Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 20
Heat Transfer in Finned Surfaces

Good morning. We had been talking about the finned surfaces Heat Transfer in Finned Surface and perhaps we said that and the finned last class today we would like to also correlate the fin efficiency and some other vital parameter on which the different graphs have been plotted against fin efficiency and how those parameters came up today would like to also highlight on that. And, as we are saying repeatedly as I am saying repeatedly that please also look into your problems and solutions.

It is not that whatever problems we are solving here you only concentrate on that, but also try to solve some many other problems associated with the topics we were covering. That will help you in understanding the subject in a better way because solving problems gives you confidence, 'right'. So, we come back to heat transfer in finned surface in lecture number 20, 'right'.

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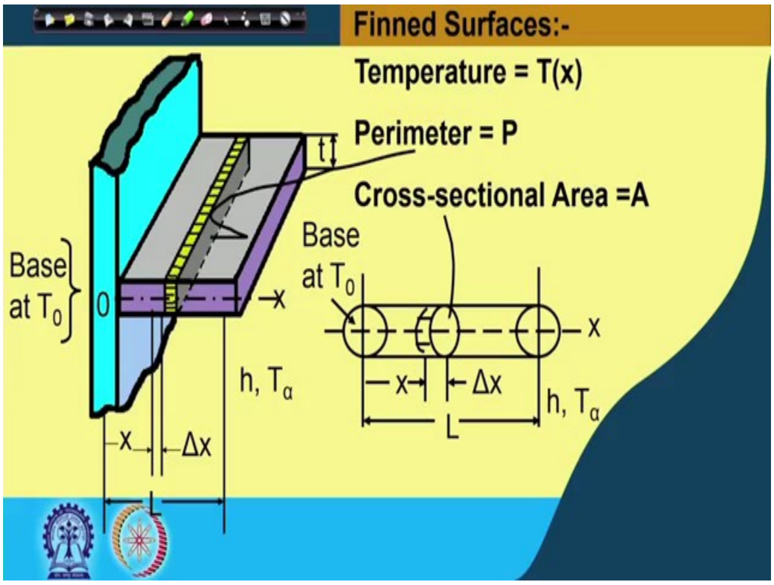

So, this was our finned surface, 'right' and as we said that there are different there are this was the fin base, 'right' and this was the fin, 'right' and this is on rectangular coordinate and this was on cylindrical coordinate, 'right'.

So, we know the sectional area A; we know the perimeter P; we know delta X or X; we know heat transfer a coefficient and we know the ambient heat transfer ambient temperature, 'right'. So, it can be T_∞ or T_0 as I said earlier that in many cases people do nominate to people do nomenclature in such a way that T_∞ is also equivalent to T_{en} environment, 'right'. So, this is the thing on which we are working, 'right'.

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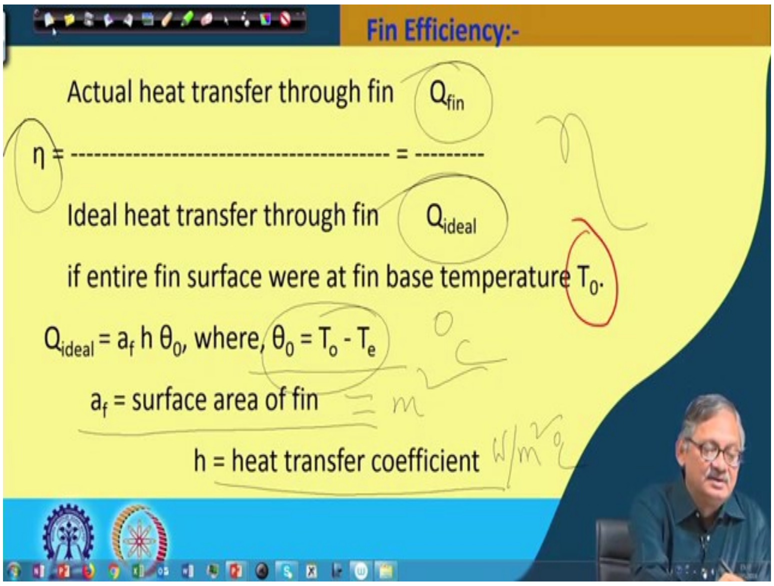

