

**Fundamentals of Food Process Engineering**  
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**Lecture – 42**  
**Mechanical Separation Techniques (Contd.)**

Hello, everyone. Welcome to NPTEL online certification course on Fundamentals of Food Process Engineering. In the last class, we have started the chapter on Mechanical Separation Technique, today we will continue in that. So, mechanical separation technique deals with the separation of different phases of the material by the use of some mechanical force. And it has a difference with the separation based on the size that we did in screening or with the membrane separation where the selected permission is takes place normally. So, in that mechanical separation we have discussed in the last class and today we will continue with the topics first that is filtration.

Filtration has enormous used in food processing industry many methods for example, filtration of juice, different kind of slurry and although this kind of filtration when we use a slurry and we want to separate it from the solid and the liquid part. So, based on the requirement sometime the slurry is important, sometime the liquid which is after filtering we are getting that is important or sometimes both are important.

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### Filtration-introduction

- ✓ Filtration may be defined as the separation of solids from liquids by passing through a permeable medium.
- ✓ The suspension of solid and liquid to be filtered is known as the **slurry**. The filter medium used to retain the solids.
- ✓ The accumulation of solids on the filter is referred to as the **filter cake**.
- ✓ the valuable stream from the filter may be the fluid, or the solids, or both.

LIQUID FILTRATION

Suspension

Filter housing

Medium

Driving force (pressure drop  $\Delta P$ )

Filtrate flow Q

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So, filtration may be defined as the separation of solid from liquid by passing through a

permeable medium. So, we can see how the arrangement look like this is the a frame is there or support is there for filter medium and there is a filter medium this one, this one is the filter medium. And as the slurry comes over the filter medium the liquid will filtered through it through the filter medium and the slurry will be deposited on the on top of the filter medium and as time will continue the deposition of slurry will, deposition of cake will increase and because of that the pressure differential will also change.

So, initially since the there was no deposition so, filter rate of filtration was very high and as the deposition increases, so, the rate will further lower. So, therefore, which time we need to remove the filter cake that is being deposited on the filter medium.

The suspension of solid and the liquid to be filtered is known as the slurry and the filter medium used to retain the solid that is in the form of cake. So, the valuable stream for the filter maybe the fluid or the solid or both as I said and the driving force across the filter medium and the cake is basically responsible for the filtration process to occur.

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**Rate of filtration:**

- ✓The fluid passes through the filter medium, which offers resistance to its passage, under the influence of a force which is the pressure differential across the filter.

**rate of filtration = driving force/resistance**

- ✓Resistance arises from the filter cloth, mesh, or bed, and to this is added the **resistance of the filter cake** as it accumulates.
- ✓The filter-cake resistance is obtained by multiplying the specific resistance of the filter cake, that is its resistance per unit thickness, by the thickness of the cake.

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So, next we will see that rate of filtration which is actually the most important parameter for us who wants to know the process or want to design any process. So, the rate of filtration should be maintained properly the fluid passes through filter medium which offers resistance to its passage, under the influence of a force which is the pressure differential across the filter. So, to cause the rate of filtration we need driving force

divided by resistance. So, if the driving force increases. So, rate of filtration will increase and if the resistance increases driving rate of filtration will decrease.

Resistance arises from the filter cloth or mesh or filter bed and to this the added resistance occur because of the deposition of the cake. Filter cake resistance is obtained by multiplying the specific resistance of the filter cake that is the resistance per unit thickness because as the deposition of cake increases so, it offers resistance in a higher scale from the initial condition. So, if we know the specific cake resistance we can analyse better that how it going to affect the rate of filtration.

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**Theory of filtration:**

- ✓ Pressure drop of fluid through filter cake: 
$$\frac{\Delta P}{L} = \frac{32\mu v}{D^2}$$
- ✓ The linear velocity based on filter area is given as follows:

section through a filter cake and filter medium at time  $t$  s from the start of the flow of filtrate. thickness of the cake is  $L$  m. The filter cross-sectional area is  $A$  m<sup>2</sup>, and the linear velocity of the filtrate in the  $L$  direction is  $v$  m/s.

