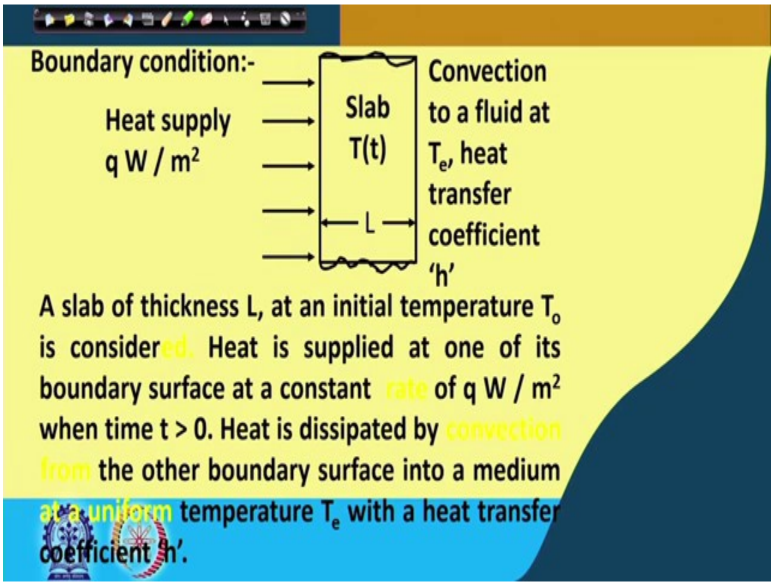
Thermal Operations in Food Process Engineering: Theory and Applications
Prof. Tridib Kumar Goswami
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

Lecture - 23
Transient Heat Transfer (Contd.)

In the previous class we have seen that lumped system analysis how it can be done, 'right'. And, today we will do another that is where the boundary conditions are given, 'right'. Boundary conditions if are given, then we can solve it analytically. We will do that today and in the previous class we were discussing about the problem, that a slab is bounded in the two sides; one with a convective boundary condition and the other with the constant heat flux boundary condition, 'right'.

So, under that situation if the slab is having a thickness L, then what will be the temperature distribution in the body that we can find out by solving it, 'right'.

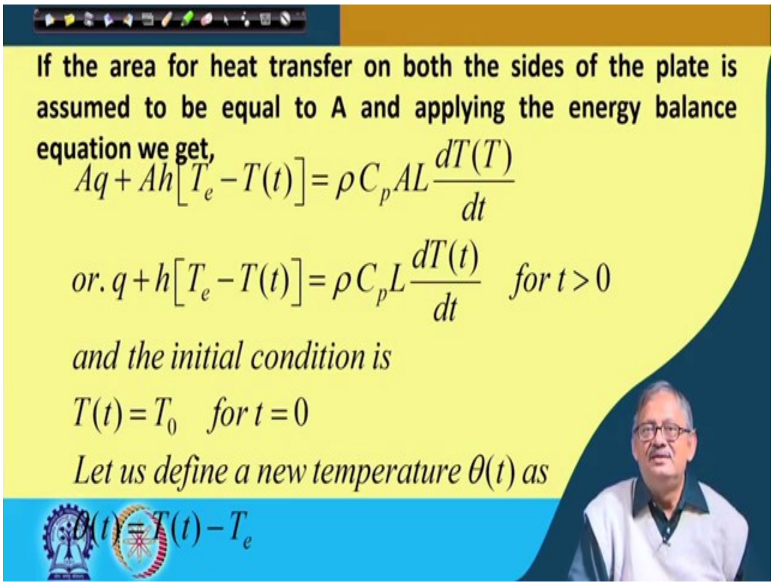
(Refer Slide Time: 01:39)



So, we come to this that if the boundary conditions are given, that we have a heat flux of q watt per meter square, in a slab of thickness L and the other boundary is a convective boundary with the heat transfer coefficient of h and the temperature is T_e.

Then a slab of constant with the thickness of L at an initial temperature of T_0 is considered and heat is supplied at one end of it is boundary surface at the constant rate of heat flux that is q watt per meter square, when time is greater than 0 or t is greater than 0. Heat is dissipated by convection into the other boundary surface into your medium at uniform temperature of T_e with a heat transfer coefficient of h, 'right'.

