Fundamentals of Food Process Engineering
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Lecture – 38
Size Reduction (Contd.)

Hello everyone. Welcome to the NPTEL online certification course on Fundamentals of Food Process Engineering. So, we will continue with Size Reduction today.

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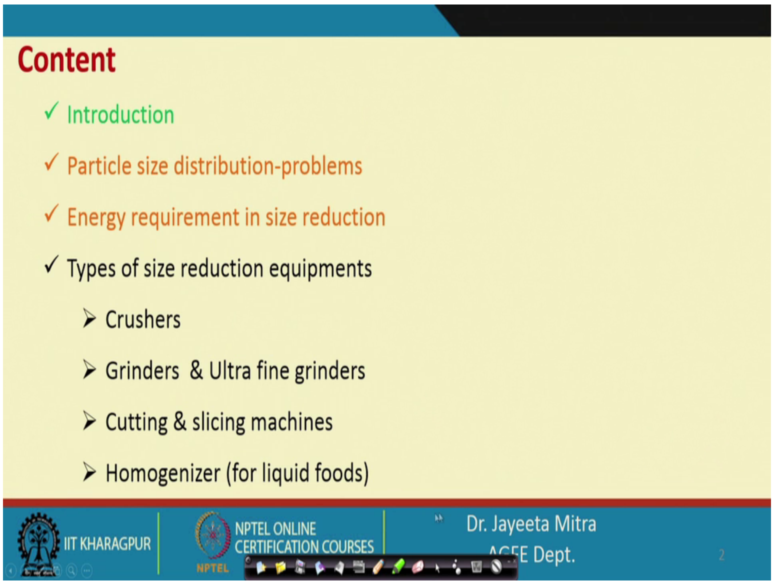

Content

- ✓ Introduction
- ✓ Particle size distribution-problems
- ✓ Energy requirement in size reduction
- ✓ Types of size reduction equipments
  - ➢ Crushers
  - ➢ Grinders & Ultra fine grinders
  - ➢ Cutting & slicing machines
  - ➢ Homogenizer (for liquid foods)

In our last class, we have discussed the particle size distribution and today will solve 1 or 2 problem on particle size distribution and also we will see the energy requirement in size reduction.

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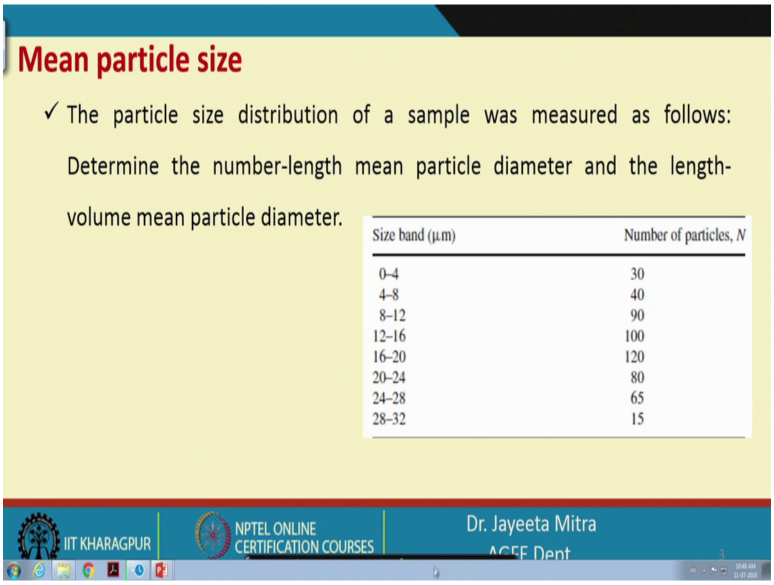

Mean particle size

- ✓ The particle size distribution of a sample was measured as follows: Determine the number-length mean particle diameter and the length-volume mean particle diameter.

Size band (μm)	Number of particles, *N*
0–4	30
4–8	40
8–12	90
12–16	100
16–20	120
20–24	80
24–28	65
28–32	15

So, we have seen that, what are the different equivalent diameter and what are the different method of particle size analysis. Out of that, the most common which we often used in food industry or in the lab analysis of the food sample for the particulate size distribution. So, we refer the sieve analysis mostly and we have mentioned that how the sieve sieves are stacked in one another.