So now, let us define another important very much parameter; very much important parameter that is fin efficiency, 'right'. So, fin efficiency is defined as the actual heat transfer through fin divided by ideal heat transfer through fin if the entire fin surface were at the fin base temperature T_0, 'right'. So, entire fin surface is at the fin base temperature T_0, if that is there then the heat transfer through the fin can be assumed to be the ideal heat transfer, 'right'. So, actual by ideal that we have to find out and that is equivalent to the fin eta that is the fin efficiency.

Now, then what is then actual heat transfer and ideal heat transfer? Ideal heat transfer will be equivalent to $a_f \times h \times \theta_0$, 'right' to where θ_0 is the T_0-T_e or T_∞, whatever we call. T_0 - T_e that is the environmental temperature 'right' that is theta 0; a_f is the surface area of the fin and h is the heat transfer coefficient. Now, all are in SI units. So, we are not separately letting you know what is the SI unit of h? So, it is, we know, W/(m^{2o}C), 'right', and surface area is in m^{2}n 'right'. And like this delta I mean θ_0 is in ºC, 'right'.

Now, this fin efficiency is obviously, that you are referring as Q fin over Q ideal, 'right'. So, whatever fin has actually transferred heat through the fin that is actual heat transfer through the fin over the ideal heat transfer through the fin if the entire fin would have been at the fin based temperature of T_0, 'right'. So, if this is there then and we said that Q ideal is equivalent to $a_f \times h \times \theta_0$.

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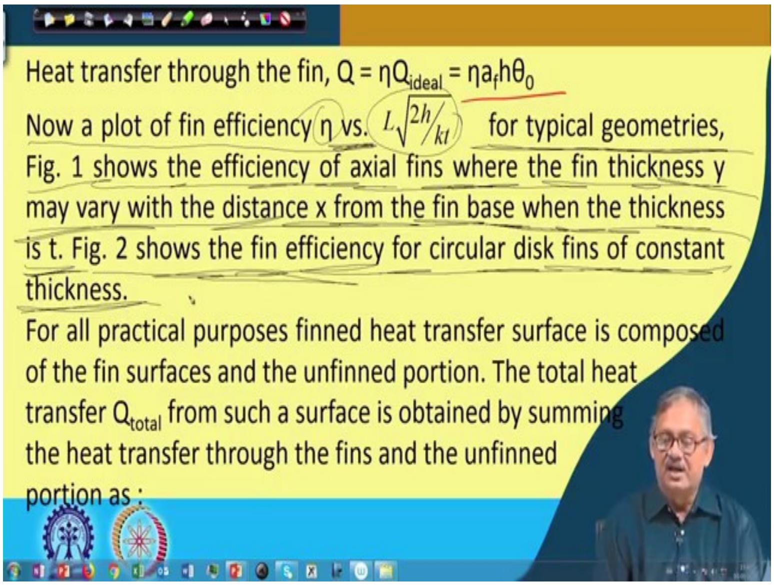

So, then what is the fin real that is actual? What is the fin actual? Fin actual will be like this that the heat transfer through the fin is $Q= \eta Q_{ideal}$, 'right'; $\eta \times Q_{ideal}$ that is fin efficiency into Q_{ideal} that is what it is because here we said that Q_{fin} here we said that Q_{fin} this is equal to eta times Q_{ideal}, 'right'. That is the Q_{fin} actual. So, if we can make Q_{ideal} then we can also frame what is the Q actual or real heat transfer 'right'. So, this eta Q_{ideal} is $\eta \times a_f \times h \times \theta_0$, 'right'.

So, if we have a plot of fin efficiency η versus $L\sqrt{(2h/kt)}$, L times under root 2h by kt. If we have a plot of this, 'right' for typical geometries, 'right' this is also geometry dependent like we will show it in a figure say 1, that shows the fin efficiency of actual fins where the fin thickness y may vary with the distance x from the fin base when the thickness is T, 'right'.

