

**Fundamentals of Food Process Engineering**  
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**Lecture – 09**  
**Rheological Properties of Viscoelastic Food**

Hello everyone, welcome to the NPTEL online certification course on Fundamentals of Food Process Engineering. We will continue with the viscoelastic properties of food today. So, Rheological Properties of Viscoelastic Food that we have started in the last class will continue with that. As I said that viscoelastic food can be characterized by the stress relaxation behavior, creep behavior. So, the test of stress relaxation and creep can be performed to understand the behavior of viscoelastic food. And along with that another study is the oscillatory study, so that we will also see in this class.

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**Viscoelastic properties**

✓ **Stress relaxation test:**

$$\sigma = \sigma_0 \cdot e^{-\frac{t}{\tau_R}}$$

✓ **When  $t = \tau_R$**   $\sigma = \frac{\sigma_0}{e}$

$$\sigma = 0.368 \cdot \sigma_0 \quad \frac{1}{e} = 36.8\%$$

**Time constant:** Mathematically, the time constant is the reciprocal of the first order rate constant in a simple first order exponential decay reaction.

**Elastic and viscous effect on material**

The slide includes a graph with 'Strain (%)' on the y-axis and 'Time' on the x-axis. A step function shows strain increasing at  $t=0$  and remaining constant. Below this, four curves show stress relaxation: 'Ideal elastic material' (horizontal line), 'Viscoelastic solid' (exponential decay), 'Ideal viscous material' (linear decay), and 'Viscoelastic liquid' (exponential decay to zero). A vertical line at  $t = \tau_R$  intersects the 'Viscoelastic solid' curve at a stress level  $\sigma_0/e$ .

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Stress relaxation test; now, what is stress relaxation? Viscoelastic material, if we apply constant strain on it for a long time, then what happens that with time the stress required to maintain that strain will die down gradually or that will decrease gradually. So that means, the stress we required to maintain the strain initially will be reduced as the time will pass, so some deformation or some flow will be there. So, this time is called the time required to get the full relaxation behavior is called the relaxation time and that time varies from different food products.

So, let us see this figure first, what we are doing is initially from the 0 condition, we are moving onto in the time scale in the x direction at  $t$  equal to 0 we are applying a constant strain  $\gamma_0$  and we are maintaining that strain throughout. What will happen in the stress behavior? So, an ideal elastic material will require the constant stress to maintain the strength because in case of pure elastic material or ideal elastic material stress is proportional to strain, so the stress will be constant as long as the strain is maintained as  $\gamma_0$ .

Now, if we see the ideal viscous material, so ideal viscous material will start flowing instantly and if it is viscoelastic material with elastic property in higher degree, so what will happen that initially when the strain is applied to  $\gamma_0$  stress is been reached to the maximum value and then gradually this stress will decrease ok. So, this stress will decrease to some extent not fully and then it will maintain or try to come to a constant level after that. Where ever if it is viscoelastic liquid; that means, the viscous component is in high degree then what will happen that the curve will exponentially dk to a higher degree compared to the viscoelastic solid and eventually come to 0, 0 condition of the stress.

So, what happened is that since the viscous material which are ideally ideal viscous or pure viscous material respond immediately with applied stress and start flowing. So, what is actually happened during flow that means that when the material flows it tries to dissipate that energy that that comes into it and it flows completely it does not come back to it is initial situation; that means, it does not have the property of storing that energy ok.

So, it will try to dissipate that and eventually come to the 0 stress condition whereas, the material which has a high solid content compared to the viscous material. So, in that case some amount of decay we will observe some amount of exponential decay we will observe; however, it will retain some amount of energy it will it will store some amount of and it will recover to some degree compared to recover to some degree with respect to the elastic property in it and  $\sigma_e$  that is the residual stress that will remain there that we can observe.