There are different standard. Mostly we follow the (Refer Time: 01:24) series standard sieve analysis method by a set of sieves are there. The largest perforated perforation largest openings are at the top and the lowest opening we are getting at the bottom. Then, at the bottom end, we are getting a pan and which is just collector. There is no perforation in that.

So, also this all search screens they are categorised that mesh per square inch size for the opening per square inch thereby they are defined. So now, the particle size distribution of a sample was measured and do the data are given ok. So, we need to calculate different you know diameters and the different quantitative parameters of those distribution.

So, here we will determine the number length mean particle diameter and length volume mean particle diameter. So, the expression of this are given in the previous slide. So, will just apply the equations today; here, this is the data given the size band in micrometre and number of particles and that is written on the size 0 to 4, 30 numbers, 4 to 8 microns

40 numbers, 8 to 12 microns 90 numbers and so on. Last, we are getting 28 to 32 we are getting 15.

So, if you look into the distribution, you can see that there is unimodal frequency distribution you will get. If you try to draw the frequency with respect to the particle size diameter plot. So, that is how it looks like.

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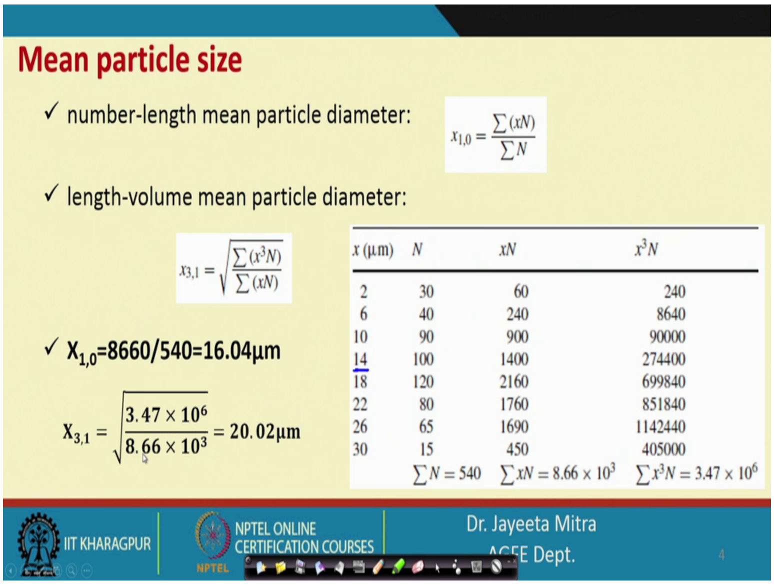

Mean particle size

- ✓ number-length mean particle diameter: $x_{1,0} = \frac{\sum(xN)}{\sum N}$
- ✓ length-volume mean particle diameter:

$$x_{3,1} = \sqrt{\frac{\sum(x^3N)}{\sum(xN)}}$$

- ✓ $X_{1,0}$=8660/540=16.04μm

$$X_{3,1} = \sqrt{\frac{3.47 \times 10^6}{8.66 \times 10^3}} = 20.02\mu m$$

x (μm)	N	xN	x^3N
2	30	60	240
6	40	240	8640
10	90	900	90000
14	100	1400	274400
18	120	2160	699840
22	80	1760	851840
26	65	1690	1142440
30	15	450	405000
	$\sum N = 540$	$\sum xN = 8.66 \times 10^3$	$\sum x^3N = 3.47 \times 10^6$

Now, number length mean particle diameter is expressed by summation of x N by summation of N ok. So, N is the N is a number. Now, length volume mean particle diameter that is summation root over summation of x cube N by submission of x N. So, these are the formula. Now, we need to calculate this parameter. We have x that is the size we are having in micrometre ok.

Now, if you look into that, we have we have this size fraction ok. So, we are taking the mean of this size fraction because we need to have one particular size that that we need to take as d or x ok. So, and this is the number given to that. So, we will always take the mid value or the mean value of this size distribution. So, that is why we have taken here 2 6 10 14 18 22 26 and 30. Now, corresponding number of the particles in the earlier equation in the earlier class, sometime you may get this x in terms of d and x were used to represent the fraction.

