Thermal Operations in Food Process Engineering: Theory and Applications
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Lecture - 11
One Dimensional Heat Transfer through Cylinders

Good afternoon. We have done the one-dimensional heat transfer in a Cartesian coordinate x, y, z. So, today we will do One-Dimensional Heat Transfer through Cylinders. You remember that we had said earlier that whenever we started with the generalized equation which was independent of the dimensions, 'right' x, y, z or r theta q, r theta z or r pi theta whatever it be means with a cylindrical or spherical or Cartesian whatever be the coordinate system we had developed the generalized equation remember.

So, will also utilize that generalized equation as and when it is required, 'right'. So, here we are saying that let us take cylindrical coordinate where for energy generation in the solid at a rate of say E at any r in W/m^3 with a constant thermal conductivity k we have to find out the temperature distribution T within any r at any r, T r in the solid by conduction and this we can start with from the generalized equation that $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{1}{k}E(r)=0$ Why it is 0? Because we have said that this is the steady state conduction since, it is a steady state conduction we can say that, the right side is 0, 'right'.

And, the left side so far I recall that it was $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{1}{k}E(r)$ and in the right side

we had $\frac{1}{\alpha}\frac{\partial T}{\partial t}$ means temperature and time, numerator temperature, denominator time,

'right'. So, $\frac{\partial T}{\partial t}$ at any (r,t) it was o, general equation or from there since it is given

steady state and, there is internal energy generation as well the conductivity is constant.

So, if it would have been r is equal to x is equal to 0, then it would have been in the x Cartesian coordinate for cylindrical coordinate if you remember you said r to the power n that n is equal to 1, then it is cylindrical coordinate. So, that corresponds to 1 by r delete r of rdT/dr +E_r/k =0, 'right'.

Now, the radial heat that is heat flux the radial heat flux is what that we can say. So, if it is the cylinder. So, whatever heat flux is going heat is being transferred in this, 'right'. The other two-dimensions this and the third dimension this are infinite, 'right'. So, these two dimensions are infinite or much larger than this. So, we can say that one dimensional heat transfer is occurring. So, the radial heat flux which is occurring is anywhere in the solid or in this cylindrical coordinate, 'right' in the cylinder anywhere in this cylinder we can write that $q(r) = -k\frac{dT(r)}{dr} \quad W/m^3$ This is square watt per meter square; 'right' that is q_r or flux.

So, heat flux radial heat flux we can write that q(r) anywhere in the solid that can be written, 'right' from the definition of the heat flux called q is equal to minus k dT da 'right'. So, dT/dx it was for Cartesian coordinates, so, in cylindrical coordinate is dT/dr. So, -k dT/dr at any position r so much watt per meter square, 'right'. So, let us write again watt per meter square. So, this was by mistake cube means perhaps here it yes this is watt per meter cube. So, perhaps that cut and paste made that mistake. However, so, we have corrected watt per meter square.

Now, this is the general equation for conduction in radial coordinate or in cylindrical coordinate there should be the, 'right' term to use and when we have this equation we now need to solve it, 'right'. So, solution of this can be obviously done, from differential equations. So, on two successive integrations, it will give a solution, 'right'.

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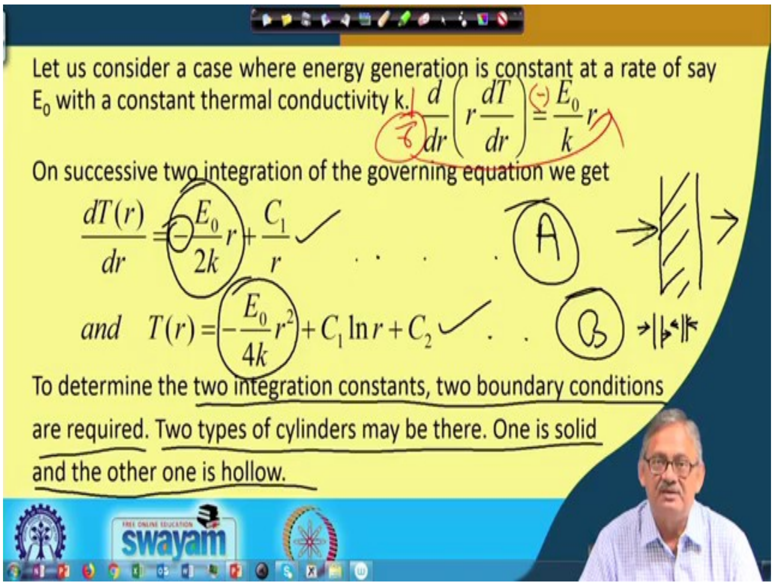

So, let us look into that. So, let us consider a case where energy generation is constant, 'right'. Energy generation is constant at a rate of say E_0 with a constant thermal conductivity, k then this generalized equation we can write $\frac{d}{dr}\left(r\frac{dT}{dr}\right)$ this is equal to minus $\frac{1}{k}E(r)$. Now, if you remember that in the previous slide here it was $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{1}{k}E(r)$. Now, we said this E, we are taking as E_0 and this r is then going and this conductivity is constant and this is also constant E_0.