Now, coming to the theory of filtration, that is what is the mechanism behind filtration. So, here we will assume that with start of the filtration if we continue it for a time, let us say  $t$  second then the deposition of cake will occur and the length of that deposition let us say  $L$  meter.

So, in that we have taken a section a very small section of having length  $dL$  in the cake across the area  $A$  of the of the filter medium. So, the slurry is coming and it is you know flowing in a cross flow situation as we have taken. So, deposition of slurry will be more in this case. So, in that deposition which has occurred in time  $t$  second we have taken a small section  $dL$  across the area  $A$  in meter square and also we want to interested in the calculation of the linear velocity of the filtrate which is here  $V$  meter per second, linear velocity.

Now, with that we will start calculating the case for a for pressure drop across the filter cake. Now, for that we will start from the approach that we normally take when we try to analyse any pipe flow. So, when we want to analyse pipe flow for any fluid in laminar condition, and in that we want to have a distance pressure drop across a distance delta L. So, then we use Hagen–Poiseuille equation for laminar flow in case of a pipe and there we use there we try to make a relation between the pressure drop delta P which is P 1 minus P 2, pressure drop with the diameter of the pipe and the velocity of the fluid and also the viscosity of the fluid.

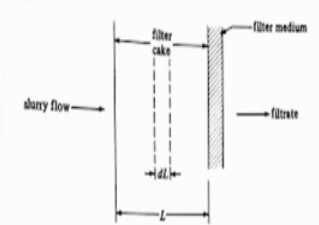
So, similarly here what we will do is we try to first you know equate this with that particular case that is delta P across the length L that is equal to  $32 \mu v$  by  $D^2$  which is Hagen–Poiseuille law. So, the basis is first this law, then what we assume that is instead of flowing a fluid through the whole pipe if we fill it with fill it with the particle of similar dimension and pack it completely if you pack this completely and we assume that there is a we assume that the pores between them will frame like a channel linear channel and through that we want to flow the fluid and also we want to measure that if the laminar flow prevails then what will be the pressure drop.

So, this kind of analysis you will read in transport phenomena when you try to analyse the porous media flow etcetera. So, from that concept we will try to relate this pressure drop for the filter cake.


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**Theory of filtration:**






- ✓ Pressure drop of fluid through filter cake: 
$$-\frac{\Delta P}{L} = \frac{32\mu v}{D^2}$$
- ✓ The linear velocity based on filter area is given as follows: 
$$v = \frac{dV/dt}{A}$$




section through a filter cake and filter medium at time  $t$  s from the start of the flow of filtrate. thickness of the cake is  $L$  m. The filter cross-sectional area is  $A$  m<sup>2</sup>, and the linear velocity of the filtrate in the  $L$  direction is  $v$  m/s.


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So, now, if we will relate it by this equation that is  $v$  will be equal to  $dV$  very small amount of liquid which has been filtered in time  $dt$  divided by the cross sectional area  $A$ . So, this will give us the idea of linear velocity.

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**Theory of filtration:**

✓ We know for velocity through a packed bed, the flow is given by **Carman-kozney** equation, given as follows:

$$\frac{\Delta P_c}{L} = \frac{k_1 \mu v (1 - \epsilon)^2 S_0^2}{\epsilon^3}$$

Where,

$K_1$  (constant)= 4.17 for random particles of definite size and shape,  
 $\mu$  is viscosity of filtrate in Pa.s,  
 $v$  is linear velocity based on filter area in m/s,  
 $\epsilon$  is void fraction or porosity of cake,  
 $L$  is thickness of cake in m,  $S_0$  is specific surface area of particle in  $m^2$  of particle area per  $m^3$  volume of solid particle, and  $\Delta P_c$  is pressure drop in the cake in  $N/m^2$ .

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So, then what we will do is, now the velocity through a packed bed that has been given by the Carman-kozeny equation as I mentioned that when you deal the porous media flow you will get to know this equation Carman-kozeny equation and Blake-Kozeny equation. So, there are different equation organ equation which relates the pressure drop across the length of  $L$  in a porous media and the velocity, also if this equations has been developed for laminar flow as well as for the turbulent flow.

