Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 18
Finned Surfaces (Contd.)

Good morning. So, we are dealing with that Finned Surface, 'right', the analysis of finned surface heat transfer. So, here we have this is on 18th class finned surface continuation, 'right'.

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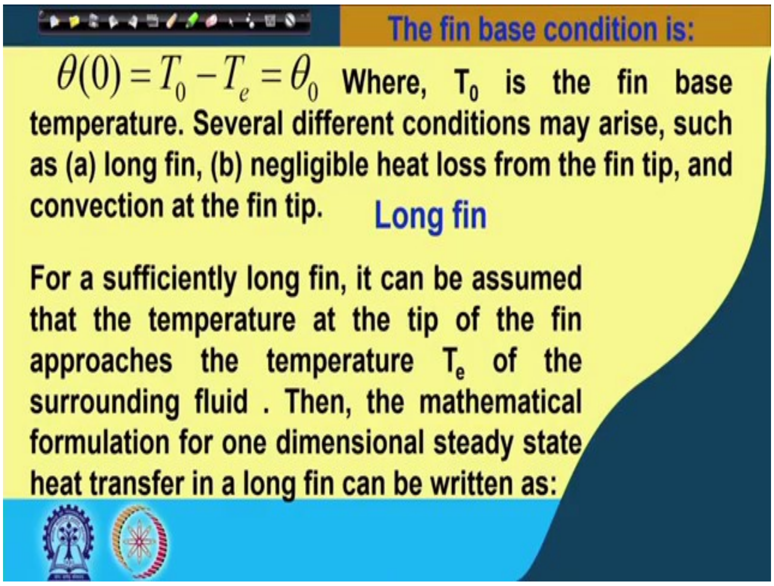

So, we go now, to that long fin we had said that the base temperature must be known, 'right'. So, that we had defined to be $\theta_0 = T_0 - T_e$ and that is θ_0, 'right' theta at x is equal to 0, 'right'. So, we said that to find out these constants that is C_1 and C_2 we need to know boundaries to out of which one conventionally or generally it is taken as the fin base temperature to be known.

And thus to find out the second case that second boundary there could be three cases appearing. One it can be a long fin or second it can be a; it can be negligible heat loss from the tip of the fin and third one is maybe the convective boundary at the fin tip, 'right'. So, these three cases may arise and let us look at one by one into them, 'right'.

So, long fin for that we can say that for a sufficiently long fin, sufficiently long fin means there are step like this, 'right'. So, if this is that material through which it is flowing and if this is the fin which is attached to it, 'right'. So, then this can be considered to be a sufficiently long fin, 'right'. So, similar to that when the fin is sufficiently long then we can say it can be assumed that the temperature at the tip, the fin tip was this is the fin tip, 'right'.

So, this is the fin base where my this hand is holding it, this is the fin base and this is the fin tip, 'right'. So, we can assume that the temperature at the fin tip of the fin approaches the temperature of the environment that is T_e, 'right'. So, this surrounding temperature T_e is assumed by the fin tip because, it is a long fin why the mechanism how it can be? So, we have this and we are attaching this one, 'right' and the heat transfer is associated with this one only, 'right' and this is the environment which is at T infinity, 'right'. So, suppose for our main thing is this body where the heat is being transferred, 'right'.

We have added this fin so, that the heat transfer is enhanced, 'right', now environment is here. So, by the time any heat flowing through this and coming to the tip or the reverse this fin is going and making it. So, the temperature which is associated at the tip that will not be affected much by the base or by the basic where the fin is attached to, 'right'. That is why it can be said that the fin tip may attain the temperature of the environment easily. So, that is the assumption for the long tip and this is how we have explained, 'right'.