(Refer Slide Time: 02:36)



Perhaps a similar problem we have done earlier also, 'right';. If the area for heat transfer on both the sides of this plate is assumed to be equal to A and applying the energy balance equation we can write, that

$$Aq + Ah\left[T_e - T(t)\right] = \rho C_p AL \frac{dT(T)}{dt}$$

$$or.\, q + h\left[T_e - T(t)\right] = \rho C_p L \frac{dT(t)}{dt} \quad for\ t > 0$$

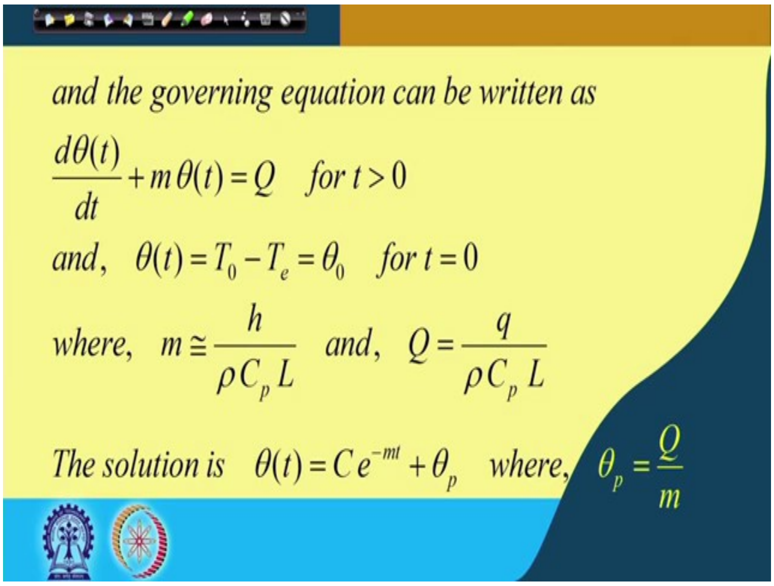
and the initial condition is

$$T(t) = T_0 \quad for\ t = 0$$

Let us define a new temperature $\theta(t)$ *as*

$$\theta(t) = T(t) - T_e$$

(Refer Slide Time: 03:47)



and the governing equation can be written as

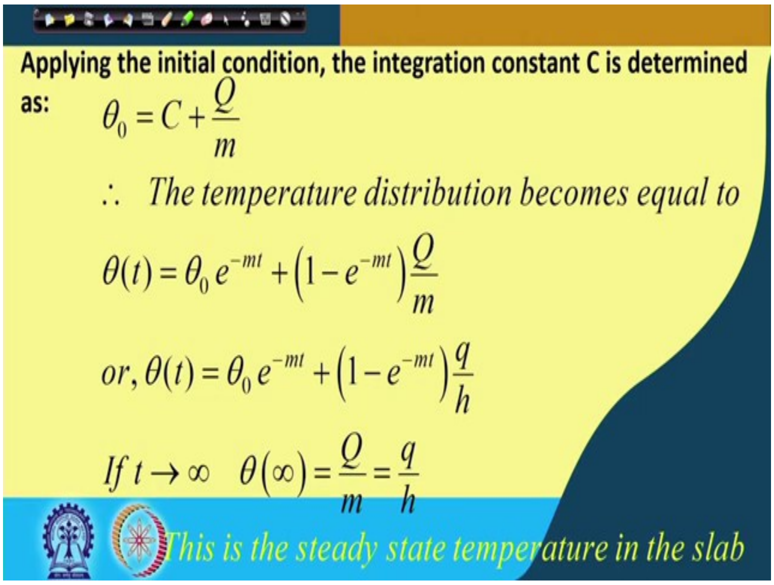
$$\frac{d\theta(t)}{dt} + m\,\theta(t) = Q \quad for\ t > 0$$

$$and, \quad \theta(t) = T_0 - T_e = \theta_0 \quad for\ t = 0$$

$$where, \quad m \cong \frac{h}{\rho C_p L} \quad and, \quad Q = \frac{q}{\rho C_p L}$$

$$The\ solution\ is \quad \theta(t) = C\,e^{-mt} + \theta_p \quad where, \quad \theta_p = \frac{Q}{m}$$

