Thermal Operations in Food Process Engineering: Theory and Applications
Prof. Tridib Kumar Goswami
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

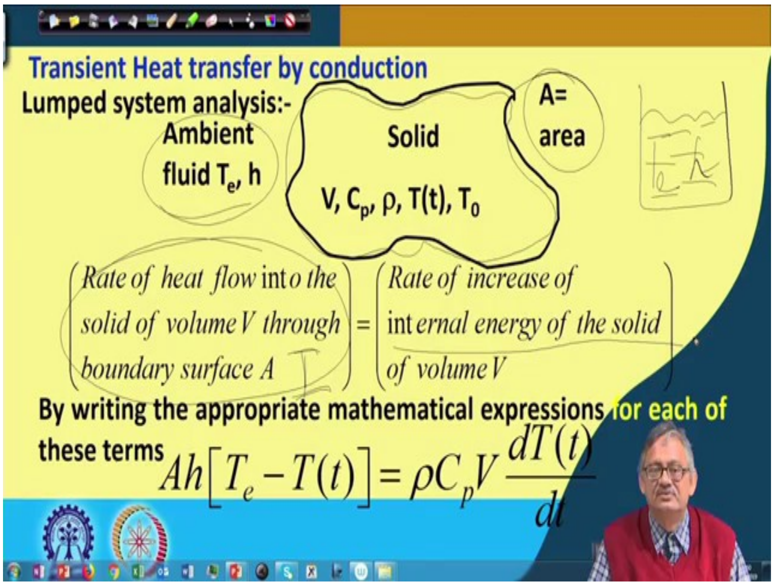
Lecture - 21
Transient Heat Transfer

So, good afternoon; again, we are coming we have finished more or less conduction heat transferred, but on the steady state condition 'right'. So, now, let us go into Heat Transfer for Transient condition that is this is called time dependent heat transfer 'right'. Earlier it was time independent; that means, under steady state, but when there is unsteady state, then there is all altogether different heat transfer and the solutions are also different.

And I let us give an example an example is that say you have a boiled water boiling water, 'right'. And you have a ball say maybe; maybe; maybe made of say iron ball of this kind of or the playing that duce ball by which cricket is being played, 'right', that is made of say iron and you dip it into that boiling water. The moment you have dipped into that boiling water initially towards under room temperature now you have given it under boiling condition. So, the inside of this ball the temperature will be going on increasing till it is attaining a steady state temperature, 'right'.

So, under this situation the heat transfer is known as transient heat transfer, 'right' or heat transfer during transient condition or also known as unsteady state heat transfer, 'right'.

(Refer Slide Time: 02:27)



So, if we look at that for the solution of heat, 'right' for it is solution let us look into that this is called a lumped system heat transfer conduction, 'right' lumped system analysis that is as I said that it being cricket ball or kind of thing say it being a ball. So, you are dipping it into hot water or boiling water the temperature inside will go on changing. So, that while it is giving on changing till it is attaining a steady temperature. So, that period is known as transient heat transfer or the heat transfer during this process is called unsteady state heat transfer, 'right'.

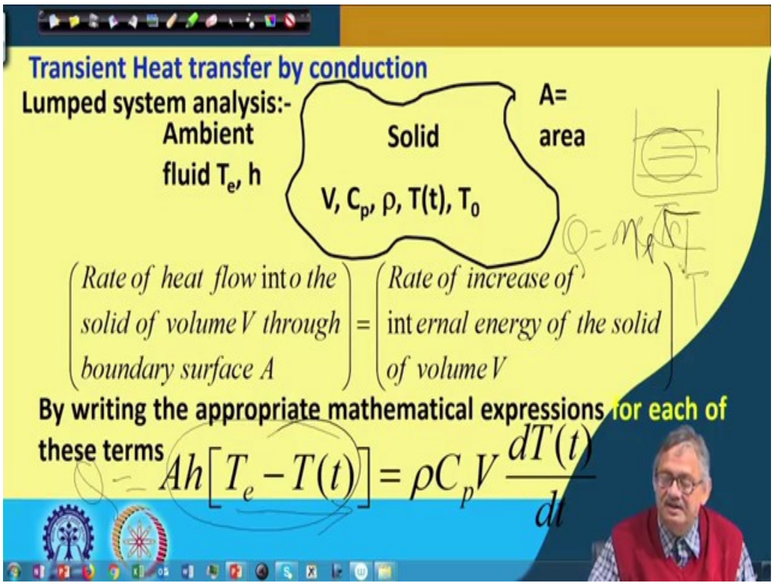
So, we are taking in that case as I said you have taken on ball of iron. So, that is a spherical, but to do the analysis let us not take any specific geometry, 'right'. So, let us take an arbitrary geometry where there is no shape and size, 'right' as we have shown in this that this is the; this is the shape of I mean a non specific shape of solid having volume V, specific heat C_p, density rho and temperature at any time T (t) and initial temperature T_0 with a heat transfer area A and ambient condition is that ambient is at a temperature T_e with a heat transfer coefficient of h, 'right'.