Similarly, another figure that will show the fin efficiency for the circular disk fins of constant thickness, 'right'. So, this, if we look at the first one is eta versus $L\sqrt{(2h/kt)}$ for different fins, 'right'.

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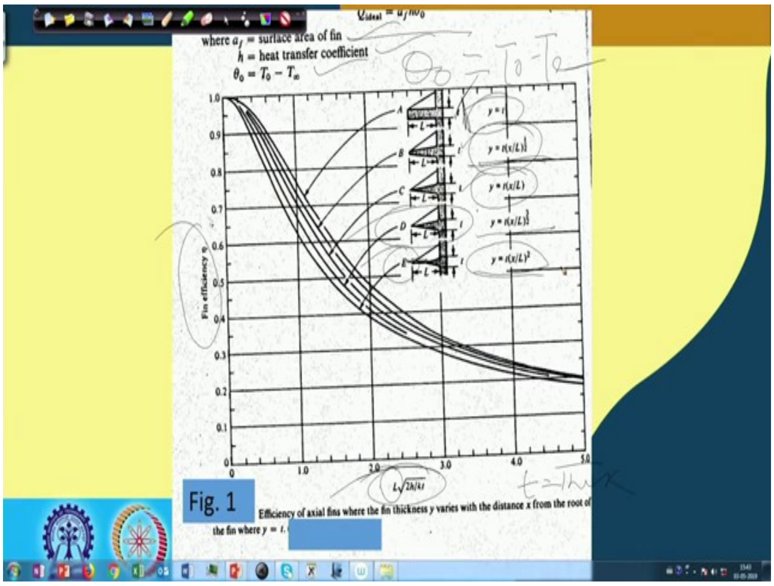

That you see in this figure we are plotting L√(2h/kt), 'right' we have said that t is the thickness, 'right', t is the thickness; h is heat transfer coefficient; k is the conductivity of the fin material, 'right' and L obviously, that is the length of the fin, 'right'. So, here this versus fin efficiency that Q ideal which we have said is equal to $a_f \times h \times \theta_0$. A_f is the surface area of the fin; h is the heat transfer coefficient and θ_0 is T_0-T_∞, 'right' or T_e whatever we name it, 'right'. It can be θ_0 equal to T_0 -T_e or T_∞ whatever, 'right'.

So, for different geometries we said there. So, here y is equal to t, 'right' this is the t, 'right' and this y is equal to t. Here y is a function of t as t into x by L to the power half, 'right'. For this is 'A' geometry; this is 'B' geometry; for 'C' geometry you see it is more sharp at the end (Pl. refer to the above figure). This is y is a function of t, but it is equal to t x by L 'right' and for the 'D' this is another type of fin where it is y is equal to t into x by L to the power 3 by 2. And for 'E', 'E' is another type of where you see it is absolutely a long portion is very much sharpened, 'right' or pointed 'right'. So, where y is equal to t x L x by L to the power 2, 'right'.

If these are different fin types then we have plotted this fin efficiency versus L under root 2h / kt 'right' where efficiency of axial fins where the fin thickness y varies with the distance x from the root of the fin where y is equal to t, 'right'. So, this y is equal to t, that is this thickness, 'right' and if we see the next one this figure says like that.

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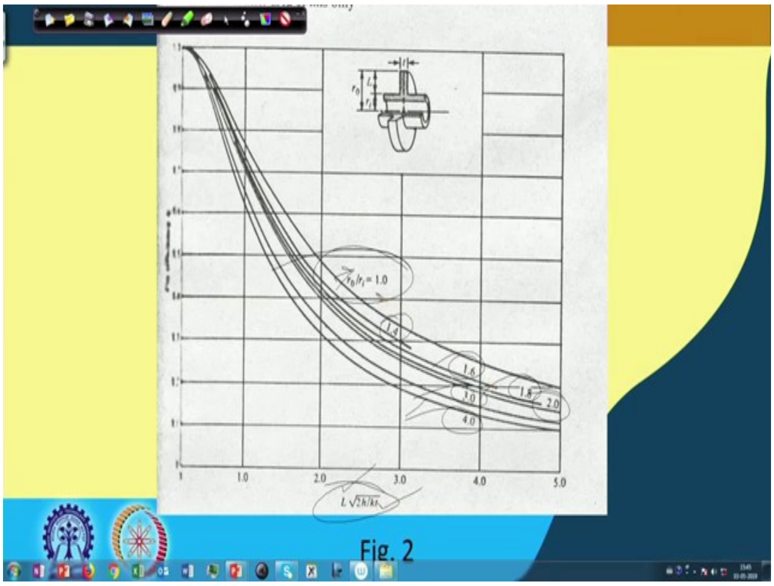