(Refer Slide Time: 04:51)



$$\theta_0 = C + \frac{Q}{m}$$

$\therefore$ *The temperature distribution becomes equal to*

$$\theta(t) = \theta_0 e^{-mt} + \left(1 - e^{-mt}\right)\frac{Q}{m}$$

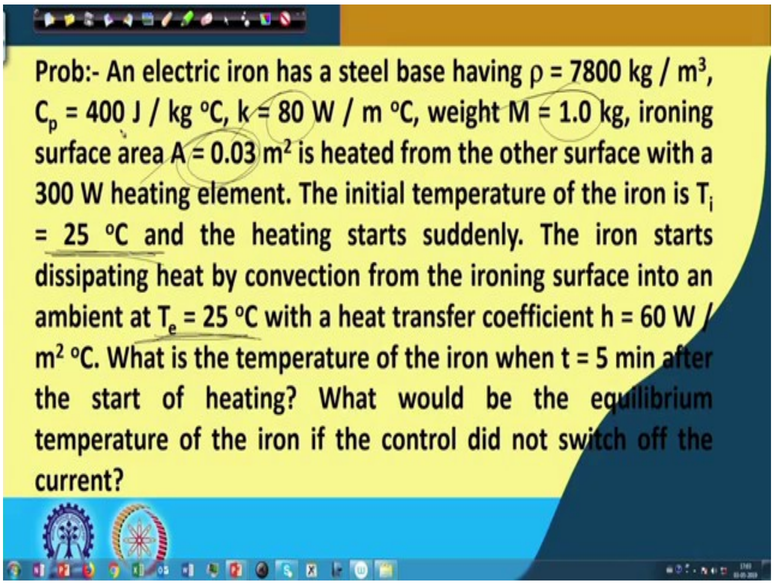
$$or, \theta(t) = \theta_0 e^{-mt} + \left(1 - e^{-mt}\right)\frac{q}{h}$$

$$If\ t \to \infty \quad \theta\left(\infty\right) = \frac{Q}{m} = \frac{q}{h}$$

This is the steady state temperature in the slab

And, this is the steady state temperature in the slab that is where theta tends to infinity is the steady state temperature.

(Refer Slide Time: 05:53)



An electric iron has a steel base having density $\rho = 7800$ kg/m^3, C_p that is specific heat $C_p = 400$ J / kg °C, conductivity = 80 W / m^2 °C, weight M is 1.2 kg, ironing surface area A is 0.03 meter square, perhaps this problem we have not done. A similar solution might have been, but this problem we have not done, 0.03 meter square is heated from the other surface with a 300 watt per watt heating element.

The initial temperature of the iron is $T_i = 25$°C and the heating starts suddenly. The iron starts dissipating heat by convection from the ironing surface into an ambient at T_e is equal to 25°C with a heat transfer coefficient h equal to 60 W / m^2 °C. What is the temperature of the iron when t is 5 minute after the start of the heating? What would be the equilibrium temperature of the iron if the control did not switch off the current?

How it is coming into this unsteady, because you see after time 0 suddenly you have put one iron, which is starting heating, ‘right’. That you everybody you or every one of us do that while ironing you put the switch on, ‘right’. So, you in that iron down below, there will be some electrical resistance through which this electricity process and by which that i square r is that is that element is heated, ‘right’.

So, you were heating and then doing the ironing and after doing you forgot to switch it off, ‘right’. So, what will be the effect? is it that if that iron will go on increasing the temperature? Perhaps not i.e., what is the problem, that is why it is that a little unsteady and the solution we have to find out through the analytical solution. Because, in this type of thing Biot number will not come into, because hl/ k is that if you see it will be very high, it is not less than 0.1. Though here we have not shown typically in this problem, but the solution itself says that it is almost equal to that solution of unsteady and the iron is the example for that, ‘right’.