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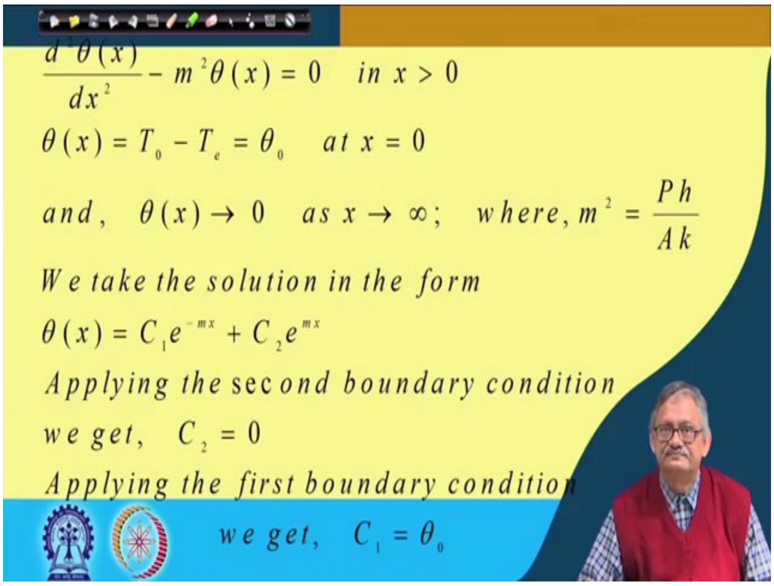

So, the mathematical formulation for this one dimensional steady state heat transfer in a long fin this can be written as; $$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0$$ This was our governing equation and valid for x greater than 0, 'right' and the boundary condition first which we have assumed that theta x is equal to T_0 -T_∞ there or T is T_e rather we should say infinity normally we denote. But, here it is give you we are giving T_e that is environmental temperature, 'right'.

In many cases you may see that the environmental temperature is denoted as T infinity also; however, since here we are precisely depending it to be T_e let us stick to that this is T_e. So, θ_x is T_0 -T_e at x is equal to 0 and is equal to θ_0, 'right' and third one is θ_x, when it is approaching a 0 as x tends to infinity, 'right'. Now, as theta x this was the first one x equal to 0, second is theta x tends to 0 as x tends to infinity because, if this x is tending to long fin that is infinity, 'right'. So, in that case this becomes equal to theta x becomes equal to 0, 'right'. So, T_x also becomes equal to T_0 and this becomes equal it is also tending towards 0, 'right'. So, this is the second boundary second condition, 'right'.

Because, this is T infinity not Tx minus T0 Tx minus Te so, this also is approaching Te. So, Te minus Te so, that becomes equal to 0, 'right'. So, that is how we are making the second boundary that theta x is tending towards 0 as x is tending towards infinity, 'right' and m is or m square is Ph by Ak, 'right'. So, this by definition we had given earlier Ph by Ak where we are said the unit of m is meter inverse, we have shown how it has come as meter inverse, 'right'.

Now, we take the solution in the form that because we had said earlier that this can have several types of solutions. It can be exponential, it can be hyperbolic any kind, 'right' though according to your suit, according to your requirement you can choose which type of solution you are asked looking for. So, here in this case let us take the solution in the form of exponential and then theta x becomes equal to $C_1e^{-mx} + C_2e^{mx}$. This is the solution where we have the constants C_1 and C_2 to be found out, 'right'.

So, we can apply this boundary that the second boundary, if we apply we get here that which said x tends to infinity theta x tends to 0, 'right'. So, this theta x tends to 0 as x tends to infinity then C_2 becomes equal to 0, 'right'. So, C_2 becomes equal to 0 and applying the first boundary condition, first boundary was that theta x is equal to theta 0 at x is equal to 0. So, this becomes theta 0 as x becomes 0 so, this C_1 becomes equal to theta 0, 'right'.

So, we got C_1 to be theta 0 and C_2 to be 0. So, substituting this into the general solution which we got this that theta x is equal to $C_1e^{-mx} + C_2e^{mx}$. So, C1 has become equal to to θ_0 and C_2 has become 0. So, we can write theta x equal to $\theta_0\times e^{-mx} + C_2$. So, the solution has become theta 0 theta x is equal to $\theta_0\times e^{-mx}$.