This is similar to that we said that in a pot we have boiling heat water and you have dipped one ball into it, 'right' who is this conditions are known that is its volume is known; its specific heat is known; its density is known and its initial temperature is known we have to find out at definite time what is the temperature. If that be true this we are going to do it here, 'right'. Here instead of taking this spherical shape let us take one geometry which is not a specific this like this one and also here we would like to say that this environment that is the water boiling has a temperature known as T_e with a heat transfer coefficient of h, 'right'.

If that be true then the fluid with heat transfer coefficient of h and the temperature T_e, if that is surrounded by and the body is having a volume as we said with an area A, then the governing equation which will govern the heat transfer we can say that the rate of heat flow into the volume into the solid of volume V through the boundary surface A, if we take that to be equal to capital I or roman I and that must be equal to rate of increase of internal energy of the solid of volume V, 'right'.

So, we are equating the rate of heat flow into the solid of volume V through the boundary A or surface A that should equal to the rate of increase of internal energy of the solid of volume V, 'right'.

(Refer Slide Time: 07:50)

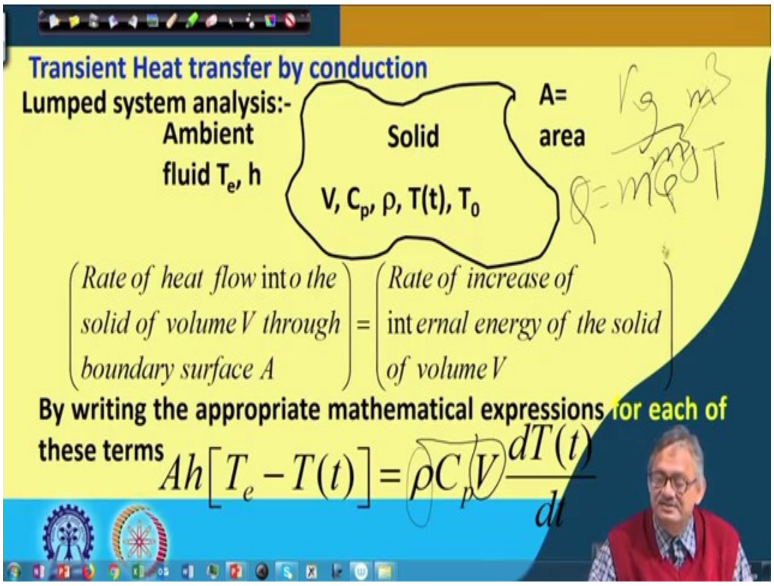


So, this is the governing equation and by writing the appropriate mathematical expression for each of this termed we can say that the heat which is being transferred is by convection from this medium 'right' which is equal to $Ah\left[T_e - T(t)\right]$ T_e in this case if it is a boiling water then it is higher T_e than the temperature of the body, 'right'. So, heat is being transferred from the environment to the body, 'right' with the area A and the heat transfer coefficient being h, 'right'. So, this must be equal to

$$\rho C_p V \frac{dT(t)}{dt}$$

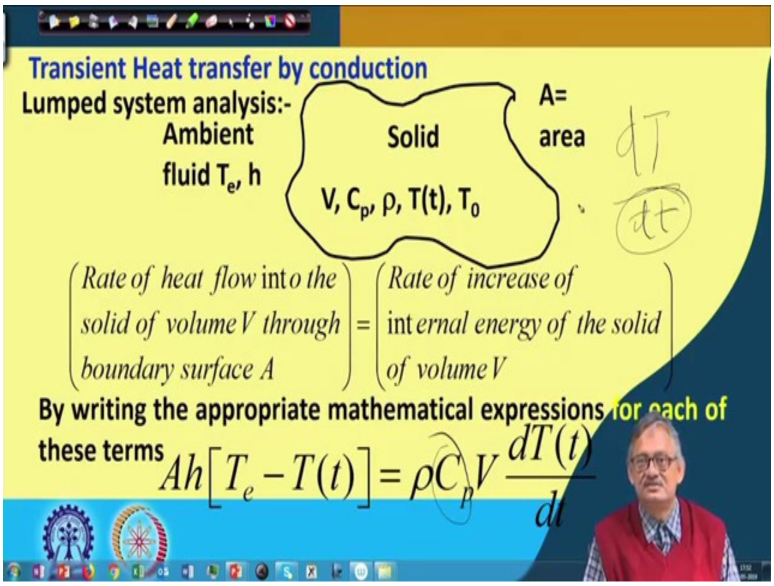
we know Q is equal to $mC_p\Delta T$, 'right'.