So, here since it is used to see the express the diameters, so, I am just mentioning it before that you will not confused with that. So, here x is the dimension characteristic dimension for the equivalent dia of the particle rose has been written on the particular sieve size.

So, 2 is having 30 numbers, 6 having 40 numbers, 10 micrometer having 90 numbers and so on. So, summation in is the total number of particle that is 540 and x into N. We need to use it here for length mean particle diameter calculation x into N, will multiply this with that and make the summation. Similarly, x cube N again will multiply cube with N and to will get the x cube N this one, 3.47 into 10 to the power 6; so, then what we need to do is we need to plot this value in the equation. So, number length mean particle diameter is coming as 8660 by 540.

So, 16.04 micrometer. So, if you go by length mean diameter, so we are getting the you know mean particle size somewhere here between 14 and 16. So, 100 and 120 particles there and if this is the number of particles are in between somewhere, the size fraction will come and if you go by length volume mean particle diameter. So, you are getting root over this x cube N 1. So, this was 3.47 into 10 by 6 divided by summation x N 8.66 into 10 cube. So, 20.02 micro metre.

So, 20.02 micrometer. So, this is the line somewhere the, this side ok. So, we can see that the based on different diameter, we are getting slightly different you know mean size and the number fraction. So, here will get somewhere in between these 2. So, this is how we can represent our data.

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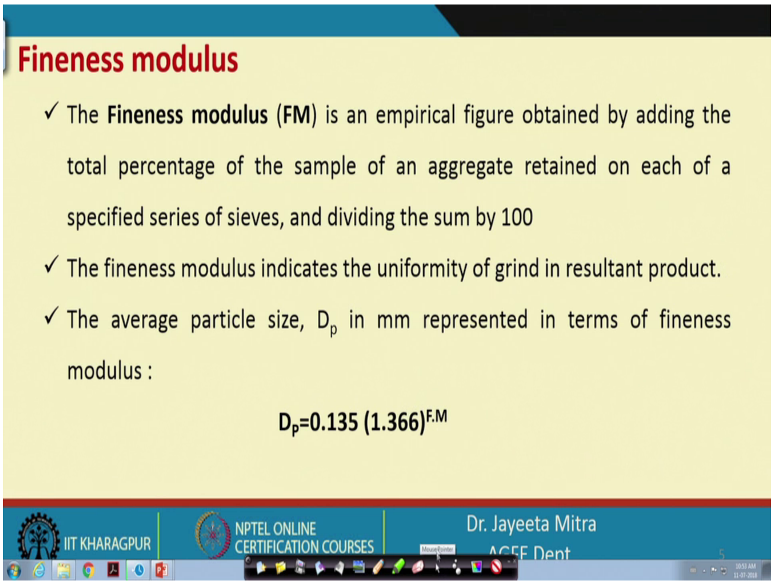

Fineness modulus

- ✓ The **Fineness modulus** (FM) is an empirical figure obtained by adding the total percentage of the sample of an aggregate retained on each of a specified series of sieves, and dividing the sum by 100
- ✓ The fineness modulus indicates the uniformity of grind in resultant product.
- ✓ The average particle size, D_p in mm represented in terms of fineness modulus :

$$D_p=0.135\,(1.366)^{F.M}$$

Now, fineness modulus again one very important concept, which is very important in particular solid analysis. So, fineness modulus is an empirical figure obtained by adding the total percentage of the sample of an aggregate retained on each of the specified series of sieve and dividing the sum by 100 ok. Adding the total percentage of the sample of an aggregate retained on each of the sieve of a specified series of sieves and then dividing by 100. That will give the value of fineness modulus.

Fineness modulus indicate the uniformity of grind in the end resultant product. So, the average particle size of D P in mm represented in terms of fineness modulus has D P will be equal to 0.135 into 1.366 to the power fineness modulus. So, fineness modulus which indicates the uniformity of grind in resultant product. So, uniformity is defined by fineness modulus. Now, once we have the value of fineness modulus, the average particle size D P can be determined by this equation 0.135 into 1.366 to the power fineness modulus ok.