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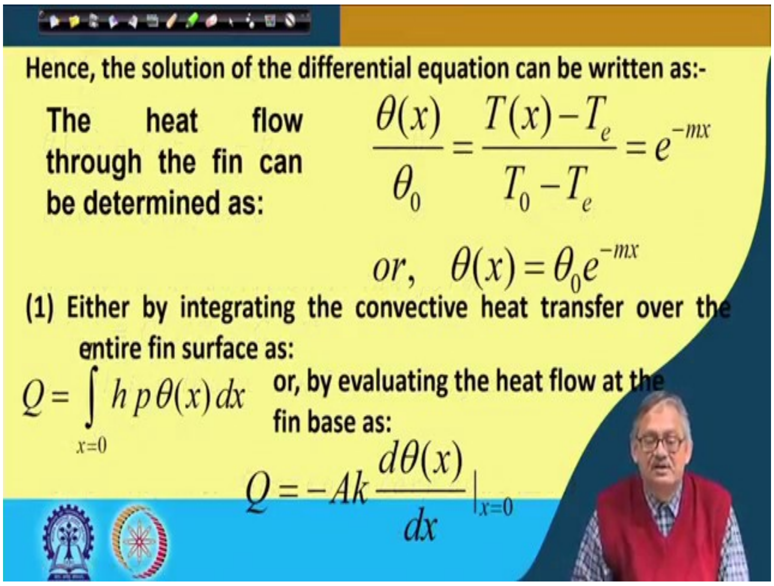

Hence, the solution of the differential equation can be written in the form that the heat flow through the fin can be determined,

$$\frac{\theta(x)}{\theta_0} = \frac{T(x)-T_e}{T_0-T_e} = e^{-mx}$$

$$or, \quad \theta(x) = \theta_0 e^{-mx}$$

'right', this is one solution, 'right'. So, second either by integrating the convective heat transfer over the entire fin surface as Q is equal to

$$Q = \int_{x=0}^{L} h\,p\,\theta(x)\,dx$$

evaluating heat transfer at the fin base that is

$$Q = -Ak\frac{d\theta(x)}{dx}|_{x=0}$$

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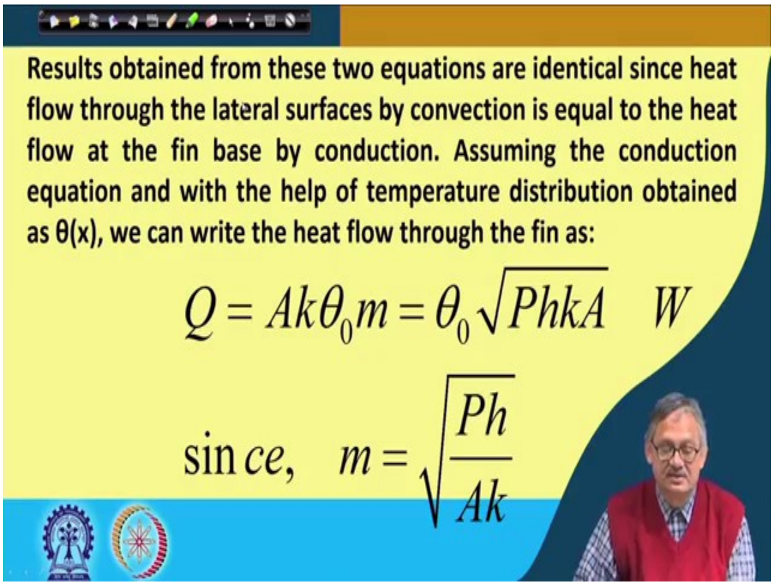