Here we will use the equation for the laminar flow and the relation is such that  $\Delta P_c$  divided by  $L$  we consider the cake as the as the packed bed. So,  $\Delta P_c$  divided by  $L$  that is the pressure drop across the bed, or cake  $L$  is the length that has been deposited  $k_1$  into  $\mu$  into  $v$   $(1 - \epsilon)^2 S_0^2$  by  $\epsilon^3$ . So, this parameters are  $k_1$  is the constant whose value generally taken as 4.17 for random particles of definite size and shape.  $\mu$  is the viscosity of the filtrate in Pascal second  $v$  is the linear velocity based on filter area in meter per second that we have just calculated that is  $dv$  by  $dt$  upon  $A$ .

Then, we have  $\epsilon$  which is void fraction or porosity of the cake. So, this is  $\epsilon$ , void fraction or porosity of the cake, then  $L$  we know the thickness of the cake in meter,

$S_0$  is the specific surface area of particle in meter square of the particle area per meter cube. So, specific surface area; that means, meter square of the surface area per meter cube of volume of the solid particle, we just square area per meter cube of the volume of solid particle.

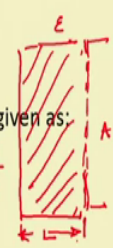
$\Delta P_c$  is the pressure drop in the cake in Newton per meter square or Pascal. So, this is the relation now we try to relate this with other equation that we can derive from the particular case where the figure we have drawn that the deposition of the cake and the filtrate. So, relating those equation we will try to get some idea of the pressure drop across the cake.



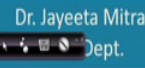
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**Theory of filtration:**

- ✓ A material balance through the filter cake is given by:
 
$$\rho_p LA(1 - \epsilon) = (V + \epsilon LA) \rho_s$$
- ✓ Using all the above equations the final form of equation can be given as:
 
$$v = \frac{dV/dt}{A} = \frac{-\Delta P_c}{k_1(1 - \epsilon)S_0^2 \frac{\mu c_s V}{\rho_p \epsilon^3}} = \frac{-\Delta P_c}{\alpha \frac{\mu c_s V}{A}}$$

Driving Force  
Resistance
- ✓ where  $\alpha$  is the specific cake resistance in m/kg, defined as
 
$$\frac{dV}{dt} = \frac{A(-\Delta P)}{R} \quad \alpha = \frac{k_1(1 - \epsilon)S_0^2}{\rho_p \epsilon^3}$$



So, how we can develop that first we have seen that what will be the flow of fluid through that; that means, about the filtrate that has been passed through. Now, we try to establish a material balance to get some idea then how the deposition of cake has occurred.

So, for that a material balance through the filter cake we have developed that is you know  $L$  is the length of the cake. So, if I draw the figure here, this is the filtered medium, and in that  $L$  is the length of that deposition, this area is  $A$ , this is the whole area of the cake and there is a porosity that is  $\epsilon$ . So, this  $L$  into  $A$  giving the total volume into  $1 - \epsilon$ , so, this is the void fraction. So,  $\epsilon$  is the void through which the liquid will pass.

So,  $1 - \epsilon$ ; that means, the cake the fraction of the cake so, that will be equal to the deposition  $V$ , the volume  $V$  that has been pass through plus the volume that has been you know that has been there in the cake. So, in the cake volume of the of the fluid is  $\epsilon$  into  $L$  into  $A$ , and here if I write now that  $c_s$  and here  $\rho_p$ , where  $\rho_p$  is the density of the cake that has been deposited and  $c_s$  is the kg of the solid per meter cube of the filtrate. So, then we will get this material balance.

So, the length of  $L$  into professional area that is the volume  $1 - \epsilon$  that is giving the cake which is there into density that is the mass of deposition of the cake will be equal to the volume that has been passed through because this volume also contain  $c_s$  kg per meter cube of the of the solid material. So, this volume plus the volume which is you know retaining the cake in the force  $\epsilon$  into  $L$  a plus  $V$  into  $c_s$ . So, this is the idea that we are getting from the mass balance.

