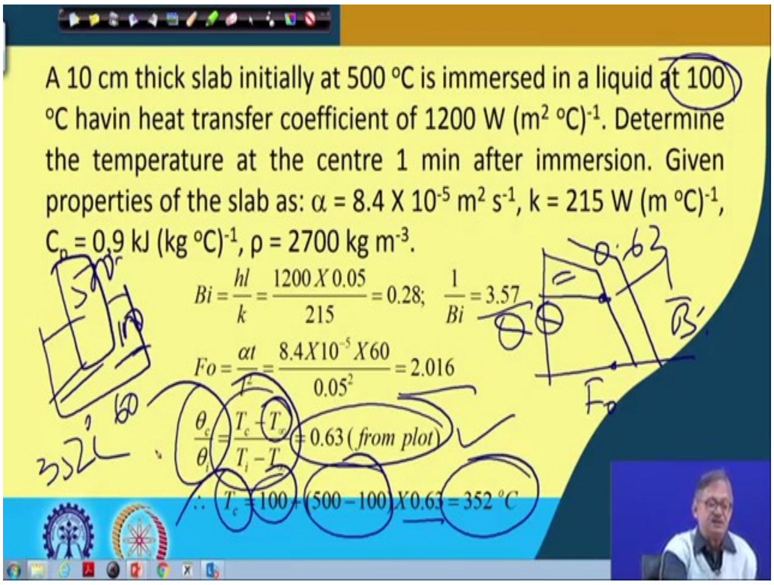


Then again what we need to do? We need to go to that theta versus Fourier number that plot, ‘right’ which was like this for different 1 by Bi. So, for a given Fourier number that is 2.06 for a given 3.57 that is this. So, we look at the point and then find out the value of θ, ‘right’. $\frac{\theta_c}{\theta_i} = \frac{T_c - T_\infty}{T_i - T_\infty} = 0.63$. So, this value of theta from the plot we get 0.63.

So, if we get from the plot this value equal to 0.63 then we can find out the value of $T_c$ or centre temperature that is $T_\infty$ is already given; $T_\infty$ is outside, $T_c = 100 + (500 - 100) X 0.63 = 352\ ^oC$ So, after 1 minute which you emerged into the solution that solution this from 500 degree centigrade, if it is 100 degree centigrade the after 60 seconds the temperature has come down to 352 degree centigrade, centre temperature, 'right'.

So, similarly if this is centre temperature similarly at any position if it would have been asked, you would have found out that what is the positional temperature using the second curve, 'right'. So, this is one example where what we have done? We have first checked what is the value of Biot number? That value is less than 0.1 or not that we have checked because if it is less than 0.1 then directly we can use the easy solution as lumped solution. But, since it is we saw that it is not point less than 0.1 it is more than 0.1. So, we are not able to use lumped system directly.

So, if we are not using lumped system directly another solution could have been analytical solution based on the boundary conditions given from the basic governing equation. But, that may take some time or some analysis or some block complication depending on the situation. That is why we are trying to find out the centre temperature from the Heisler chart.

For the Heisler chart to find out the centre temperature what we need? We have to find out what is the Fourier number that is $Fo = \frac{\alpha t}{l^2}$; the values are given, t is given for 1 minute that is 60 seconds, alpha is given and the l characteristic length dimension also given 10 centimetre thick. So, t/2 is that; so, it is 0.05 centimetre 0.05 meter, 'right' 5 centimetre 0.05 meter.

So, from there we found out Fourier number, you have found out already Biot number. So, inverse of the Biot number you know; so, these are the two points from where you can point out on the graph of Heisler chart Fourier number versus non-dimensional temperature against 1 by Bi. That point from there you find out what is the value of θ and from that θ value you now expand that θ in non-dimensional temperature into that into the form of T that is $\frac{\theta_c}{\theta_i} = \frac{T_c - T_\infty}{T_i - T_\infty}$. So, other two are given, T centre you find out and it became to be in this problem 352 degree centigrade, 'right'. Similarly, other problems also can be solved.