If we look at the next figure that is figure 2 which we referred to earlier this is that for the circular fins, 'right'. Here also you see that the L under root 2h / kt, 'right' this is plotted against the fin efficiency, 'right'. For different r_o/ r_i 'right'. So, this first one is r_o/ r_i 1 and this last one is r_o/r_i is 4 in between 1.4, 1.6, 1.8, 2.0, 3.0 etcetera are also there, but the thing is that what we need to know that what are the different parameters like L, like t and here r_o and r_i, 'right'.

This to understand these things we need to see this picture, 'right' where you see r_o is the outer radius, this is the r_o, 'right' and r_i is the inner radius that is this one, 'right'. So, r_o and r_i are known and L is this, 'right', L is this that is this is the L, 'right'. So, if L is this, 'right' r_i is this one and r_o is this one, 'right' L is this and t is the thickness which one is this one, 'right'. So, r_o, r_i is known, t here is known, k is already k and h are different k is the conductivity of the material h is the transfer coefficient and L that we have shown this to be like this, 'right' L to be like this.

So, once you know then you can find out by knowing this parameter for different r_o over r_i you can find out the fin efficiency for circular fins which is like this, 'right'. You remember the other day, I said that putting something like that on which we call 2 d per hour. So, that kind of thing when you are making a shrink fitting and that shrink fitting is happening here for circular fins like that. Whereas, the other one which we showed there it is not so much where the contact resistance thermal contact resistance could be factor if not properly done like this one, ‘right’.

Here you see on this base you have put the fin in such a way that this is the thickness this is the thickness and this all are t, ‘right’ this all are t’s and this is the L, ‘right’ from this to that; from this to that; from this to that; from this to that and from this to that.

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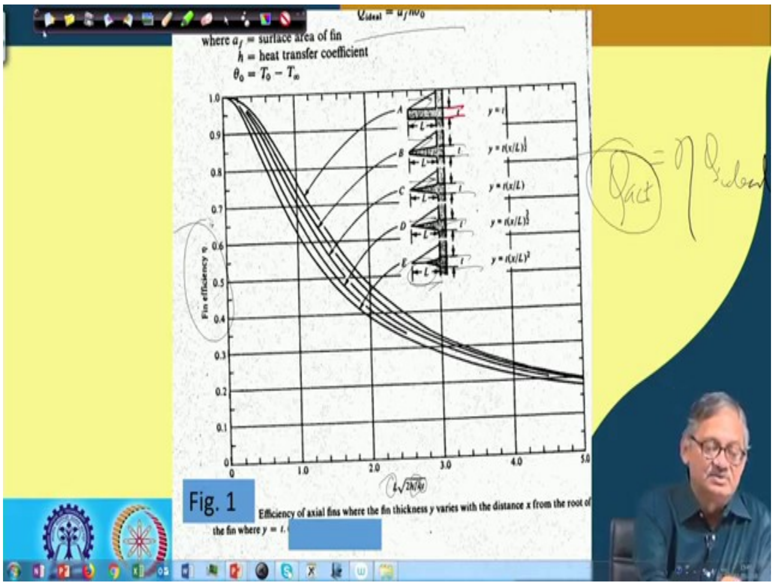

So, you know t, you know L and h and k are given. So, you can find out fin efficiency and if you know the fin efficiency then if you know the Q ideal which is a f h theta 0 then you can find out Q_{real} or Q_{actual} as Q_{actual} is equal to $\eta \times Q_{ideal}$, ‘right’. So, this way you can find out what is the Q for the actual already you have found out Q for ideal cases, ‘right’.