So, if we look at the problem once more then it becomes easier, more clear to you that an electrical iron has a steel base having density of 7800 kg/m^3, specific heat of 400 J / kg ^oC. So, conductivity k is 80 W/m^{o}C weight of the material is 1.2 kg, ironing surface area through which we are ironing you see most of the iron is like that, this end is little cons-equated and gradually it is expanding, ‘right’ one end is narrow or pointed so, ‘right’.

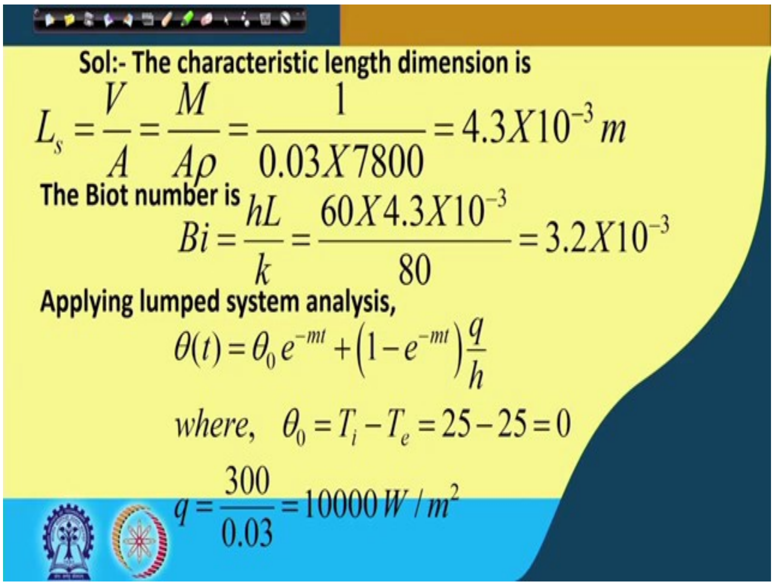
So, the total area of iron is 0.03 m^2, heated from the out other surface with a 300 watt heating element. Initial temperature of the iron is 25 ^oC and the heating starts suddenly, because you switched off suddenly or switched on suddenly so, that you can iron it, ‘right’.

The iron starts dissipating heat by convection from the ironing surface into an ambient where the temperature is 25 ^oC with a transfer coefficient of 60 W/m^{2o}C. What is the temperature of the iron when t is 5 minutes you have been ironing you have been put on the switch for 5 minutes your system is on, what will be the temperature?

That is number 1, after the start of the heating, ok. What it would be the equilibrium temperature of the iron, if the control did not switch off the current? If the problem we have defined in a different way actually it is like that.

So, you were doing that ironing sometime after doing that few you have suddenly forgotten to switch off and you left the place, 'right', that is what equivalent to that, what will be the equilibrium temperature of the iron if the control did not switch off the current? That means, you have not switched off the current. If that be the condition what will be the maximum and that is what we call it to be the steady state temperature, because that iron will not go on increasing the temperature, rather then it will become red hot and then suddenly burst like that it does not happen, 'right'. So, that is a real example or real problem which we are solving, 'right'.

(Refer Slide Time: 12:08)



So, we have understood the problem and the solution of heat we can say, that here also we are finding out first

$$L_s = \frac{V}{A} = \frac{M}{A\rho} = \frac{1}{0.03X7800} = 4.3X10^{-3}\,m$$

So, Biot number is 0.003. So, if it is 0.003 similar to that earlier we can use the lamp system, 'right'.