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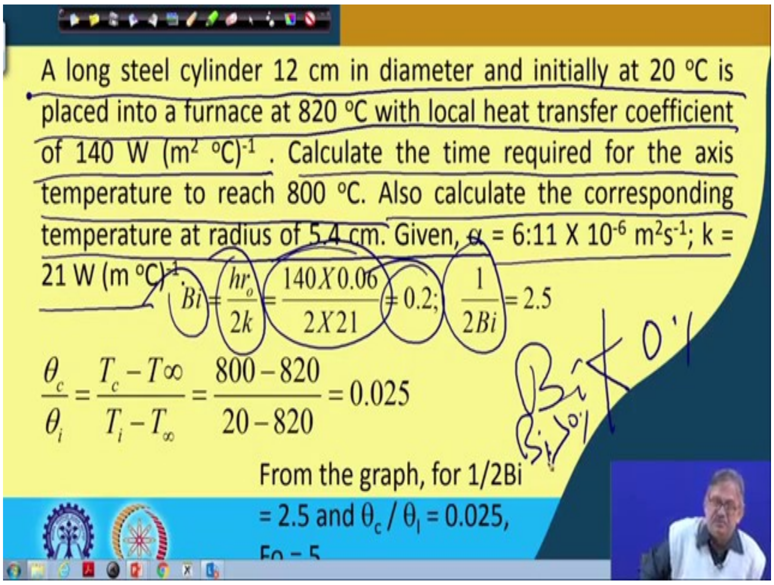


Let us go into that this is the second problem; that was with the slab, this is with the cylinder. A long cylinder 12 centimetre in diameter and initially at 20ºC is placed into a furnace at 820ºC with local heat transfer coefficient of 140 W(m$^2$ ºC)$^{-1}$ Calculate the time required for the access temperature to reach 800 degree centigrade. Also calculate the corresponding temperature at radius of 0.5 centimetre.

Given alpha 6.11 X 10$^{-6}$ m$^2$s$^{-1}$; conductivity 21 W (m ºC)$^{-1}$. So, if this is known then we can find out, again I will read long steel cylinder 12 centimetre in diameter and initially at 20ºC, placed into a furnace at 820ºC with local heat transfer coefficient of 140W(m$^2$ ºC)$^{-1}$.

Calculate the time required for the access temperature to reach 800ºC also calculate corresponding temperature at radius of 5.4 centimetre. Given alpha is equal to 6.11 X 10$^{-6}$ m$^2$s$^{-1}$; conductivity k 21W (m ºC)$^{-1}$, 'right'.

So, it is also again coming that first we have to find out Biot number. So, Biot number is

$$Bi = \frac{hr_o}{2k} = \frac{140X0.06}{2X21} = 0.2; \quad \frac{1}{2Bi} = 2.5$$

So, if Bi is greater than 0.1 we cannot use lumped system.

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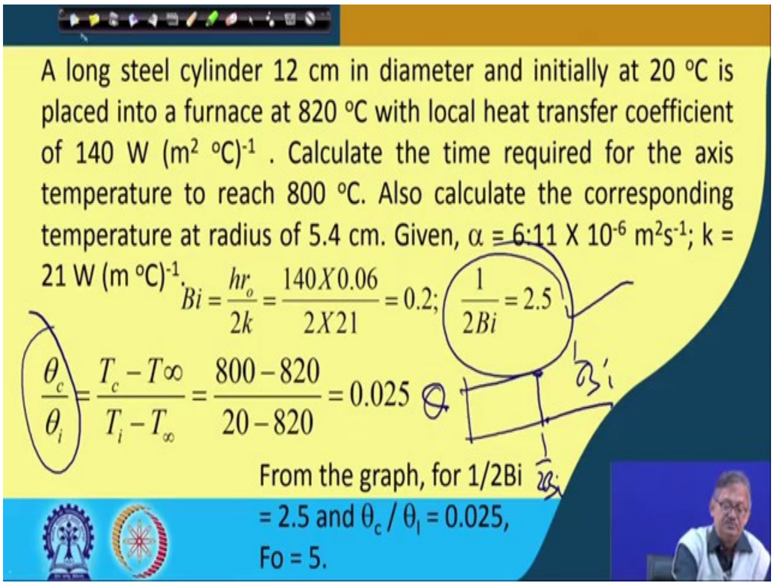


So, we have found out what is the value of 1 by 2 Bi which is 2.5. So, we have found out this; so, theta c by theta i. Now again we go to that plot where we have to find out 1/2Bi, 'right'; so, 1/2Bi versus $r_o$/ri so, that here it is centre temperature. So, centre temperature is 1 by Bi so, from there we get this value and get the value of theta, 'right'. So, the $\theta_c$ / $\theta_I$ that has come to be 0.025, 'right'.