So, let us look into that where we where we left. So, there if we go that fin we were here, ‘right’. So, for from the plots of η versus $L\sqrt{2h/kt}$ and for different geometries we can find out the values of eta for different geometries or for the for the circular fins also, ‘right’; circular this type fins also where the thickness was constant, ‘right’.

For all practical purposes the fin heat transfer surface is composed of the fin surfaces and the un finned portion, ‘right’. So, obviously, if we if you remember the previous graph which we had shown that fin surfaces where this was like this let us say in one case this was like this. So, and if this is the fin then the surface area of this plus un-finned portion.

So, un-finned plus fin portion is the total surface area and the total heat transfer Q total from such a surface is obtained by summing the heat transfer through the fins and the on fin portions, 'right' and this can be done by summing finned and un finned portion as it is shown like this.

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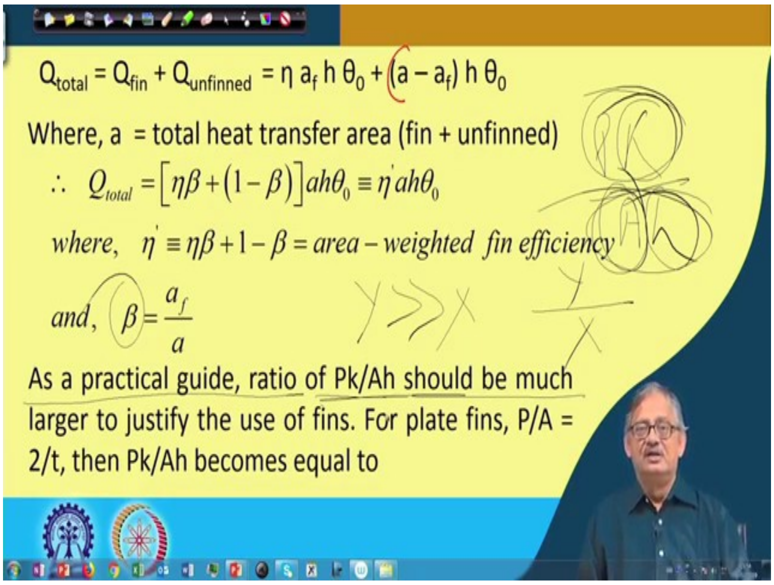

$Q_{total} = Q_{fin} + Q_{unfinned} = \eta\, a_f\, h\, \theta_0 + (a - a_f)\, h\, \theta_0$; obviously, a_f is the finned area where, fin is there and 'a' is the total heat transfer area that is fin plus un finned, 'right'. So, from the total heat transfer area finned and un-finned if we subtract the finned area then we get the un-finned area which is there, 'right'. So, if we know that total surface area where the finned or un-finned together then.

$$\therefore\ Q_{total} = \left[\eta\beta + (1-\beta)\right] ah\theta_0 \equiv \eta' ah\theta_0$$

$$where,\ \ \eta' \equiv \eta\beta + 1 - \beta = area - weighted\ \ fin\ efficiency$$

$$and,\ \ \beta = \frac{a_f}{a}$$

So, as a practical guide ratio of Pk by should be much larger to justify the use of fins that is Pk / ah, you also see that for a given area and for a given material, k is known and for the given system, h is known. So, if area times h, 'right' if that product is X and the perimeter times conductivity, if that is Y then Y has to be much greater than X then only you can justify adding fin.

Otherwise, if you are adding finned for no reason then the additional cost involved in it that will not be justified with respect to heat transfer. If heat transfer is not improved then what is the point of what is the point of extending the surface area because that is also cost involved.