And solving this the value of Q comes to equal to this that Q is equal to the result obtained from these two equations are identical since, heat flow through the internal surfaces by convection is equal to the heat flow at the fin base by conduction. Assuming the conduction equation and with the help of the temperature distribution obtained as theta x we can write the heat flow as Q is equal to

$$Q = Ak\theta_0 m = \theta_0\sqrt{PhkA} \quad W$$

$$\sin ce, \quad m = \sqrt{\frac{Ph}{Ak}}$$

this Ak when is going inside then it is on the top Ak square k square. So, this Ak and this square goes out and then it becomes under root hP Ak, ‘right’ that is what we have written that instead of Ak m under root hP Ak. So, the Q has become equal to

$$\theta_0\sqrt{PhkA}$$

This is the cube, ‘right’, since m was under root Ph/Ak. This is how we have found out that for a long fin the temperature distribution within the body, within the fin it was theta 0 theta x is equal to $\theta_0 \times e\text{-}^{mx}$, that was one solution.

This was the temperature distribution within the fin and the heat flow we have taken that Q is becoming equal to theta 0 under root PhkA so much watt, ‘right’. So, this is for long fin, ‘right’, now let us look into the other one that is if the fin at the tip as negligible fin, I mean heat loss, ‘right’.

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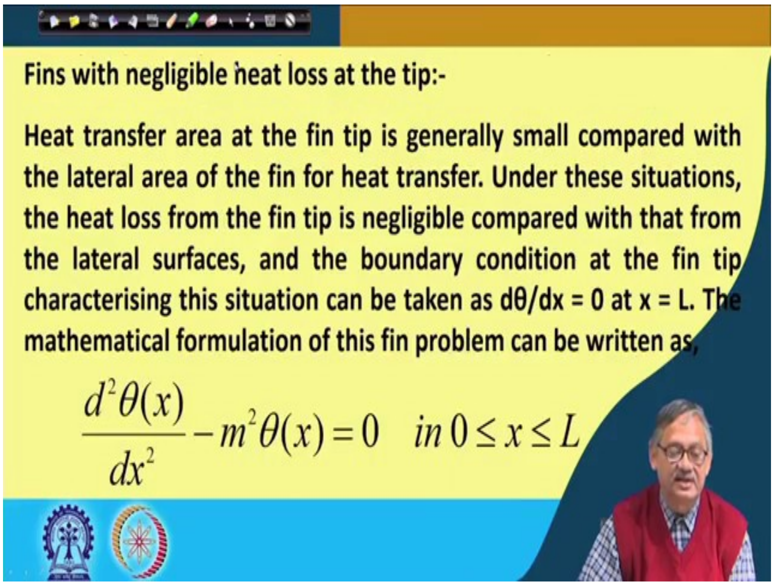

So, if we take the second thing that fins with negligible heat loss at the fin tip; here heat transfer area at the fin tip is generally small compared with the lateral area that is what we had shown repeatedly. So, this was our base and this was our fin. So, this lateral area, which we have compared to that of the fin tip area, is very small negligible, 'right'. So, heat transfer area at the fin tip is generally small compared with the lateral area of the fin for heat transfer, 'right'. So, under these situations the heat loss from the fin tip is negligible compared with that from the heat loss from the fin tip from the lateral surfaces, 'right'.

And the boundary condition at the fin tip characterizing this situation this can be written as $d\theta/dx = 0$ at $x = L$, 'right'. The mathematical formulation of this fin problem this can be written as $\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad in\ 0 \le x \le L$, that is 0 less than equal to x less than equal to L. This is valid this governing equation is valid between 0 to L of x. So, $\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0$ is the governing equation. Now, we need the boundary conditions, 'right'.