(Refer Slide Time: 08:48)



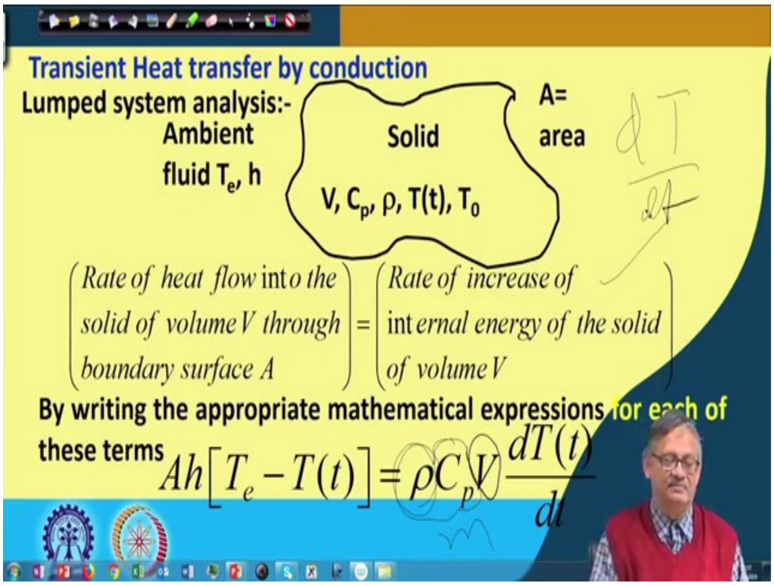
So, Q is equal to $mC_p\Delta T$ that we know. So, to have the m we are making ρ times V, 'right'; ρ is in kg per meter cube and V is in meter cube, so, it is kg-m, 'right'.

(Refer Slide Time: 09:12)



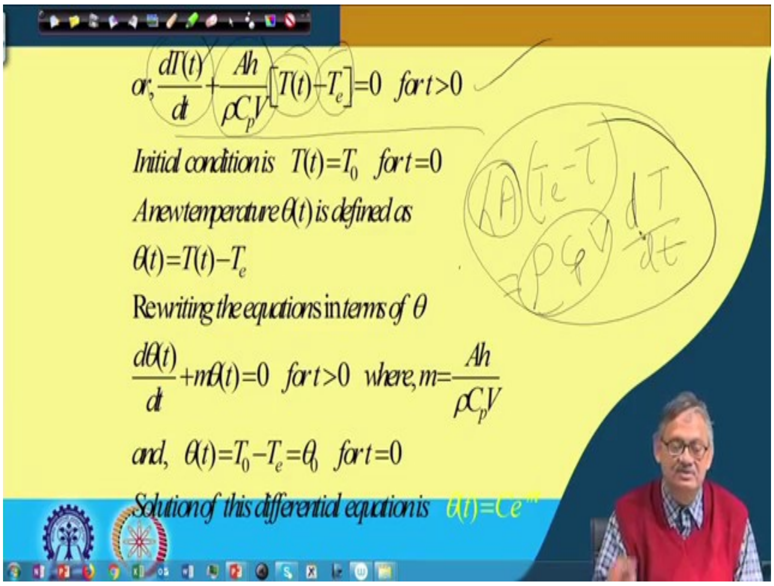
So, $mC_p\Delta T$ and this ΔT, is a function of time, 'right' that is why we write dT/dt that is del del T of t, that is, rate of change of temperature with respect to time. So, this equation we can solve that the governing equation which we have said rate of heat flow into the solid of volume V through the boundary surface A must be equal to the rate of increase of internal energy.

(Refer Slide Time: 09:53)



And this internal energy we have calculated as $mC_p\Delta T$, where m corresponded to ρ times V and this is m and C_p and ΔT and this ΔT is a function of time. So, that is why it is dT/dt that is rate of change of temperature with respect to time, 'right'.

(Refer Slide Time: 10:38)



So, once we have this the solution of heat can be written as

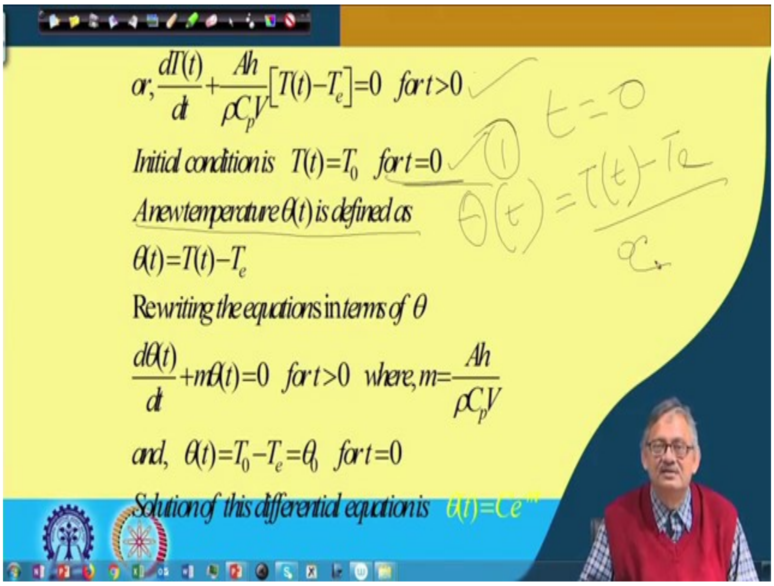
$$\frac{dT(t)}{dt}+\frac{Ah}{\rho C_p V}\left[T(t)-T_e\right]=0 \quad for\, t>0$$