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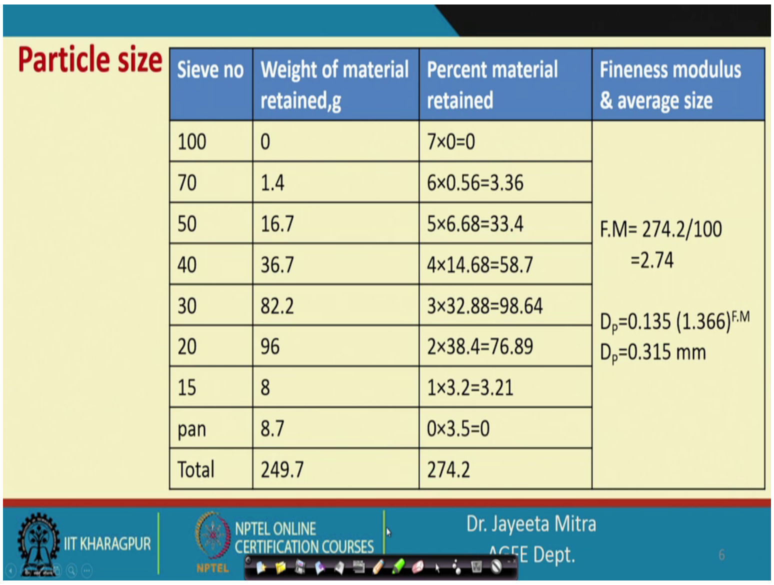

Particle size

Sieve no	Weight of material retained,g	Percent material retained	Fineness modulus & average size
100	0	7×0=0	F.M= 274.2/100 =2.74 D_p=0.135 (1.366)$^{F.M}$ D_p=0.315 mm
70	1.4	6×0.56=3.36	
50	16.7	5×6.68=33.4	
40	36.7	4×14.68=58.7	
30	82.2	3×32.88=98.64	
20	96	2×38.4=76.89	
15	8	1×3.2=3.21	
pan	8.7	0×3.5=0	
Total	249.7	274.2	

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So, let us have one case analysis for that, we have a sieve number 100, 70, 50, 40, 30, 20 15 and pan. So, 7 sieves and a pan this is the standard configuration that we are having for this particular sieve, then weight of the material retained on it in the gram. So, all have passed through the 100 number sieve. So, this is 0, then 1.0 gram retained on 17 number sieves 16.7 gram retained on 15 number sieves 36.7 gram retained on 40 number sieves 82.2 gram retained on 30 number sieves and 96 retained on 20 and so on finally, in the pan we are getting 8.7.

So, total is 249.7 this much gram. Now, then what we need to do is we need to see that from the total how much fraction or how much percentage is this 1.4 ok. So, first if we go from the bottom pan, we can be considered as 0 and number each of the consecutive a person upper sieves 1 2 3 up to 7. So, this 1 2 3 4 5 6 and 7 defining the number of sieve from the starting from the pan, ok.

Now, in the pan what is retained that is 8.7. So, if we calculate the percentage retained based on the total sample that is 3.5 percent, then the 8 it will be 3.2 percent, 96 gram will be 38.4 percentage 82.2 is coming 32.88 percentage, ok. So, like that will calculate all the percentage that is retained (Refer Time: 11:44) material retained. So, all the material that has been retained on that on the sieve, total metal that is retained on the sieve, ok. So, that we will count here only for only those retained on the sieve will take the total based on that and calculate the percentage material retained on each sieve, then we will add then. So, it is coming 274.2 total percent material retained, ok.

So, this exclude the material that is on the pan that is why we are getting 274.2. Now, fineness modulus and average size will be 274.2 divided by 100. So, we are getting 2.74. So, if this is the fineness modulus. So, what will be the D P? D P will be the 0.135 into 1.366 to the part 2.74. So, we are getting 0.315 mm.