So, stress in case of viscoelastic material, will follow this pattern  $\sigma_0$  which is initial stress into  $e$  to the power minus  $t$  by  $t_R$ , where  $t_R$  is the stress relaxation time. So, if  $t$

equals to  $t_R$  what will happen then? We can write  $\sigma$  equal to  $\sigma_0$  by  $e$  and 1 by  $e$  is 36.8 percent; that means,  $\sigma$  equal to  $\sigma_0 \cdot 0.368$  ok. So, we can say that when the stress has reduced to 36.8 percent of initial stress, we can measure the relaxation time. So, relaxation time will be observed when the material when the stress in case of a viscoelastic material has reached to 36.8 percent of it is initial stress.

So, time constant mathematically that time constant is the reciprocal of the first order rate constant in a simple first order exponential decay reaction. So, elastic and viscous effect on material we can perceive that in such a manner like; in elastic material the molecules that are you know combined to form that elastic material, so, they have a strong attraction force, they have a strong bond forming forces. And when we strain them or we apply a constant stress, they try to opposite by having strained in the material, but if we apply the stress within the elastic limit; that means, no rupture or break will be observed, this energy is stored in the material and when we remove that stress, it is come to it is initial condition that is the elasticity ok.

Now, when we consider the viscoelastic material or in that the viscous effect will be visible along with the elastic property. So, in case of viscous material, the molecules in the viscous material are non covalent molecules are connected by the non covalent forces Van Der Waal forces and when some stress we apply on them, a constant stress we apply. So, these bond energy will try to reorient and they form some different kind of Van Der Waal forces non covalent forces again, but they also release some amount of energy ok.

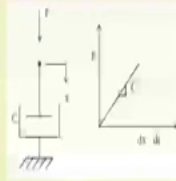
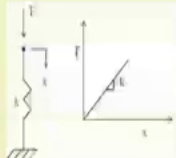
So, that energy dissipates in the form of heat; the heat is not perceivable, but they are not recoverable they are lost in the process. That is why we while try to regain the initial condition or the stress is removed, we cannot get back to the initial situation because the lost energy is not recoverable; however, to some extent the bonds will have certain amount of energy so, that is can be get back in case of the viscoelastic material. So, we will see how we can express the behavior of relaxation test by via mechanical models.

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**Mechanical model to represent viscoelastic material**

- ✓ **Elastic (Spring) Model** :  $F=Kx$ ,  $K$  is spring constant
- ✓ an ideal solid element obeying Hooke's Law,  $\tau=G\gamma$

**Viscous (Dashpot) Model:**  $F = C \frac{dx}{dt}$   
The rate of extension varies linearly with the force acting on the system.  
an ideal fluid element obeying Newton's law of viscosity

$$\tau = \mu \frac{d\gamma}{dt} \quad \text{or} \quad \tau = \mu \cdot \dot{\gamma}$$


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So, as we know that viscoelastic material will have both the properties of elastic nature and viscous nature. So, elastic model that is spring model, it has been considered that spring can express the ideal elastic nature. So, spring when you know is being applied by a constant force  $F$ , it expands and the deformation  $x$  can be related with the applied force in this manner;  $F$  equal to  $Kx$  where  $K$  is the spring constant that is almost equivalent to elasticity. When we remove the force  $F$  the spring tries to come to its initial condition, it recalls back and come to initial condition. So, an ideal solid element obeys Hooke's law that is shear stress proportional to strain  $\gamma$  and there is a proportionality constant  $G$  that is called the elasticity or elastic modulus.

Now, viscous property; so, viscous property is explained by dashpot and what happened in case of a dashpot that, when the force is being applied on it tries to move, tries to flow and dissipate that energy. So, the behavior that we can get here that is the force will be proportional to constant  $C$  into  $dx$  by  $dt$  ok. So, the flow is being expressed by  $dx$  by  $dt$ , the rate of extension varies linearly with force  $F$  acting on the system the rate of extension varies linearly with the force  $F$  acting on the system.

So, this will follow the Newton's law of viscosity ok. So, dashpot model will follow the Newton's law of viscosity, as shear stress will be proportional to shear rate  $d\gamma$  by  $dt$  or  $\gamma$  dot and  $\mu$  is the dynamic viscosity ok. So, combinedly spring and the dashpot may be used to describe different kind of viscoelastic behavior.