This is the same as the previous one which we had said that $Ah\left[T_e - T(t)\right]$ at any t, this is the body temperature, this was equal to ρ times Cp times V times dT/dt, 'right'. So, this we are rearranging and writing like this and.

$$or, \frac{dT(t)}{dt}+\frac{Ah}{\rho C_p V}\left[T(t)-T_e\right]=0 \quad for\, t>0$$

$$Initial\ condition\ is \quad T(t)=T_0 \quad for\, t=0$$

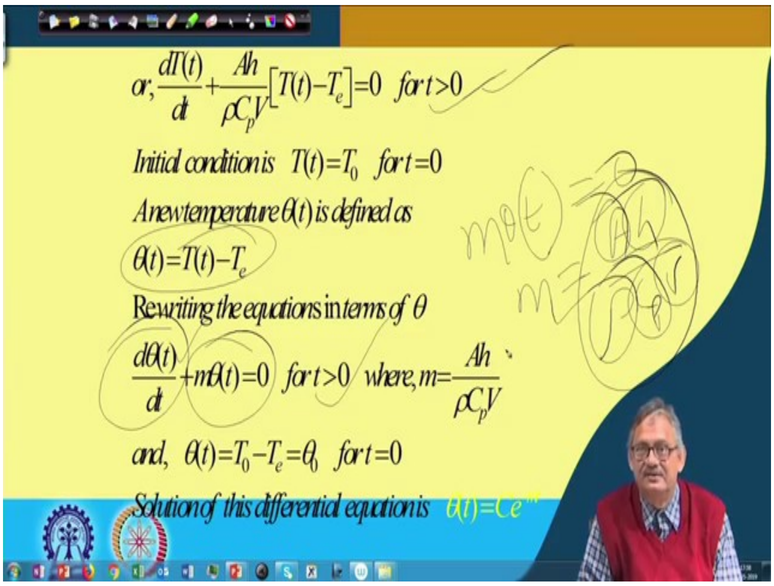
(Refer Slide Time: 12:05)



Not at time t is equal to 0 because time t is equal to 0 is the initial condition. So, this is valid for time t greater than 0. And initial condition is like this that T at any time t is equal to T_0 for t is greater is equal to 0, 'right' this is the initial condition, 'right' and new temperature definition we let us define θ_t, 'right' θ_t let us define as a new temperature and that is to be defined as T_t-T_e, 'right'.

So, this we are depending as T_t -T_e like this temperature is also having a unit of temperature that is either degree centigrade or any equivalent temperature unit, 'right'.

(Refer Slide Time: 13:26)



Now, if these be true then we can write that this equation we can rewrite as in terms of theta as

$$\text{Re}\,\textit{writing the equation}\text{s in } \textit{terms of } \theta$$

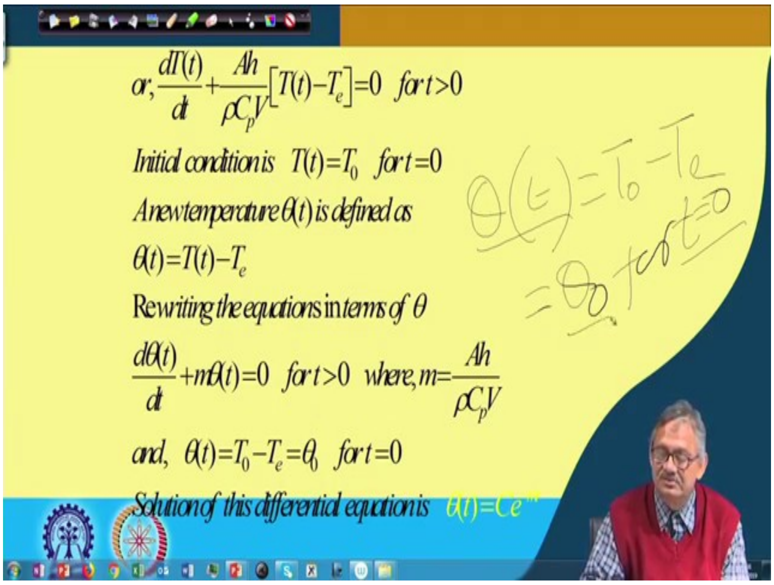
$$\frac{d\theta(t)}{dt} + m\theta(t) = 0 \quad for\ t > 0 \quad where,\ m = \frac{Ah}{\rho C_p V}$$