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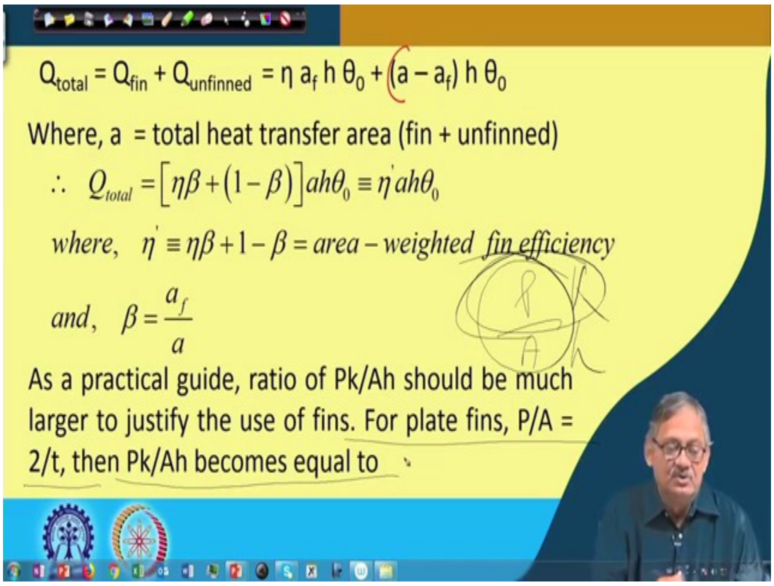

So, in that way if we look at that Pk / ah, this value should be much larger and to become it much larger for a given P, for a given ‘a’ or Pk / ah, ‘right’ P is the perimeter, ‘a’ is the area for a given P and for a given h, if k is becoming higher, the higher the value of k the more is the value of Pk / ah, ‘right’. So, more it will be justified or fine you can you can say that now adding fin is helpful for the heat transfer, ‘right’.

So, for plate fins P/A is nothing, but 2/t that is there.

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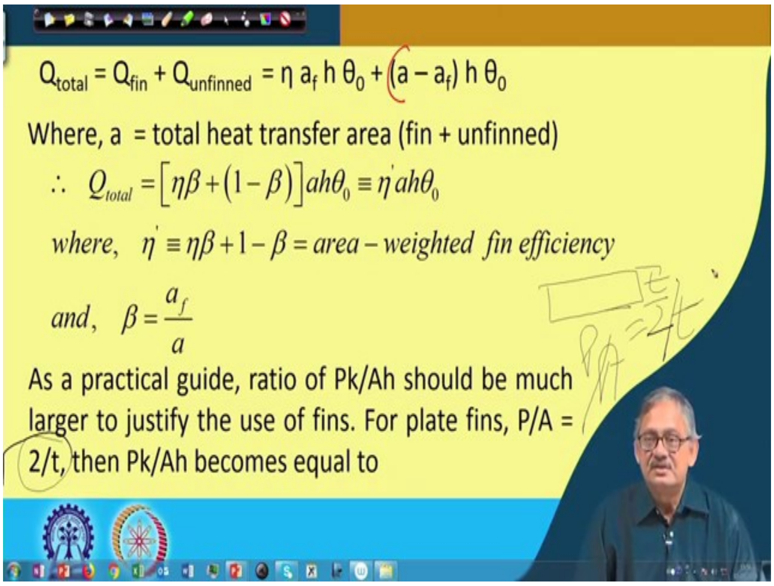

So, if the fins are like that plate then it is that thickness if there this is the thickness then P/a that is perimeter work the area that is becomes equal to 2/t,'right'.

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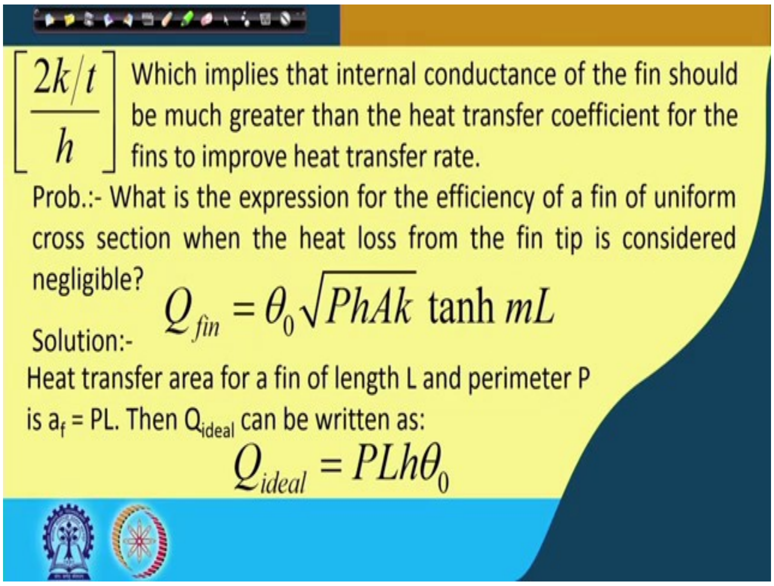