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**Combined mechanical Models**

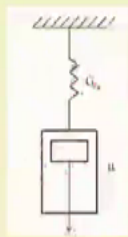
✓ **Maxwell model :**  
 stress relaxation of viscoelastic liquids, especially polymeric liquid.  
 series arrangement = total shear strain can be expressed as the summation of strain in the spring and dashpot.

$\gamma = \gamma_{spring} + \gamma_{dashpot}$

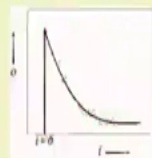
$\frac{d\gamma}{dt} = \dot{\gamma} = \frac{1}{G} \left( \frac{d\tau}{dt} \right) + \frac{\tau}{\mu}$

$\mu \dot{\gamma} = \tau + \lambda_{rel} \left( \frac{d\tau}{dt} \right) \quad \lambda_{rel} = \frac{\mu}{G}$

*Handwritten notes:*  
 $\tau = G \delta$   
 $\dot{\gamma} = \dot{\epsilon}$   
 $\frac{d\delta}{dt} = \frac{1}{G} \frac{d\tau}{dt}$



**Maxwell models**



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So, Maxwell model a very important model, where spring and dashpot are connected linearly in a series. So, here stress relaxation of viscoelastic liquid especially polymeric liquid can be express what will happen that, initially when the stress is applied so it will come to the spring, so, spring will expand following the elasticity.

So, it is directly proportional to the applied stress, so, stress will be felt by straight increase in the in the stress value as and when the force is come coming on it. Now as this force will be transferred to the dashpot, and dashpot as we know it is tries to flow and tries to dissipate that energy so, gradually it will come to a constant state asymptotically exponentially decay will be observed over time ok.

So, this is a Maxwell model that can express the stress relaxation behavior of viscoelastic liquids. So, first because of this elastic property and then viscoelastic liquids has as we say viscoelastic liquids; that means, it has higher viscous property. So, almost complete dissipation is there, little portion is not you know that the stress is not coming to 0 because some elastic behavior will be there.

So, in the series arrangement what will happen that when we apply any stress here stress is equally transmitted to the dashpot as well as the spring, but the strain will be because of spring and because of the dashpot ok. So, we can say that in the series arrangement, total shear strain can be expressed as the summation of the strain in the spring and the dashpot. So, gamma that is the total strain that is equal to gamma spring plus gamma

dashpot; and we can with time differentiate this to get the rate. So, gamma dot will be equal to 1 by G d tau by dt plus tau by mu; because it will have both elastic and viscous property.

So, we know that for the elastic material we have seen that tau equal to G into gamma that we have taken for the elastic material and for the viscous material we had mu into gamma dot. So, if in this equation we do the with respect to time we do differentiation, we will get we will get this as d gamma by dt that is equal to 1 by G into d tau by dt. So, ok. So, let us take as d only ok. So, the same thing we have put here gamma spring that is of elastic nature. So, 1 by G d tau by dt plus this will simply be tau by mu ok.

Now, considering this equation, we can we can express this in this form, we can take the mu out this side left side. So, we are getting mu into gamma dot, that is equal to shear stress tau here plus lambda relative lambda rel that is the relaxation time and that is lambda rel into d tau by dt ok. So, this is the relation time and this will be signified this will be signified as mu by G the relaxation behavior. So, mu by G where mu is the viscosity of the material, and G will be the viscosity of the dashpot and G will be the elastic modulus of the spring.

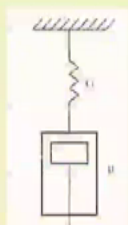
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**Combined mechanical Models**

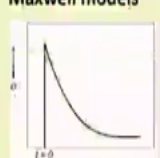
✓ **Maxwell model :**  $\mu \dot{\gamma} = \tau + \lambda_{rel} \left( \frac{d\tau}{dt} \right)$

□ For a constant strain  $\gamma$ , shear rate,  $\dot{\gamma}$  becomes zero  
 integrating with respect to time,  $\tau = \tau_0 \exp\left(-\frac{t}{\lambda_{rel}}\right)$

□ Describes the gradual relaxation of stress (from  $\tau_0$  to zero) after the application of a sudden strain. This relationship helps determining the relaxation time ( $\lambda_{rel}$ ), which is the time it takes for the stress to decay to 1/e (36.8%) of its initial value.