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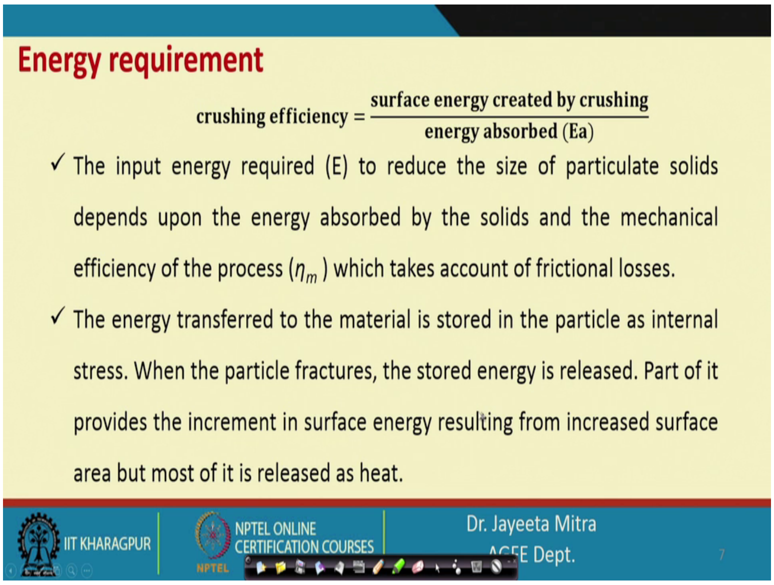

Energy requirement

$$\text{crushing efficiency} = \frac{\text{surface energy created by crushing}}{\text{energy absorbed (Ea)}}$$

- ✓ The input energy required (E) to reduce the size of particulate solids depends upon the energy absorbed by the solids and the mechanical efficiency of the process (η_m) which takes account of frictional losses.
- ✓ The energy transferred to the material is stored in the particle as internal stress. When the particle fractures, the stored energy is released. Part of it provides the increment in surface energy resulting from increased surface area but most of it is released as heat.

Now, energy requirement. So, since we are increasing the surface area by reduction of the size of the particle, in so, in the very beginning section where were discussing the introduction of size reduction, we mention that this is our most important goal that is to increase the surface area ok. Now, increase the surface area involved energy requirement right, because whenever we want to do any sort of work, so, we are we are giving certain amount of energy in the particle, initially which is make them strain.

So, that energy which we are applying that is stored in the particle and when it breaks into smaller particles so, that energy is released in terms of creating the surface area, and also in terms of creating the heat ok. So, we if we want to analyse the energy requirement we can calculate first one parameter that is the crushing efficiency which is very often we used and it defined as the surface energy created by crushing divided by the energy absorbed because of crushing ok.

So, the input energy required to reduce the size of the particles solid depends on the energy absorbed by the solid, and also there are certain frictional losses will be there and certain material, which is which is the equipment or the other rotating part of the machines that will absorb certain amount of you know applied energy. So, because of those frictional losses will take also the mechanical efficiency eta m into consideration.

So, the input energy that will give that will reduce the size of the particle ok depending on the energy absorbed for creating the surface and also the mechanical efficiency of the process, which takes into account of frictional losses. Now the energy transferred to the material is stored in the particle. As I say internal stresses, when the stored energy is released because of particle fracture part of it increase the surface you know increment in the surface energy resulting from the surface area and some are released as the heat energy.

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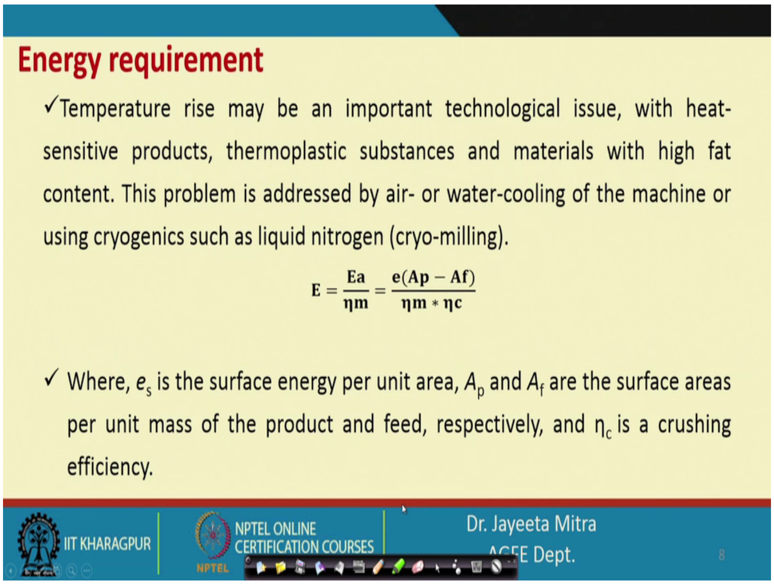