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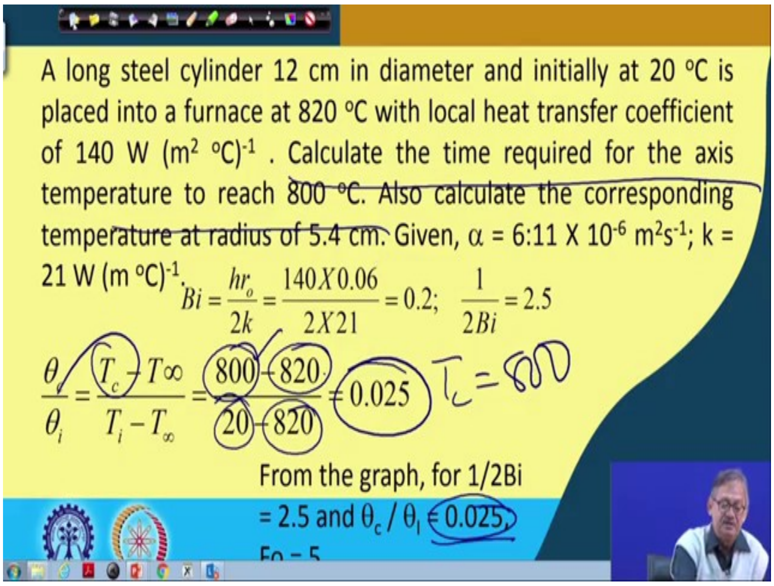


So, 0.025 we have found out and from there we have this centre temperature we can find out there is from the graph of this, this is 0.025, 'right' and this why we have made 800 minus 820 this is the environmental temperature, initial temperature, environmental temperature, 'right'. So, here we should first find out because 800 degree centigrade we have not shown.

So, however, this is like that that 0.025 was that from the graph and we can find out $T_c$ and that $T_c$ comes to equal to 800, 'right' $T_c$ comes to 800, because calculate the time required for the access temperature to become 800 I made mistake I thought that time is given.

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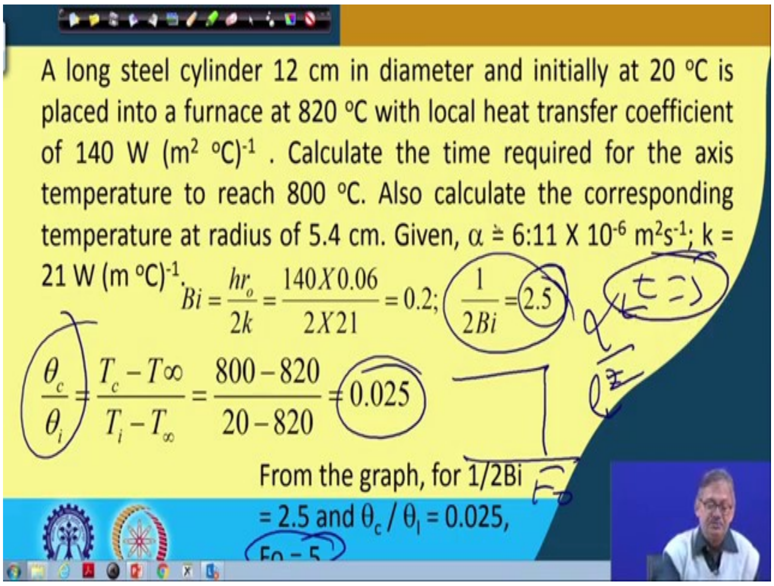


Since time was not given so, we have found out theta c over theta i and that has become 0.025, $F_o$ is not given. So, from this value and from 1/2 this value we find out $F_o$ value. So, $F_o$ value came out to be 5, 'right'; $F_o$ value Fourier number came out to be 5. So, if it is 5 then we know it is $\frac{\alpha t}{l^2}$ and this is $r_o^2$, 'right'; $\alpha t$ /ro$^2$ $F_o$ value 5 we can find out value of t. So, which we have done subsequently. Let us see that subsequently we have done this.

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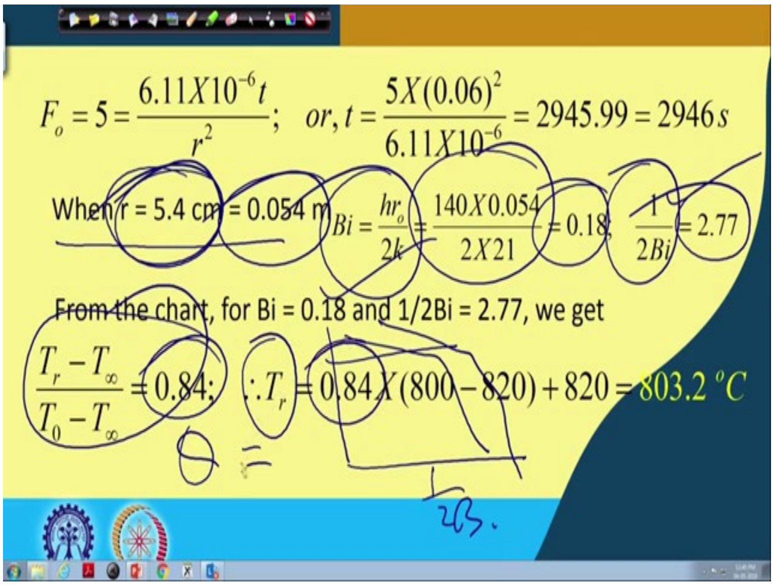


$F_o$ is 5, 'right' and

$$F_o = 5 = \frac{6.11X10^{-6}t}{r^2}; \quad or, t = \frac{5X(0.06)^2}{6.11X10^{-6}} = 2945.99 = 2946s$$

, 'right'. So, you remember that the value of r was given the value of r or 12 centimetre, 'right'. So, 12/2 that is r is 6 centimetre, 'right'.