$$and, \quad \theta(t) = T_0 - T_e = \theta_0 \quad for\ t = 0$$

$$Solution\ of\ this\ differential\ equation\ is \quad \theta(t) = C\,e^{-mt}$$

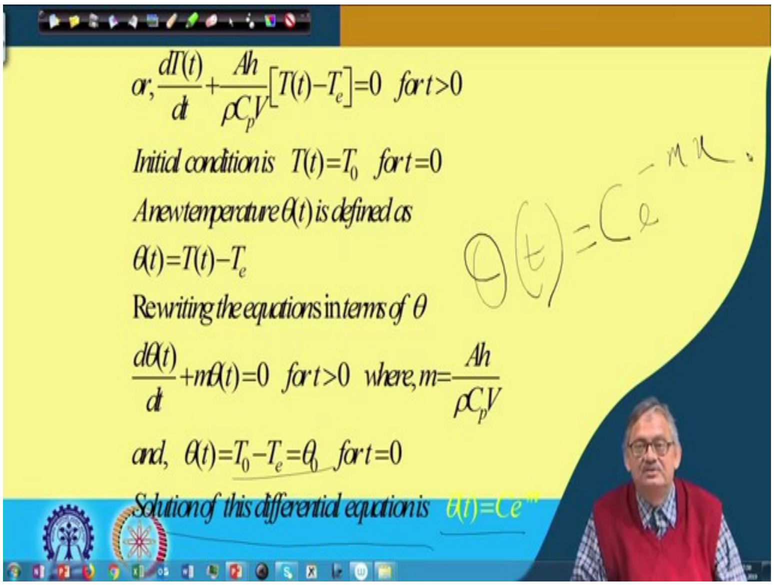
A is the surface area, h is the heat transfer coefficient ρ is the density C_p is the specific heat and V is the volume of the body through which that is being transferred 'right'.

(Refer Slide Time: 14:46)



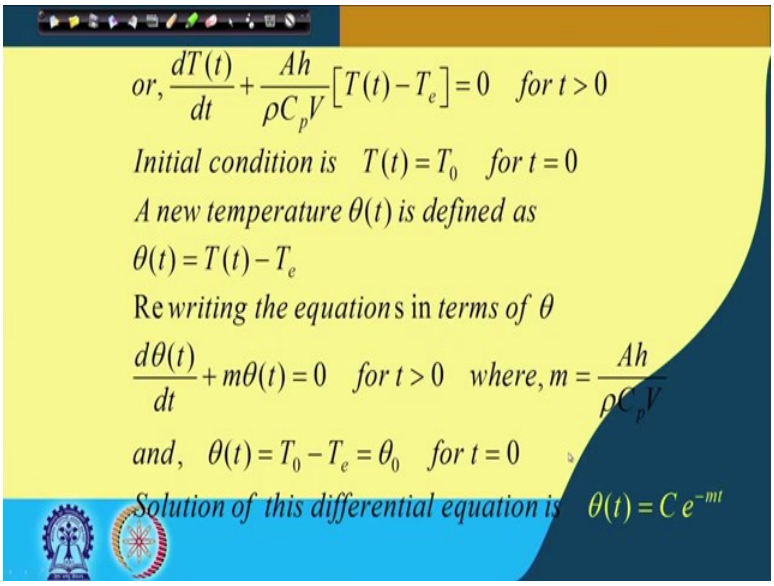
So, if this be true the we can write that as we have said the initial condition is θ_t is equal to T_t -T_e which is equal to θ_0 and this is valid for t time is equal to 0; that is why it is called initial condition at time t is equal to 0, 'right'. So, at time t is equal to 0, θ_t is equal to θ_0 and defined as T_t-T_e, 'right'.

(Refer Slide Time: 15:24)



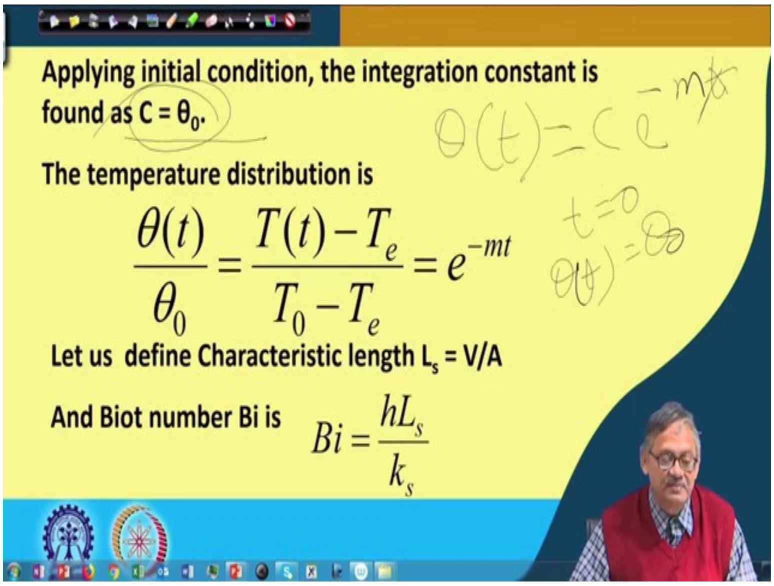
And solution of this differential equation gives in the form θ_t, 'right', θ_t is equal to Ce^{-mx}