Now, using this equation and the previous equation where we have got the, you know Blake-Kozeny equation relating the pressure drop with the length and other parameter. So, with this equation we are getting this final equation which is  $dV$  by  $dt$  divided by  $A$  that is the velocity. So, velocity will be equal to minus  $\Delta P_c$  that is the pressure drop across this cake divided by  $k_1$  into  $1 - \epsilon$  into  $S_0$  square by  $\rho_p$  into  $\epsilon^3$  into  $\mu$  into  $c_s V$  by  $A$ .

So, what we have did, what we have done is that we have replaced this  $L$  from here and from the previous equation the Blake-Kozeny equation we have replaced  $L$  and then we have tried to express this in terms of  $v$ , which is the linear velocity. So, then we are getting this equation. So, all the parameter  $k_1 \epsilon S_0 \mu$  that we have expressed  $c_s$  area  $A \rho_p$ .

Now, this equation can be further represented as  $\Delta P_c$  which is the driving force divided by resistance. So, resistance is  $\alpha$  into  $\mu c_s V$  by  $A$ . So, if you look in to this two parameters, first one is this one and second one is this one so,  $k_1$  specifically for the bed of particle or the size of the particle if they are uniform. So, based on that cake property only porosity also of the property for cake only,  $\rho_p$  is the density of the cake,  $S_0$  is the specific surface area of the particle per unit volume. So, all these represent the property of the cake specifically. Therefore, we have combiningly express this in terms of  $\alpha$  and call it specific cake resistance, multiplied with  $\mu$  that is the property of the

filtrate that is going out into  $c_s$  that is kg of solid per meter cube of the filtrate into volume  $V$  by  $A$ .

So, this is how we can express this as driving force by resistance. So, therefore, we have taken all these factor as  $\alpha$  and  $\alpha$  can be defined as this way  $k(1 - \epsilon)^2 / \rho_p \epsilon^3$ . So,  $dV/dt$  that is the that is from here I want to write that  $dV/dt$  can be written as  $A$  multiplied with  $\Delta P$  this is pressure drop. So, this is reducing in the that that way we have kept minus sign by  $R$  which is the resistance.

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**Theory of filtration:**

For the filter medium resistance, we can write  $v = \frac{dV/dt}{A} = \frac{-\Delta P_f}{\mu R_m}$

where  $R_m$  is the resistance of the filter medium to filtrate flow in  $m^{-1}$ . It includes the resistance to flow of the piping leads to and from the filter and the filter medium resistance and  $\Delta p_f$  is the pressure drop

Since the resistances of the cake and the filter medium are in series,

$$v = \frac{dV/dt}{A} = \frac{-\Delta P}{\mu \left( \frac{\alpha c_s V}{A} + R_m \right)}$$

$$\Delta P = \Delta P_c + \Delta P_f$$

The slide also features a diagram of a filter cake with arrows indicating flow direction and a small video inset of the presenter.

Now, for the filter medium resistance what we can write? We have seen now then how filter cake resistance can be formed, but definitely some resistance will be provided by the filter medium as well. So, for that what we will do is we use a factor  $R_m$ . So, similar manner that way we did we calculated for the filter cake that is  $v$  is equal to  $dV/dt$  divided by  $A$  that is equal to minus  $\Delta P_c$  by  $\alpha$  into  $\mu c_s v$  by  $A$ .

Similarly, we have calculated for the filter medium resistance as  $dV/dt$  by  $A$ . Similarly, because that linear velocity we assume that will be fixed from the cake also and as well as for the filter medium as well. So, that will be instead of  $\Delta P_c$  now  $\Delta P_f$  that is because of the filter medium divided by  $\mu$  into  $R_m$ . So,  $\mu$  is the viscosity and  $R_m$  is the resistance of the filter medium to filtrate the flow in per meter. So, this is the unit of this  $R_m$  and this includes the resistance to flow of the piping leads to and



from the filter and the filter medium resistance,  $\Delta P_f$  is the total pressure drop across it.

Now, since the resistance of the cake and the filter medium are in series because the slurry is coming into first the deposition of the cake is there, then there is the filter medium is there, and then the clear liquid is going out. So, they are in series, so, we can club those two pressure drop  $\Delta P_c$  and  $\Delta P_f$  to find the total pressure drop across it.