So, if we take that then we come to this that dT/dr, 'right'. So, we can write that k dT/dr are dT/dr is a is equal to E_0/k. You remember that it was $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{1}{k}E(r)$. So, this was the thing. So, now, we have taken to that side r, so, we are writing with the thermal constant thermal conductivity. So, $\frac{d}{dr}\left(r\frac{dT}{dr}\right)$ is equal to E_0/k ×r this 1/r which we had earlier 1 by r. So, this 1 by r is changed to this position, 'right'. So, this r has gone to that position number 1.

Number 2, here perhaps one minus sign negative sign is missing because in that from the left side we have taken it to the 'right'. So, this should be taken off, 'right'. Now, if we integrate it on successive integrations we can write that dT(r)/ dr this is equal to dT(r)/dr is equal to this is that negative we have again made it, 'right', here, ok. So, dT(r)/ dr is equal. ,'right'.

$$\frac{dT(r)}{dr} = -\frac{E_0}{2k}r + \frac{C_1}{r}$$

So, we have taken this r has now come back to that C_1/r and on second integration we can write

$$T(r) = -\frac{E_0}{4k}r^2 + C_1 \ln r + C_2$$

the new constant that is there. Now, we have two equations say this is equation a 'right' if we say like that and this is if it is B, then we can we integrate equation A and equation B, 'right'. For integration of equation A and I sorry we have only integrated equation A and B we get C_1 and C_2 this got to constant we have to find out.

Now, as we earlier in the Cartesian coordinate also we have seen that whenever we are after integration we had to get two boundaries, 'right'. So, two boundary conditions defined boundary conditions we have to have. So, to determine the two integration constants, two boundary conditions are required. So, two types of cylinders may be there one is hollow and the other is solid, 'right'; that means, if it is solid like this then that will have one type and if this is a hollow. So, for the hollow we need; so, this is a hollow cylinder.

So, what we need? This will come to this here also it is there and it will come out from here or it will come out from there, 'right' this is the hollow and the other one is the solid. So, let us look into that what could be the boundary condition and to do that let us see.

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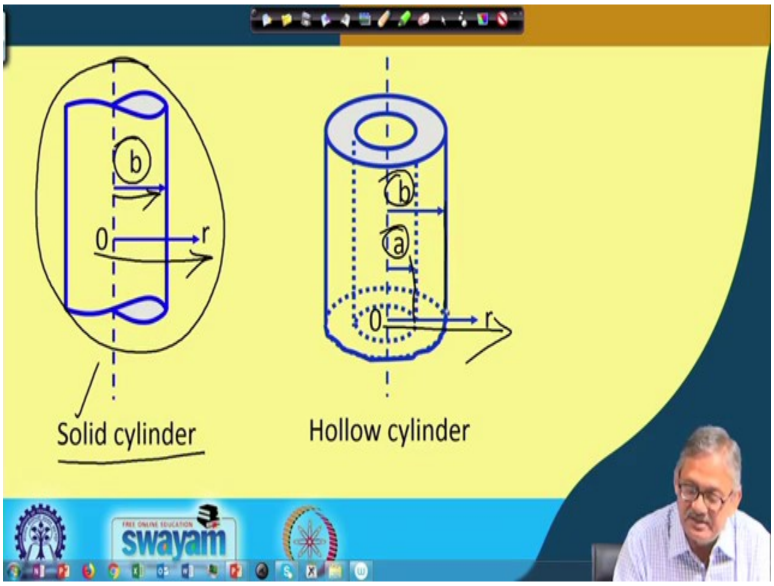

So, that this is the solid, 'right'. So, this is the solid and cylinder where we have taken this solid with 0 to r is the radius I mean this in one direction heat has been transferred and radius we have assumed to be b, 'right' for solid cylinder. For hollow cylinder, again we have 0 to r this is the axis and we have inner radius a and the outer radius b, say inner radius is this and the outer radius is this, 'right'. So, if that we true then we have these two conditions these two cylinders one is solid and other is hollow.