Energy requirement

✓Temperature rise may be an important technological issue, with heat-sensitive products, thermoplastic substances and materials with high fat content. This problem is addressed by air- or water-cooling of the machine or using cryogenics such as liquid nitrogen (cryo-milling).

$$E = \frac{Ea}{\eta m} = \frac{e(Ap - Af)}{\eta m * \eta c}$$

✓ Where, e_s is the surface energy per unit area, A_p and A_f are the surface areas per unit mass of the product and feed, respectively, and η_c is a crushing efficiency.

So, temperature rise because that heat energy is released; that means that will cause the temperature rise of the particle that are generated larger smaller particles are generated. Larger surface area created the temperature rise may be an important technological issue with heat sensitive products; thermoplastic substances and also the material with high fat content. So, this problem is address by air or water cooling of the mechanism, using the cryogenic such as the liquid nitrogen or cryo milling.

So, you might have heard that in case of mini spice grinding, this cryogenics grinding is approaches being taken because the spices are mostly having very heat sensitive components and antioxidants etcetera. So, to preserve we need an environment which is which is very chill environment for sometime pre cooling environment or in the casing we need to circulate the liquid nitrogen to reduce the effect of the temperature generation during the size reduction.

So, that total energy can be expressed as the energy absorbed by the mechanical energy, mechanical efficiency and that is energy absorb also can be represented as the energy required for the surface area creation divided by the crushing efficiency eta c.

So, e s is the surface energy per unit for surface energy per unit area; that means joule per meter square. Also, sometime it is expressed as surface tension that is Newton per meter and A p is the surface area per unit mass per unit mass of the product and A f is the feed surface area per unit mass respectively eta c is the crushing efficiency.

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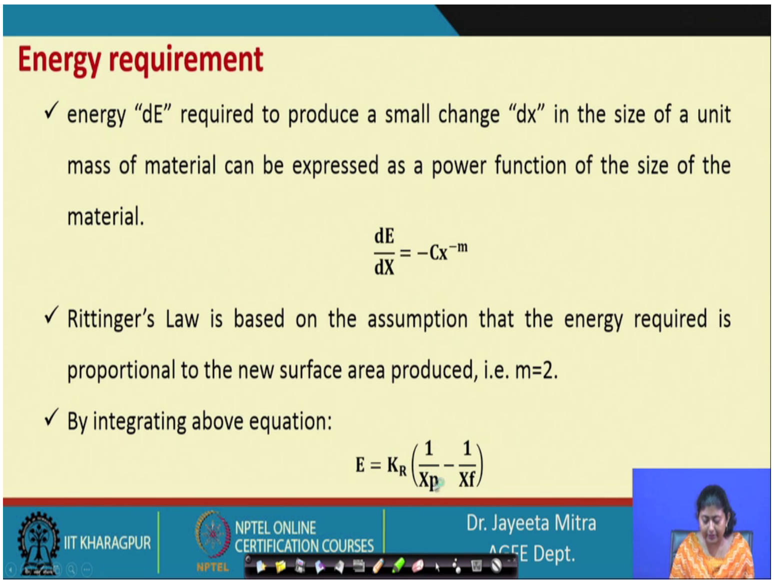

Energy requirement

- ✓ energy "dE" required to produce a small change "dx" in the size of a unit mass of material can be expressed as a power function of the size of the material.

$$\frac{dE}{dX} = -Cx^{-m}$$

- ✓ Rittinger's Law is based on the assumption that the energy required is proportional to the new surface area produced, i.e. m=2.
- ✓ By integrating above equation:

$$E = K_R\left(\frac{1}{Xp} - \frac{1}{Xf}\right)$$

So, energy d E required to produce small change dx that means, the size fraction size or the dimension of the material that is being reduced to the amount of the d X of the unit mass of material can be expressed as a power function of the size of the material. So, d E by d X will be equal to minus C into x to the power minus m. So, always it is inverse of the material size because as small we want to perform the size of the material larger amount the energy is required.