$$Bi = \frac{hL}{k} = \frac{60X4.3X10^{-3}}{80} = 3.2X10^{-3}$$

So, here that lamp system was the solution

$$\theta(t) = \theta_0 e^{-mt} + \left(1 - e^{-mt}\right)\frac{q}{h}$$

$$where,\quad \theta_0 = T_i - T_e = 25 - 25 = 0$$

$$q = \frac{300}{0.03} = 10000\,W/m^2$$

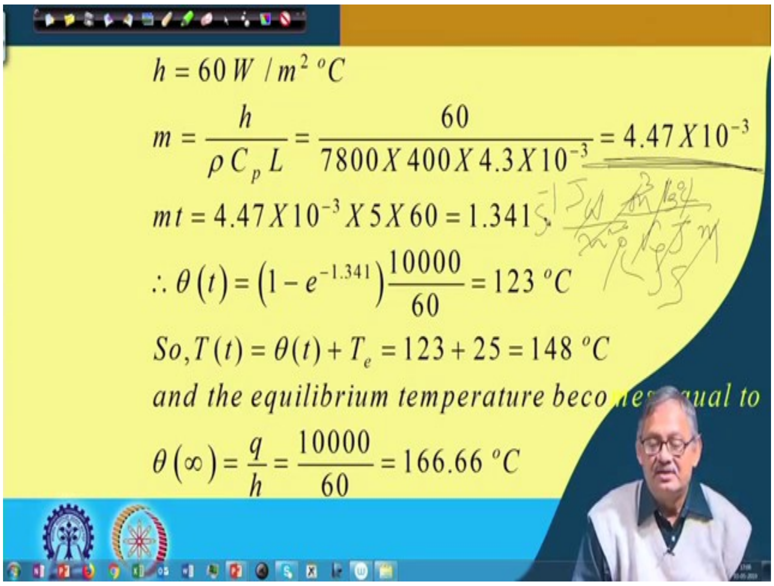
If that be true initial and the temperature which was there let me see where is the where is the pen with this side I have kept, ok. So, if we see that so, initial temperature was given 25 and environmental temperature is also 25, 'right'. And, the earlier that area was 0.03, conductivity was 80, this we have changed to m and the other things remaining same.

So, we have found out Bi and then now we are finding out the solution this solution we have already found out earlier and Bi we have found out to be equal to 0.3 0.2 10 to the power minus 3 that is 0.003, 'right'. So, 0.003 say applying the lumped system analysis, we found out theta T is theta 0, e to the power minus mt plus 1 minus e to the power minus mt into q by h, this we have just solved it with the boundary, 'right'. Where theta 0 is T_i minus T_e that is 25 minus 25 that is equal to 0, 'right'.

So, theta 0 is 0 and q the value of q which was not given which was given 300 watt that much of supply heat being supplied 300 watt, but q is a flux. So, that is watt per meter square, 'right' so, to get the q what we need to do we have to divide with the area. So, that is what we have done q is 300 by 0.03 that becomes equal to 10000, that is 10,000 watt per meter square, that we have found out. Because we were given that heat is supplied equivalent to 300 watt, 'right'.

So, that 300 watt we are now converting it to the heat flux, which is watt per meter square. So, by dividing that with the area of the heat transfer that is 0.03. So, 300 by 0.3 is nothing, but 10,000, 'right', so that we have found out.

(Refer Slide Time: 18:44)



Now, we have to find out the temperature distribution 'right'.

$$h = 60\,W/m^2\ ^oC$$

$$m = \frac{h}{\rho C_p L} = \frac{60}{7800 X 400 X 4.3 X 10^{-3}} = 4.47 X 10^{-3}$$

$$mt = 4.47 X 10^{-3} X 5 X 60 = 1.341$$

$$\therefore \theta(t) = \left(1 - e^{-1.341}\right)\frac{10000}{60} = 123\ ^oC$$

$$So, T(t) = \theta(t) + T_e = 123 + 25 = 148\ ^oC$$

and the equilibrium temperature becomes equal to

$$\theta(\infty) = \frac{q}{h} = \frac{10000}{60} = 166.66\ ^oC$$

So, value of q was watt per unit of q was watt per meter square and unit of h is watt per meter square degree centigrade. So, this goes out; so degree centigrade is the unit. So, that becomes 166.66 degree centigrade. So, the equilibrium temperature; that means, if you have not put the power supply off, then the temperature will rise from 148 to 166 and it will be at that temperature only, 'right'.

Because, why you are not changing the 300 that 3 you remember that, you are not changing the heat source that is 300 watt; that means, that it is i square r, there is i you are not changing, v you are not changing, r you are not changing, so that is fixed. So, correspondingly your watt supply is also 300.

So, it will come to your steady that is 166.66 and where it will remain, if we do not put it up it will not go beyond that temperature, ‘right’. So, hopefully we could have explained the situation and we have come to the end of the class for this. So, let us call it a day.

Thank you.