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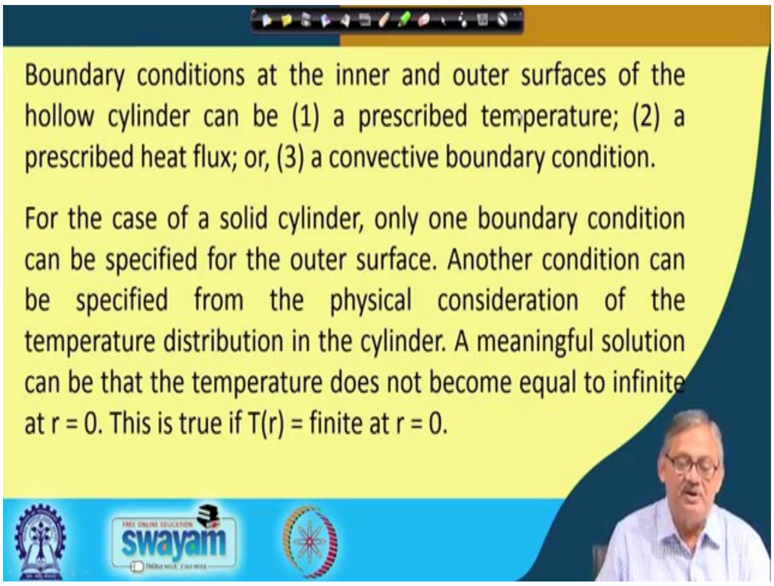

So, if that is the case then we can now, write that the boundary condition at this inner and outer surfaces of the hollow cylinder can be one either prescribed temperature boundary condition; or a prescribed heat flux boundary condition; or a convective boundary condition, 'right'. Either of this any of two of these three, I in combination or singly that can be the typical case, 'right'.

But, for the case of the solid cylinder which we had shown earlier; so, solid cylinder we had this, 'right'. So, there we have seen; so, here this is one boundary and this is another boundary. So, this boundary can be either prescribed heat flux or prescribed temperature or prescribed convective boundary, this side also either prescribed temperature or prescribed heat flux or convective boundary. Any one of these two can be the boundary and we can solve it.

But, what about the solid? Solid we will have one at this boundary. So, this boundary and this boundary is same because this is the central axis and we can say that this side is the mirror image of this side. So, this is at the center, 'right'. So, here whatever be the boundary, the same boundary is also pre building here. So, physically we get only one boundary. So, what to do with the others? So, other boundary we can say since you we said that at the center it is there, 'right' and it is the mirror image. So, the you see whatever temperature was there the same temperature is supposed to be here because it is the mirror image; whatever temperature is here the same temperature has to be here at this other end.

So, that means, that we can write del del t, 'right' we can write that r is equal to 0 ok. For the case of solid cylinder only one boundary condition which we have said can be specified for the outer surface. Another condition can be specified from the physical consideration of the physical consideration of the temperature distribution in the cylinder, 'right'. A meaningful solution can be that the temperature does not become equal to infinite at r is equal to 0. So, this is true if T of at any r t at any r is finite at r is equal to 0, 'right'.

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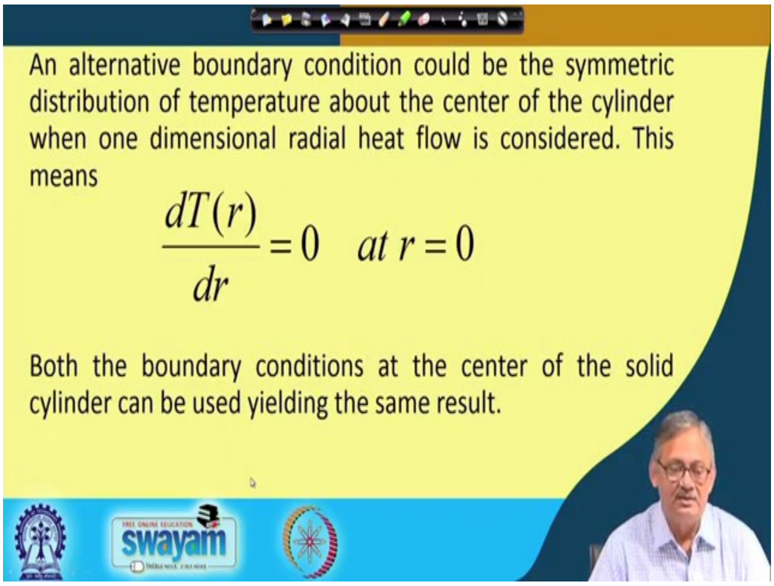