(Refer Slide Time: 15:45)



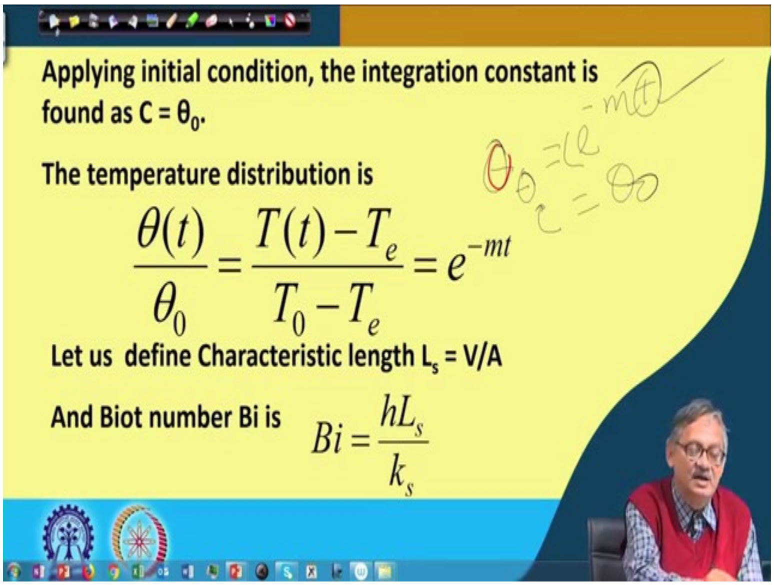
So, it is like that; θ_t is equal to Ce^{-mx}. Now, things are all clear and you can see. So, this is the solution.

(Refer Slide Time: 15:59)



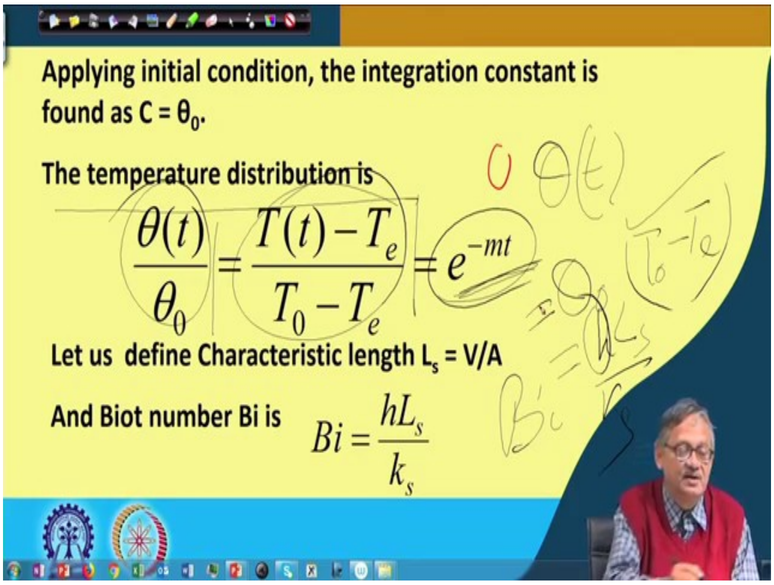
Then we can write that applying the initial condition applying the initial condition and integration of this gives that C is equal to θ_0, 'right'; C is equal to θ_0, 'right' because we had theta t is equal to Ce^{-mx}, 'right', and we said at time rather at time t is equal to 0 at time t is equal to 0, we said theta t is equal to θ_0, 'right'. So, that if we put at time t is equal to 0. So, here it was sorry not mx it was m t if we remember correctly, 'right' it was mt not mx

(Refer Slide Time: 17:26)