So, this if it is so, then we can find out the value of Pk by as 2k over t over h, 'right'. So, because P by A has been equivalent to 2 by t; so, Pk is $\left[\frac{2k/t}{h}\right]$ 'right' which implies that internal conductance of the fin should be much greater than the heat transfer coefficient for the fins to improve heat transfer rate. Internal conductance of the fin should be much greater than that is this internal conductance of the fin through you should be much greater than the external heat transfer coefficient, 'right'.

So, with this let us solve a very simple form problem that what is the expression for the efficiency of fin of uniform cross section when the heat loss from the fin tip is considered negligible? 'right', we have already seen that $Q_{fin} = \theta_0\sqrt{PhAk}\tanh mL$ that was total Q to the fin. And, and this can be solved that heat transfer area for fin of length L and perimeter P is a_f equal to PL and Q_{ideal} that can be said to be equal to $Q_{ideal} = PLh\theta_0$

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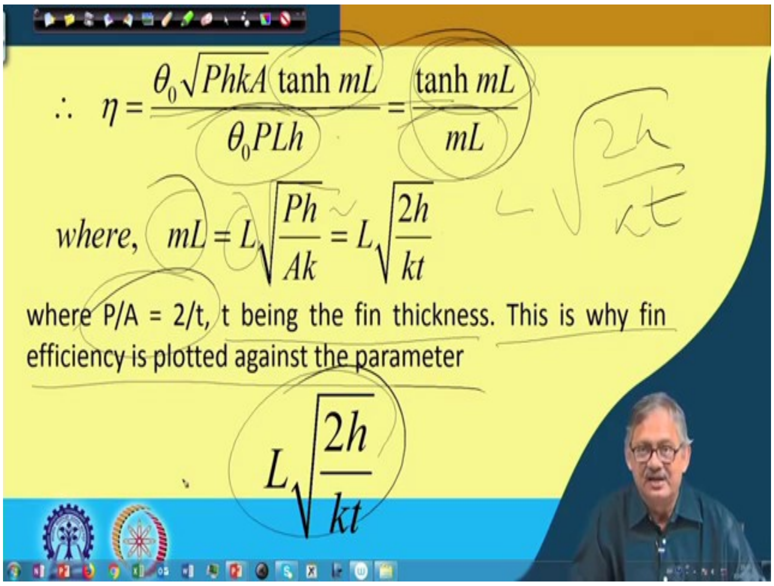

$$\therefore \quad \eta = \frac{\theta_0\sqrt{PhkA}\tanh mL}{\theta_0 PLh} = \frac{\tanh mL}{mL}$$

$$where, \quad mL = L\sqrt{\frac{Ph}{Ak}} = L\sqrt{\frac{2h}{kt}}$$

where, P/A = 2/t, t being the fin thickness. This is why fin efficiency is plotted against the parameter $L\sqrt{2h/kt}$.

So, this we said in the beginning that why this L under root 2h by k or 2h by kt or Ph by Ak. So, ultimately to it was 2h by kt. Why it is plotted against efficiency is because of this, 'right' that efficiency can be ultimately termed in terms of tan hyperbolic mL over mL where mL is nothing, but L times P and L times √(Ph/Ak) that is $L\sqrt{2h/kt}$

So, this is why we have plotted subsequently this which we have shown you, 'right' this one that $L\sqrt{2h/kt}$ and this is fin efficiency and the next one was also $L\sqrt{2h/kt}$ that the against fin efficiency, 'right'. So, with this let us conclude the heat transfer through fins, 'right' and please do some problems as and when you can.

Thank you.