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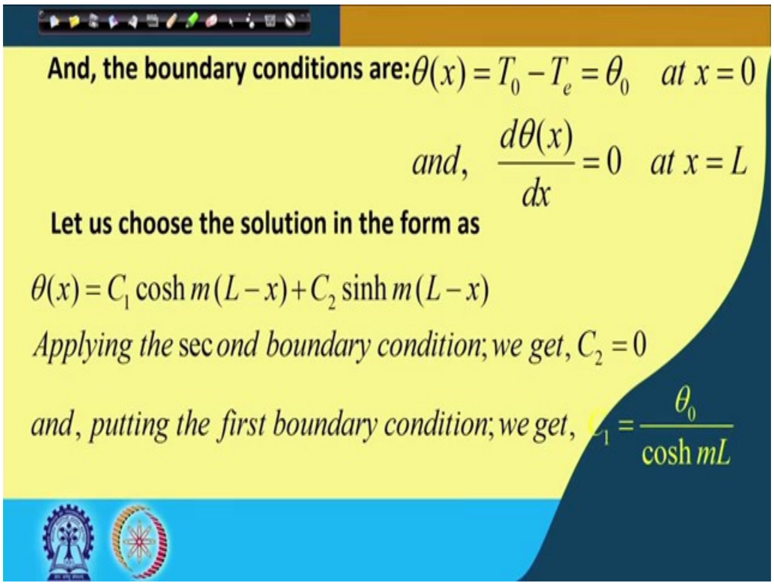

So, to have the boundary we can write that the boundary conditions are that

$$\theta(x) = T_0 - T_e = \theta_0 \quad at\ x = 0$$

$$and, \quad \frac{d\theta(x)}{dx} = 0 \quad at\ x = L$$

And theta x what we had defined? Theta x we had defined as T_x- T_e, 'right'. So that means, that at x goes to 0 change with x there is no change in the θ, 'right' dθ/dx = 0.

If this be true then the solution we can take for this is $\theta(x) = C_1 \cosh m(L-x) + C_2 \sinh m(L-x)$

And if we applied the second boundary condition, the what is the second boundary? That is at x is equal to L dθ(x)/dx is equal to 0 then we get C2 is equal to 0 from here, 'right' this is the second boundary. And, putting the first boundary we get C1 is equal to θ_0 /cosh mL that is at x goes to 0 theta x is theta 0 from that. So, by substituting into this equation we get C1 is equal to θ_0 /cosh mL, 'right'.

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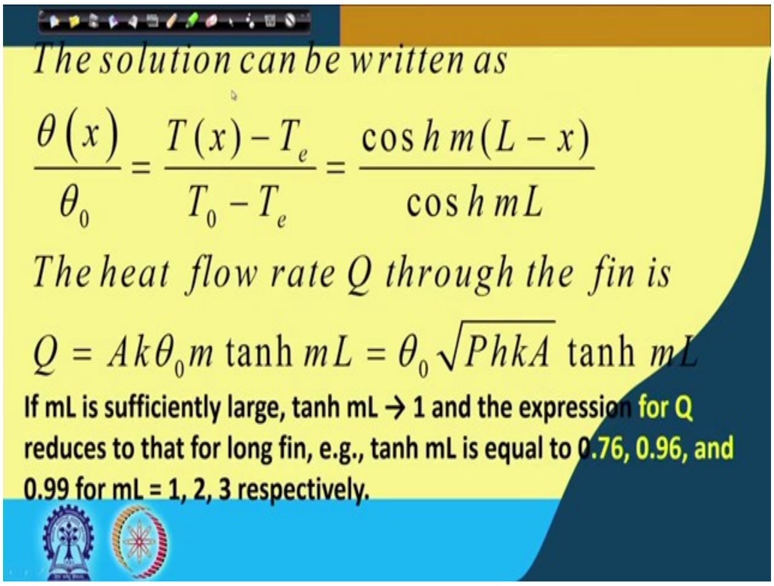

Then the solution then comes to be equal to so, the solution can be written in the form. $\frac{\theta(x)}{\theta_0} = \frac{T(x)-T_e}{T_0-T_e} = \frac{\cos h\, m(L-x)}{\cos h\, mL}$ So, our solution for the case where it is fin at the tip negligible heat loss, for that we have taken the solution to be in the form of hyperbolic solution, 'right'. So, from there the we have found out the constants with the boundary conditions boundary 1 and boundary 2, boundary 1 was theta is theta 0 at x is equal to 0.