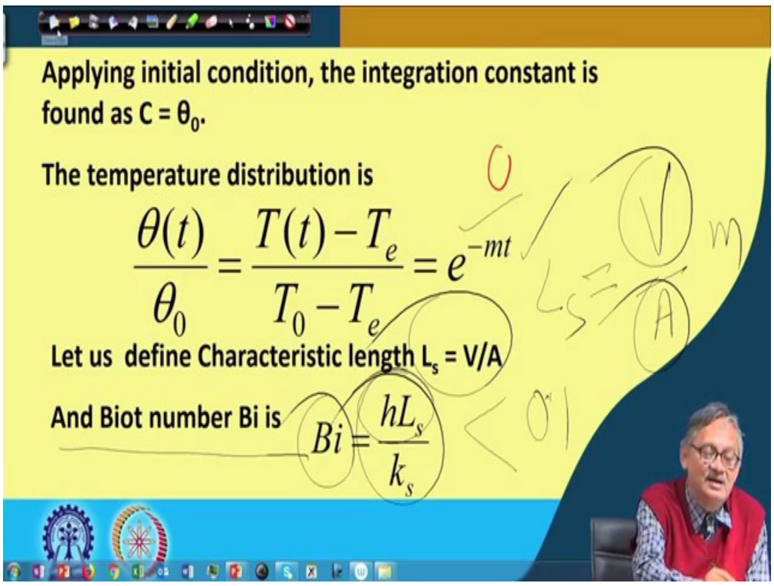
So, if it is mt then at time t is equal to 0, we can write that theta 0 theta 0 is equal to or theta t was now theta 0 and that is equal to that e to the power minus mt. So, c became equal to θ_0, 'right' because time t is equal to 0 and is θ_0. So, C became equal to it was Ce^{-mt}. So, time t is equal to 0. So, it became t theta 0 and C became equal to θ_0, 'right'.

(Refer Slide Time: 18:16)



So, if that be true then the temperature distribution is $\frac{\theta(t)}{\theta_0} = \frac{T(t) - T_e}{T_0 - T_e} = e^{-mt}$

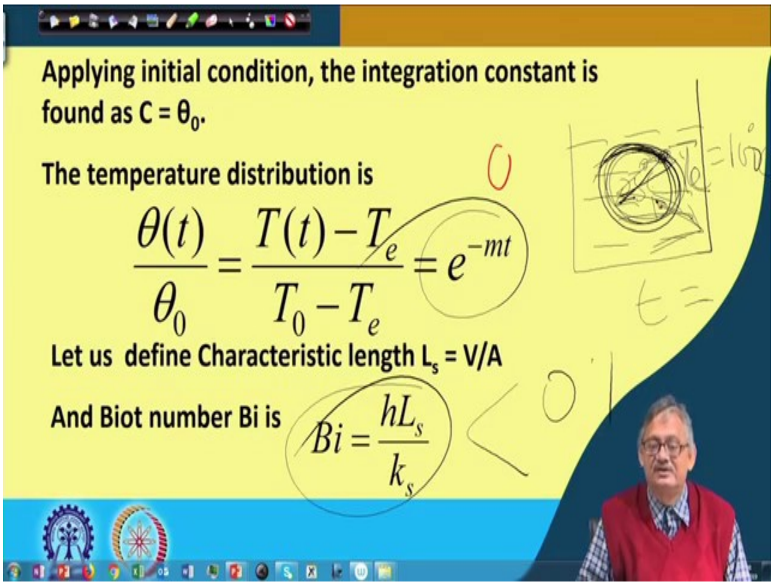
(Refer Slide Time: 19:28)



So, if this be true this is dependent on here this is valid only if that a number Biot number Bi is equal to $Bi = \frac{hL_s}{k_s}$, where h is the transfer coefficient of the environment; L_s is the characteristic length and this is defined as equal to V over A, where V is the volume of the body, A is the surface area. So, L_s is V over A and that is in meter, 'right'. So, this is called characteristics length or L_s.

So, this term that is called Biot number shortly denoted as Bi is $Bi = \frac{hL_s}{k_s}$, L_s is the characteristic length or characteristic dia dimension of the body and k_s is the thermal conductivity of the material, 'right'. So, if this $Bi = \frac{hL_s}{k_s}$ this is less than equal to 0.1, then only this solution is true means it is valid. So, unless that $\frac{hL_s}{k_s}$ this value is less than 0.1, we cannot go for this lumped system analysis.

(Refer Slide Time: 21:21)

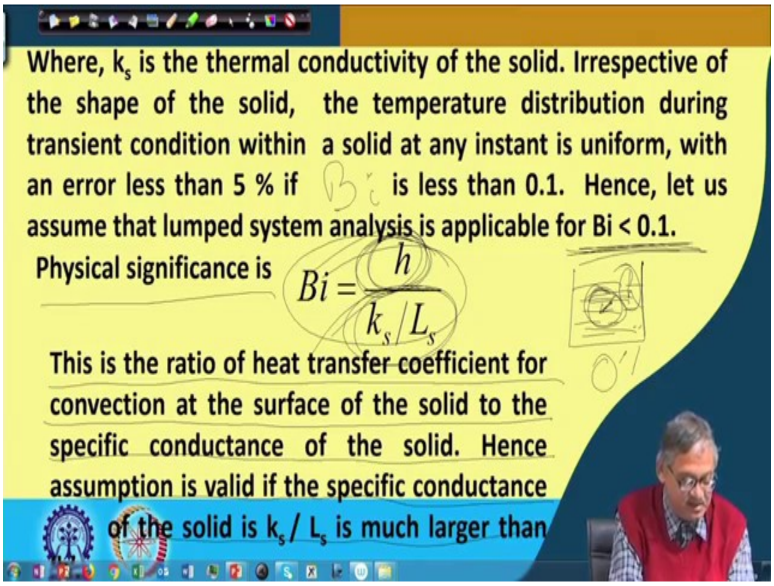


This is the very fundamental that the Biot number has to be very high which in other word is saying that is take again I gave the same example that we had bath of water under boiling condition, 'right', and we had put spherical ball in it this environmental temperature was equal to 100 degree centigrade and this initial temperature was say 25 degree centigrade.