So, an alternative can be that an alternative boundary which can we think out that dT/dr is 0 at r is equal to 0; that means, rate of change of temperature with respect to r at r is equal to 0 is 0. That is true because if we go back to that here, ‘right’ rate of change of temperature with respect to r; so, r is this way or this way with respect to r is 0 at r is equal to 0. r is equal to 0 is here because at this point from this side also whatever temperature has come from this side also the same temperature has come. So, there is no change at that point at the center position yes there will be a change if we go at delta r from this axis then there will be a change if we go even more then there may be a change, but at this center there is no change, ‘right’.

Since from the symmetry of the cylinder we can say that at the alternative boundary condition can be that since there is a symmetry and the distribution of temperature about the center of the cylinder when one dimensional radial heat transfer is considered the this can be written as dT/dr is equal to 0 at r is equal to 0, ‘right’. Hope you have understood why this we are taking. This is almost equivalent this is not almost this is equivalent to that we can assume that at r is equal to 0, this phase at r is equal to 0 there is insulation no heat is neither going higher or coming in from this side. So, this is equivalent to that this r is equal to 0 is under full insulation.

And, this insulation means it is adiabatic absolutely; no heat is getting transferred, ‘right’. So, in that case this dT/dr is 0 at r is equal to 0, ‘right’. Now, we have two boundaries both the boundary conditions at the center of this solid cylinder can be used yielding the same result. So, one we have said that since T is not infinite at r is equal to 0 it is finite so, ‘right’. So, that is one solution which we have said earlier. So, here we have said that the temperature does not become equal to infinite at r is equal to 0. So, if this is only true if T at any r is finite at r is equal to 0. This is one boundary and the second boundary as we are saying that there is no change of temperature at this center that is at r is equal to 0, dT/dr is equal to 0, ‘right’.

So, now we have both for the solid cylinder as well as the hollow cylinder two boundaries and we can solve any with any given boundary the analytical solution, ‘right’.

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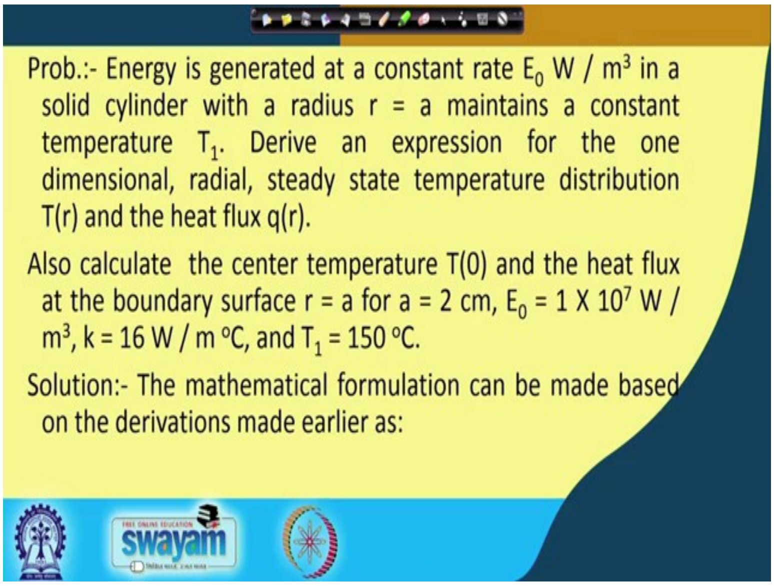
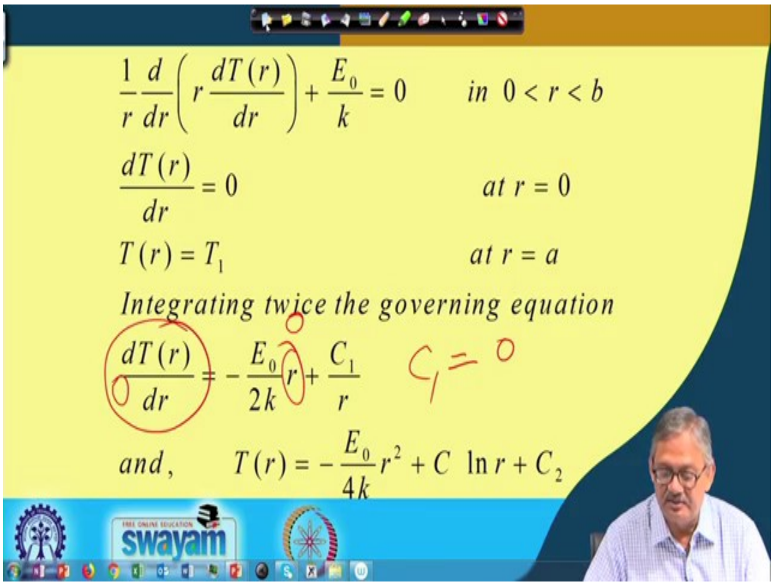