So, that is why, it is a normal convention to have the energy required per unit change dimension is always proportional to the inverse of some power of the dimension of the linear dimension of the characteristic dimensions.

So, there are 3 different laws of size reduction that is used basically for addressing the different size fraction. So, one is the Rittenger's law. This is based on the assumption that the energy required is proportional to the new surface area produced that is the surface area if d E by d X is proportional to 1 by the dimension d square. So, it will be m is equal to dimension a square m equal to 2. So, we are getting integration energy e that is equal to account standard is constant K R into 1 by X p that is the dimension of the product minus 1 by X f when the dimension of the feed.

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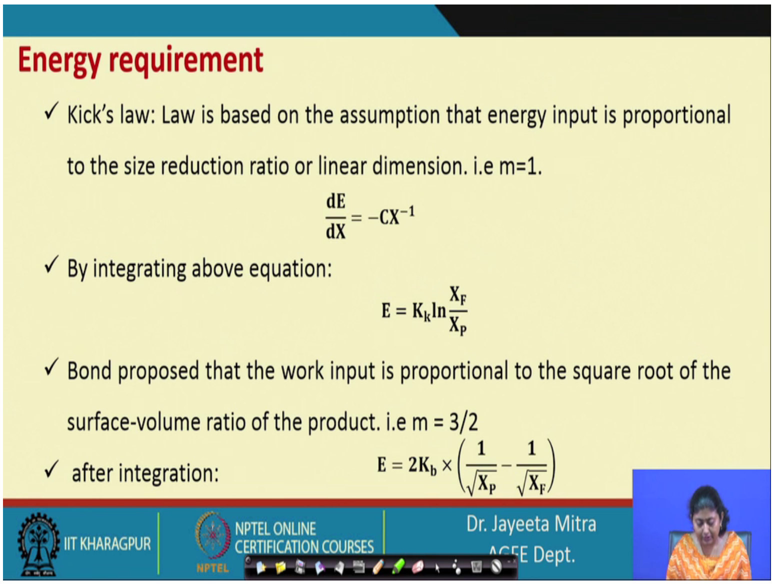

Energy requirement

- ✓ Kick's law: Law is based on the assumption that energy input is proportional to the size reduction ratio or linear dimension. i.e m=1.

$$\frac{dE}{dX} = -CX^{-1}$$

- ✓ By integrating above equation:

$$E = K_k \ln\frac{X_F}{X_P}$$

- ✓ Bond proposed that the work input is proportional to the square root of the surface-volume ratio of the product. i.e m = 3/2
- ✓ after integration: $E = 2K_b \times \left(\frac{1}{\sqrt{X_P}} - \frac{1}{\sqrt{X_F}}\right)$

So, another law which is Kick's law and in this law is based on the assumption that the energy input is proportional to the size reduction ratio or linear dimension that is m equal to 1. So, this will become d E by d X equal to minus C into X to the power minus 1. So, by integrating this, we are getting E equal K, k it is constant into ln of X f, that is the dimension, dimension of the feed to that of the product. And the last one is the Bond's equation. Bond proposed that the work input is proportional to the square root of the surface volume ratio of the product.

So, here m is taken has 3 by 2 and after integration we are getting energy requirement E is equal to 2 into K b which is want constant into 1 by root over X p minus 1 by root over

X f. So, this is just like equation d E by d X minus C into x to the power minus 3 by 2 and from that you are getting this equation by integrating.