**Maxwell models**



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So, again we have got this expression mu into gamma dot equal to tau plus lambda rel d tau by dt. So, for a constant strain gamma dot will be 0 d gamma by dt that will be 0 and integrating this equation with respect to time, we will get we will get tau equal to tau 0

into exponential minus t by lambda rel. So, again we are getting the same expression that we have expressed by looking into the graph only. So, if we can take this shear stress if we can take this as stress or tau sigma or tau, we can get that tau 0 into e to the power minus t by lambda rel so, t by stress relaxation time. So, this equation describes the gradual relaxation of stress from tau 0 initial value to 0 after the application of sudden strain, and this relationship helps determining the relaxation time lambda rel, which is the time it takes for the stress to decay to 1 by e that is 36.8 percent of its initial value.

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**Combined mechanical Models**

✓ **Maxwell model :**

- ❑ Not suitable for many viscoelastic materials since it does not include an equilibrium stress ( $\tau_e$ ).
- ❑ A parallel spring connected to the Maxwell element.
- ❑ The stress relaxation equation

$$\tau(t) = \tau_e + (\tau_0 - \tau_e) \exp\left(-\frac{t}{\lambda_{rel}}\right)$$

$$\tau_e = G_0 \gamma_0$$

$$\lambda_{rel} = \frac{\mu_1}{G_1}$$

The slide also features a schematic diagram of a mechanical model where a spring with modulus  $G_0$  is connected in parallel to a Maxwell element (a spring with modulus  $G_1$  and a dashpot with viscosity  $\mu_1$  in series). The entire assembly is suspended from a fixed point, and a downward force  $F$  is applied.

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Now, it has been found that ok. So, we have discussed now Maxwell model and then will see that Maxwell model has been modified by adding another spring another spring in parallel to analyze the behavior of equilibrium stress ok. So, here we can take the value of tau e as equilibrium stress, because the Maxwell model may not be suitable for many viscoelastic material, since it does not include the equilibrium stress.

So, therefore, this new model has been devised where the parallel spring element has been added with the 2 Maxwell element. So, the stress relaxation equation will now become tau shear stress at any time t will be equal to tau e equilibrium stress, plus tau 0 minus tau e exponential minus t by lambda relative.

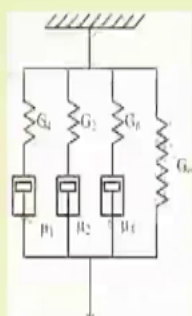
Tau e is nothing, but the stress this spring provides  $G_0$  into gamma 0;  $G_0$  is the elastic modulus or spring constant here and gamma 0 is the strain, lambda relative is mu 1 by  $G_1$  which is of this Maxwell element. Now why this is because many viscoelastic material

it has been found that the full I mean the full dissipation of energy may not happen, some may be retained inside the material because of the elastic behavior. So, if we take that tau will tau stress will become completely tau 0 it may not happen ok. So, therefore, this equilibrium stress concept has been introduced here and this is the behavior lambda relative is nothing, but the relaxation time and this equation can very well explain the behavior of this combined model.

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**Combined mechanical Models**

✓ **Maxwell model :**  
 a seven-element Maxwell model (three Maxwell elements and a free spring) used to model cooked potatoes.  
 The stress relaxation data of osmotically dehydrated apples and bananas were modeled using two terms Maxwell model.  
 Osmotic pretreatment decreased relaxation time of dehydrated samples, showing that sugar gain increases viscous nature but decreases elastic nature of fruits.