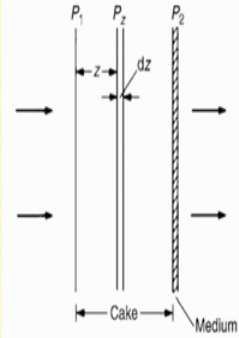
So, therefore, we can write  $dV$  by  $dt$  by  $A$  which is nothing, but linear velocity that is equal to minus  $\Delta P$  divided by we have taken  $\mu$  as common, here also  $\mu$  was there. So,  $\alpha c s V$  by  $A$  plus  $R_m$  so,  $\Delta P$  is equal to  $\Delta P_c$  plus  $\Delta P_f$ .

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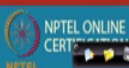
### Filter-cake Compressibility:

- With some filter cakes, the specific resistance varies with the pressure drop across it. This is because the cake becomes denser under the higher pressure and so provides fewer and smaller passages for flow. The effect is spoken of as the compressibility of the cake.
- It is usually possible to express the voidage  $e_z$  at a depth  $z$  as a function of  $(P_1 - P_z)$ .

$$\alpha = \alpha_0 (-\Delta P)^s$$



Flow through a compressible filter cake.

Now, filter cake compressibility so, all the cake formation that is deposited on the surface on the on the above of that medium and depositing in the form of a cake. So, that maybe sometime compressible as well also most of the cases we take it as incompressible, but we can find the factor of compressibility. So, what is that with some filter cake the specific cake resistance that is  $\alpha$  we have taken varies with the pressure drop across it. So, this is because the cake becomes denser under the higher pressure and so, provides fewer and smaller passage for flow. That means, the pressure drop and porosity has a relation and the packing is more I mean pressure is more. So, the porosity will be changing and the effect is spoken as the compressibility of the cake

because when the force is you know small that means it has the compressibility is there.

So, it is usually possible to express the voidage  $\epsilon$  at a depth of  $z$ . So, at a depth of  $z$  in the cake what will be the if we take a  $dz$  sections, so, what will be the porosity. So, that will be a function of the pressure drop from the top phase where the slurries coming into the  $z$  depth what is the pressure at the  $z$  depth. So, based on that this porosity will be varying.

So, therefore, we can define alpha will be equal to alpha 0 into minus delta P 2 the power s. So, this alpha what we have got this relation this relation is alpha equal to alpha 0 into minus delta P to the power s.

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**Specific cake resistance**  $\alpha = \alpha_0 (-\Delta P)^s$

- where  $\alpha$  is the specific resistance of the cake under pressure  $P$ ,  $\Delta P$  is the pressure drop across the filter,  $\alpha_0$  is the specific resistance of the cake under a pressure drop of 1 atm and  $s$  is a constant for the material, called its compressibility
- $\alpha = f(\epsilon, S_0, \Delta p)$  since pressure can affect  $\epsilon$ .
- constant-pressure experiments at various pressure drops develops the relation between  $\alpha$  and  $\Delta p$ .
- If  $\alpha$  is independent of  $-\Delta p$ , the sludge is incompressible; Usually,  $\alpha$  increases with  $-\Delta p$ , since most cakes are somewhat compressible. An empirical equation often used is  $\alpha = \alpha_0 (-\Delta P)^s$
- The compressibility constant  $s$  is zero for incompressible sludge or cakes. The constant  $s$  usually falls between 0.1 to 0.8

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So, here we can write this alpha equal to alpha 0 into minus delta P to the power s, this we have got.

Now, also if you look into the expression of the alpha that we have related with the constant  $k_1$  with the porosity which the density of the cake so, all the parameters were there. So, here alpha is the specific cake resistance under the pressure  $P$ , delta P is the pressure drop across the filter, alpha 0 is the specific resistance of the cake under the pressure drop of 1 atmosphere, and  $s$  is the constant for the material call the compressibility. So, here when the  $s$  relate the compressibility that means,  $s$  can be change for a different kind of a pressure. So, the value of  $s$  can be change if it is the

compressible cake.