Now, suddenly when we dipped into this solution the temperature of this ball will go on increasing and we said that it will go on increasing till the outside and inside temperature are under equilibrium or it is called under steady state. Till that we will go on and the time required for this we can find out from this equation. But, this is when the Biot number is less than 0.1 that is $\frac{hL_s}{k_s}$ is 0.1. Physically what it means that this we are of the metal if the temperature is here 100 degree that should immediately be conducted into the body. So, the body resistance is negligible. Body resistance for the thermal energy to be transported is negligible if that is happening then only it is this solution can be valid or you can apply this solution, 'right'.

So, the moment this is coming to the contact of the environmental temperature this will be conducted immediately inside, 'right'. Otherwise normally what is happening assume that your this as we are repeatedly saying this is the solution of the water boiling water 100 degree centigrade. And suppose you have one bowl of food material normally food material is not having high conductivity very low poorly conducting any food material, is generally poorly conducting. So, what will happen? This temperature we will come to the surface and there will be resistance and inside it will take long time to go into this, immediately this will not be going inside. So, if this is not happening then you cannot apply this situation. That is why as we have defined that $Bi = \frac{hL_s}{k_s}$ that has to be less than 0.1 which in subsequent thing we are saying.

(Refer Slide Time: 24:33)

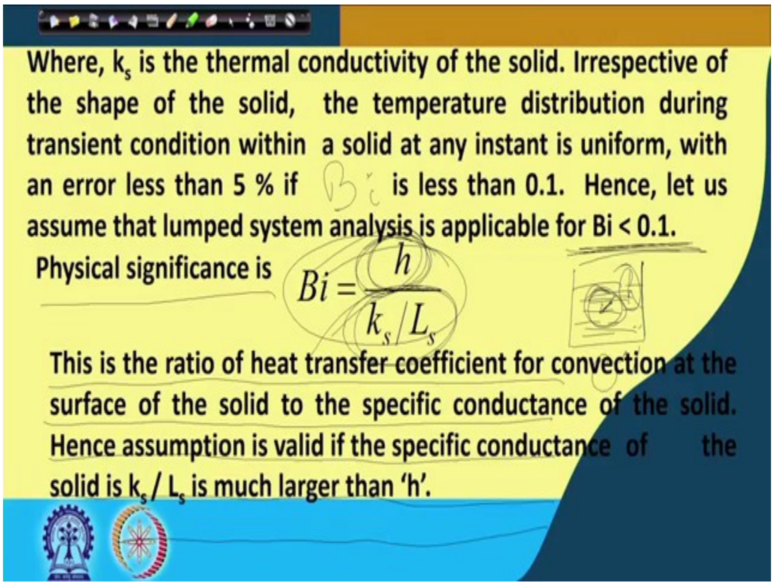


Now, where k_s is the thermal conductivity of the solid irrespective of the shape of the solid the temperature distribution during transient condition within a solid at any instant is uniform with an error less than 5 percent; if $Bi = \frac{hL_s}{k_s}$ less than 0.1, ‘right’; here it is Biot number, if Biot number is $\frac{h}{k_s/L_s}$ less than 0.1. Hence let us assume the lumped system analysis is applicable for Bi less than 0.1 only, ‘right’, and physically it is meaning that Bi is $\frac{h}{k_s/L_s}$. This means h is the convective heat transfer and k_s/L_s is the conductive heat transfer, ‘right’.

So, if you still remember that we had given this analysis or this analogy for this example that this is water and we had this one, ‘right’. So, h came from the water, ‘right’, and k_s/L_s is that conductive. So, this is a convective heat transfer and this is the conductive heat transfer, the ratio of these two, ‘right’. So, h has to be much lower than compared to k_s/L_s, then only it will be less than 0.1, ‘right’. So, that means, this is the ratio of heat transfer coefficient for convection at the surface of the solid to the specific conductance of this solid.

Hence assumption is valid if the specific conductance of the solid k_s/L_s is much larger than 0.1, ‘right’, is valid if the specific conductance of the solid that is k_s/L_s is much larger than h not 0.1 h, ‘right’. So, then only this is valid, h has come here.

(Refer Slide Time: 27:24)



So, with this let us say that we have initiated the unsteady state heat transfer and we have seen Biot number is h over k_s/L_s and the value of it has to be less than 0.1, then only we can make the lumped system analysis, 'right'.

So, today time is over. We will continue in the next class solving different problems and solutions of heat ok.

Thank you.