Now, we are asked that what is the value of theta or temperature when r is 5.4 centimetre, 'right'. So, When r = 5.4 cm = 0.054m, 'right' again Bi is

$$Bi = \frac{hr_o}{2k} = \frac{140X0.054}{2X21} = 0.18; \quad \frac{1}{2Bi} = 2.77$$

So, if 1/2Bi is known then from the chart we can find out what is the value of theta 'right'. So, theta becomes equal to from the chart of what? In this case our chart will be again 1/2Bi versus Bi. So, that Bi we have found out what is the 1/Bi what is the value of θ, 'right'. So, that θ we have found out to be 0.84, 'right'.

$$\frac{T_r - T_\infty}{T_0 - T_\infty} = 0.84; \quad \therefore T_r = 0.84X(800-820)+820 = 803.2\ ^oC$$

So, we have found out $T_r$ from the graph there is at a location 5.4 centimetre at the as the radius, 'right'. So, we have with this we can find out which was 803.2 degree centigrade, 'right'.

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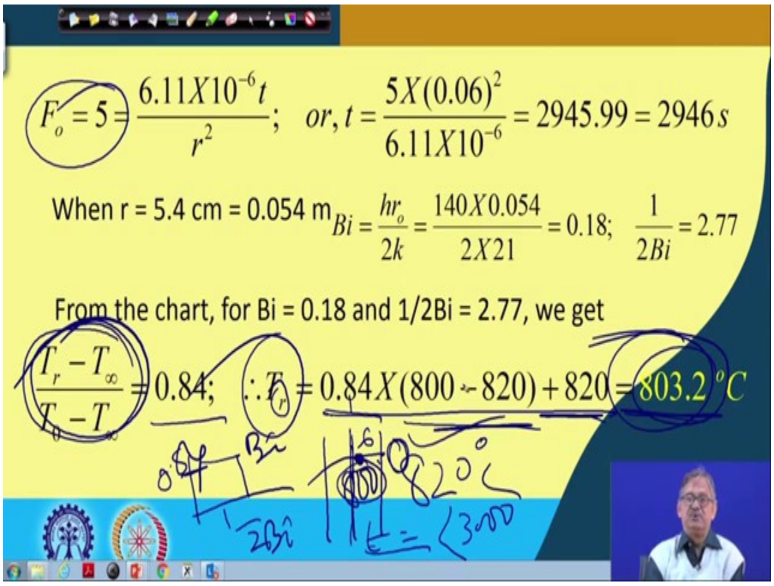


Now, you also see, your concept wise your radius was like this it was put at some 820 degree centigrade that is the your oven temperature and it came to be after some time whatever time was given after time t whatever it came centre temperature came to be 800, 'right'. You were know you were asked to find out what is the time required. So, time we had shown some almost close to 3000 seconds, 'right'. So, close to 3000 seconds we have shown.

Now, from there what we did? We now found out that at the different location, 'right'; so, at the different location to find it out as we have taken the $\dfrac{T_r - T_\infty}{T_0 - T_\infty}$ that value, 'right' and this came to be 0.84, how? This was with that 1/2Bi, 'right' versus different Bi whatever. So, this value we got from there, 'right'. So, this was 0.84; so, from this 0.84 value we then calculated the value of $T_r$ that is the value of temperature at r is equal to 5.4 centimetre, 'right' and we got it to be 820.

So, what I would like to say if the centre is 800 and if the outside is 820 then this was at r is equal to 6, 'right'. So, at r is equal to 5.4 it will be away from 6 so, somewhere here. So, not the centre, but closer to the surface than the centre, this point is closer to the surface than the centre.

So, it is likely that if the centre was 800 and if the outside was 820 this temperature will be higher than 800 that is what we got it to be 803.2 degree centigrade, ‘right’. This is how we expect that you will also do some problems perhaps the next class we will do a some problem with the sphere and then we will complete this and some more sphere or some other similar so that this Heisler chart becomes handy to you and you can easily determine the temperatures, ‘right’. So, with this let us stop to this class.

Thank you.