So, let us take first with the problem that the problem is like this: energy is generated at a constant rate E_0 watt per meter cube in a solid cylinder with a radius r is equal to that maintains a constant temperature T_1. Derive an expression for one dimensional radial steady state temperature distribution T(r) and the heat flux q(r), ‘right’. So, once you have developed this temperature distribution as well as the heat flux equation, then from that determine also calculate the center temperature T_0 and the heat flux and the heat flux at the boundary surface r is equal to a, ‘right’.

And, for ‘a’ is equal to 2 cm, $E_0 = 1\times10^7$ W/m^3, k is equal to 16 W/mºC and T_1 is equal to 150ºC, ‘right’. So, if we draw this solution that energy is generated at; so, we have a cylinder, ‘right’ at a constant energy. So, a constant energy source is getting generated E_0

W/m^3 in a solid cylinder with a radius. So, if the solid cylinder means we have. So, this is at r is equal to 'a', 'right' derive an expression for one dimensional radial heat transfer. So, we have to find out what is the T(r), 'right' this expression as well as we have to find out q(r), 'right'.



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT(r)}{dr}\right)+\frac{E_0}{k}=0 \qquad in\ \ 0<r<b$$

$$\frac{dT(r)}{dr}=0 \qquad at\ r=0$$

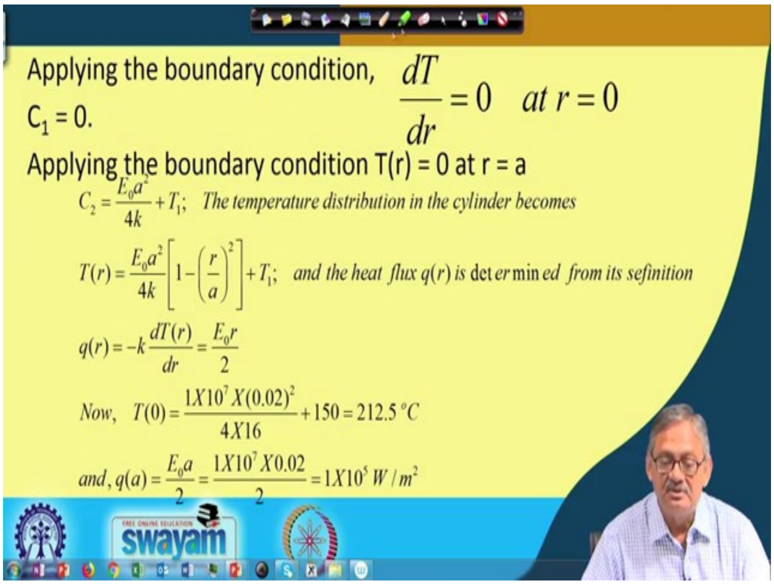
$$T(r)=T_1 \qquad at\ r=a$$

Integrating twice the governing equation

$$\frac{dT(r)}{dr}=-\frac{E_0}{2k}r+\frac{C_1}{r}$$

$$and, \qquad T(r)=-\frac{E_0}{4k}r^2+C\ \ln r+C_2$$

So, this is the temperature distribution T(r) 'right' and we can write from this two and given boundary applying, the boundary condition we can write $\frac{dT}{dr}=0 \quad at\ r=0$



If that be true from the previous equation you see we have from the previous equation what we have that dT/dr is 0, 'right' dT/dr is 0 and if this is 0 at r is equal to 0, so both this is becoming 0 this is also becoming 0. So, C_1 is becoming 0, 'right'. So, this is the case which is happening. So, if we go to the next then we see that since we have

this one dT/dr is 0 at r is equal to 0, C_1 is 0 and applying the second boundary we get T(r)

= 0 at r = a. So, this is the second boundary and we can write C_2 is

$$C_2 = \frac{E_0 a^2}{4k} + T_1;$$

Perhaps for this class we are running out of time and in the next class we will solve it and do some more problems, 'right'. So, let us stop it here because time is out or time is over.

Thank you.