Three Maxwell elements in parallel with a free spring

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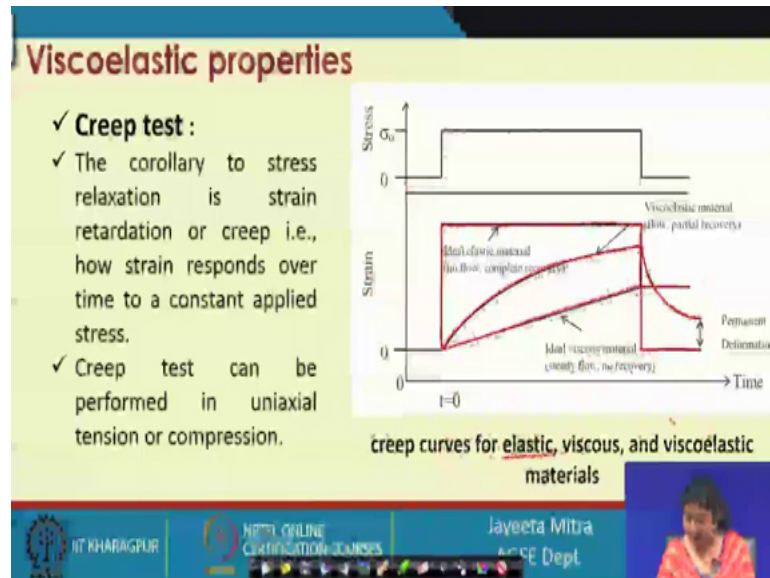
So, generalized Maxwell model where we have added 3 Maxwell component here which is 6 element plus 1 spring because of the equilibrium stress. So, total 7 elements are there in this generalized model ok. So, this model has been express the behavior of cooked potatoes. So, this different kind of complexities in a model, you can see that this will this will come when you analyze different kind of food material and their behavior, their cooking behavior, their swelling behavior, gelling behavior.

So, what happen that sometime it may not be possible to express the behavior using 2 Maxwell element. So, in that case it is very appropriate to use many models and we can see the stress relaxation data of osmotically dehydrated apples and bananas were modeled using 2 terms Maxwell model; osmotic pretreatment decrease the relaxation time. So, relaxation time decrease showing that the sugar gain increases the viscous nature, but decreases the elastic nature of the fruits, that is why relaxation time decreases.



So, many interesting phenomena of food processing can be explained by different kind of rheological model and many times we go for mathematical modeling of processing or fluid phenomena of different behavior. So, if we know the proper rheological nature of that product, we can accurately predict the behavior and the design of the various process will be correct.

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So, next important thing is creep test. So, creep is just the reverse case of stress relaxation behavior, what happen in case of stress relaxation ah?

So, what we did there we maintain a certain strain or deformation for longer period of time, and we measure the amount of stress needed to maintain that strain over the time period. Here what we do, we apply a stress we apply a constant stress and maybe this is a significant amount of higher stress not very low amount in the magnitude so, that we will apply. And with time we observe that what will be the strain in the material; will observe the strain or the response of the material in the stress right. So, this behavior will be observed.

So, what is that is we can see the creep behavior for elastic viscous and viscoelastic material. So, how strain responds over time to a constant applied stress that is the fundamental of creep test and it can be performed in uniaxial tension or compression. Now for the elastic material elastic material again we know that it in the elastic material stress is proportional to strain and when we withdraw that stress, we can gain the initial

nature of the material. So, ideal elastic material will observe like this as and when the stress is applied and it will be constant till the stress disappears right. So, then it will recover it is initial condition instantly.

Now, ideal viscous material; so, they will show steady flow behavior when the stress is constant ok. So, steady flow behavior will be there, so, strain will be in this fashion over time strain will increase, and when we remove that stress the deformation which already has happened the flow which already has happened will be there and it will maintain that one ok. So, initial condition will not get it will maintain the deformation when we take out the stress maintain that flow.

Now, if it is a viscoelastic material. So, it will have certain property for which the flow will takes place and also it has certain properties for which the elastic behavior will be visible. So, it will flow, but when we remove the stress. So, it will try to regain it is initial condition recover it is initial condition. So, partial recovery will be possible following the exponential decay curve and some amount of permanent deformation will be there because of it is viscous nature, and some amount of recovery will be possible ok.

So, how the material the strain response over time because of applied stress that is the creep behavior. So, we can see that for the viscoelastic material under the same stress for a long time, the material will deform permanently or show a creep behavior. So, we will stop here and will continue in the next class.

Thank you.