So, from the equation that we have developed for the specific cake resistance we have seen that  $\alpha$  has a relation that is  $\alpha$  is the function of porosity  $S_0$  that is specific surface area surface area of the particle per unit volume of the particle and also the  $\Delta P$ , because if  $\Delta P$  changes porosity changes and for that  $\alpha$  is also change. So, this is the factor.

So, constant pressure experiments at various pressure drop develop the relation between  $\alpha$  and  $\Delta p$ . So, if you want try to get an idea then how  $\Delta p$  is going to effect the specific cake resistance, then what we need to do at different constant pressure we need to perform the filtration operation and then we need to see that what is the value of this resistance coming. So, constant pressure experiment at various pressure drop and thereby we can develop a relation between  $\alpha$  and  $\Delta p$ .

So, if  $\alpha$  is coming independent of  $\Delta p$  that is pressure drop the sludge is incompressible and usually  $\alpha$  increases with  $\Delta p$ , specific cake resistance increases with increase in  $\Delta p$  since, most of the cake are somewhat compressible and empirical equation often, we use that is  $\alpha = \alpha_0 (\Delta P)^{-s}$ . The compressibility constant  $s$  is 0 for incompressible sludge or cake and the constant  $s$  usually falls between 0.1 to 0.8, for the incompressible cakes.

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**Factors affecting rate of filtration:**

- ✓ The rate of filtration is proportional to the pressure difference across both the filter medium and filter cake.
- **The pressure drop can be achieved in a number of ways:**
  1. **Gravity:** A pressure difference could be obtained by maintaining a head of slurry above the filter medium and depends on the density of the slurry.
  2. **Vacuum:** The pressure below the filter medium may be reduced below atmospheric pressure by connecting the filtrate receiver to a vacuum pump.
  3. **Centrifugal force:** The gravitational force could be replaced by centrifugal force in particle separation

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Now, coming to the factors affecting the rate of filtration. So, the rate of filtration is proportional to the pressure difference across both the filter medium and the filter cake. So, the pressure drop can be achieved in a number of ways first one is gravity, where a pressure difference could be obtained by maintaining a head of slurry above the filter medium and depend on the density of the slurry.

Another method is by vacuum the pressure below the filter medium may be reduced below atmospheric pressure by connecting the filtrate receiver to a vacuum pump. So, thereby we can create the pressure drop and third thing is centrifugal force. The gravitational force could be replaced by centrifugal force in particle separation. So, all three forces are huge in different kind of filtration mechanism to cause the pressure drop.

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**Factors affecting rate of filtration:**

- ✓ The area of the filtering surface.
- ✓ The viscosity of the filtrate.
  - an increase in the viscosity of the filtrate will increase the resistance of flow, so that the rate of filtration is inversely proportional to the viscosity of the fluid.
  - **Probable solution**
    - (1) The rate of filtration may be increased by raising the temperature of the liquid, which lowers its viscosity.
    - (2) Dilution is another alternative
- ✓ The resistance of the filter cake.
- ✓ The resistance of the filter medium and initial layers of cake.

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And, another thing that is the rate of the area of filtering surface that has an effect on rate of filtration and the another important parameter is the viscosity of the filtrate. So, normal assumption is that it increase in the viscosity of the filtrate the resistance of the flow will also increase, and increase in the viscosity of the filtrate will increase the resistance of flow. Obviously, because if it is a thick viscous liquid, it will not flow so easily, so that the rate of filtration is inversely proportional to the viscosity of the fluid; so, that is why rate of filtration is inversely proportional. So, if viscosity increasing rate will decrease.

Now, what could be the you know solution for this kind of problem. So, then we can do

two things; one is we can increase the temperature. So, by raising the temperature of the liquid the rate of filtration maybe increase. Now, in a practical case we should keep in mind that if we raise the temperature whether any heat sensitive compound will degrade or it will it can handle the filter medium that we are using. So, looking into all that we can apply either 0.1.

Second is dilution is another alternatives. So, in the case where we can not apply temperature we can go for dilution of the slurry. So, in that it will have the affect in the rate filtration. Another point is the resistance of the filter cake. So, definitely resistance will going to affect the rate and the resistance from the filter medium and initial layers of cake. So, this will also have affect on the factors. So, we will stop here and we will continue in the next class.

Thank you.