And second boundary was d theta dx is equal to 0 at x is equal to L, 'right', from there we found out and got that C_2 became equal to 0 and C1 became equal to C_1 become equal to cos hyperbolic; let me check that C_1 became equal to θ_0 /cosh mL, 'right'. So, if this be true then we can write that the solution of this is $\frac{T(x)-T_e}{T_0-T_e}$ this is equal to the heat flow rate Q through the fin that can be written as to $\frac{\cos h\, m(L-x)}{\cos h\, mL}$

$$Ak\theta_0 m \tanh mL = \theta_0 \sqrt{PhkA} \tanh mL$$

because, we have seen Ak times m, earlier we have seen Ak times m this is equal to tan hyperbolic Ph by a PhkA, 'right'. So, that we have shown also how it has become that Ak times m is equal to under root PhkA. So, Q has become equal to theta 0 under root PhkA tan hyperbolic mL, 'right'. Now, if this is the limiting condition because if it satisfies; that means, our solution is also correct. What is the limiting condition? If mL is sufficiently large, now what does it mean if mL is sufficiently large? That means, product of m and L, 'right' so, product of m and L. And then what is m? M was under root Ph over kA, 'right'.

So, generally this Ph over kA is small; so, this means that if L is going large, 'right'. So, if mL is sufficiently large; that means, tanh mL becomes equal to; that means, tanh mL that becomes very close to 1, 'right'. So, tan hyperbolic mL this becomes very close to 1 and this is meaning that the expression for Q which we have written here as theta 0 Ph under root PhkA tan hyperbolic mL this expression reduces to that for the long fin.

So, if you remember for long fin because tan hyperbolic m L that becomes a 1. So, theta 0 so, Q has become theta 0 under root PhkA which was the solution for Q for long fin, 'right'. So, this means that for long fin whatever Q we are getting and if the m L value is sufficiently high large then the Q also for the fin with negligible heat loss at the fin tip that is also becoming same, 'right'. And tan hyperbolic m L is equal to say 0.76, 0.96 and 0.99 for m L is equal to 1 or 2 or 3.

So, if mL value is 0.76 then this tan hyperbolic mL this a tan hyperbolic mL equal to 0.76 is the mL value is 1, it becomes tan hyperbolic mL becomes 0.96 if mL is 2 or tan hyperbolic mL becomes almost 0.99 or close to 1 if it is 3. So, beyond that it is always equal to 1, almost it can be taken like 0.9999 as many, 'right'. So, this says that we have come to the solution where that the fin with long fin and the fin with negligible heat loss at the surface, so that a fin tip, sorry, at the fin tip that has the same Q, 'right'.

This is a limiting condition which since, it has become same it has become identical with the long fin we can say the solution which we have made is correct. And the solution has been cross checked with the value of Q for both long fin and fin with negligible heat loss at the fin tip. Because, if the value of mL is becoming high, or large, so that it is converting tanh mL, very close to 1, 'right'. So, the value of Q that became equal to theta 0 under root PhkA which was also true for the long fin, 'right'.

So, with this let us stop today because our time is now over and we will have the third one that is the convective boundary, 'right', that convective boundary solution that will have still remaining for the fin or fin analysis, 'right'. But, be sure that you are also doing some problem at home so, that you can come to know and you get solving the problems gives you some idea about the numerals.

So, that is very much required. So, I hope you will do it and if we can do it here fine otherwise if the time is also limited then we will ask you to do at home. And if there is any doubt anywhere you may contact us as in the portal we have said ok.

Thank you.