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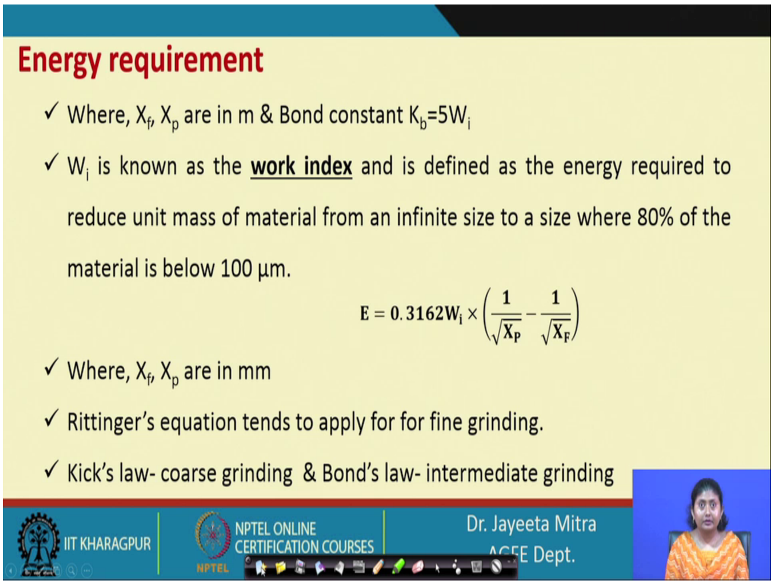

Energy requirement

- ✓ Where, X_f, X_p are in m & Bond constant K_b=5W_i
- ✓ W_i is known as the **work index** and is defined as the energy required to reduce unit mass of material from an infinite size to a size where 80% of the material is below 100 µm.

$$E = 0.3162W_i \times \left(\frac{1}{\sqrt{X_P}} - \frac{1}{\sqrt{X_F}}\right)$$

- ✓ Where, X_f, X_p are in mm
- ✓ Rittinger's equation tends to apply for for fine grinding.
- ✓ Kick's law- coarse grinding & Bond's law- intermediate grinding

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So, X f and X p are in metre Bond's constant K b that is equal to 5 into W i, W i is the work index and is defined as the energy required to reduce unit mass of a material from an infinite size to a size where 80 percent of the material is below 100 micrometre. So, E equal to 0.3162 into work index into 1 by root over X p minus 1 by root over X f X f and X p are in mm and Rittinger's equations tends to apply for fine grinding and Kick's law course grinding Bond's law valid for the intermediate grinding.

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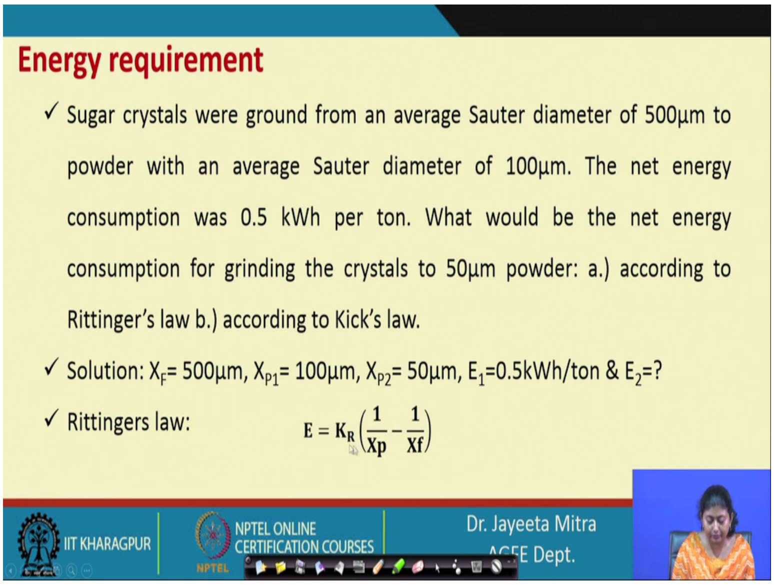

Energy requirement

- ✓ Sugar crystals were ground from an average Sauter diameter of 500µm to powder with an average Sauter diameter of 100µm. The net energy consumption was 0.5 kWh per ton. What would be the net energy consumption for grinding the crystals to 50µm powder: a.) according to Rittinger's law b.) according to Kick's law.
- ✓ Solution: X_F= 500µm, X_{P1}= 100µm, X_{P2}= 50µm, E_1=0.5kWh/ton & E_2=?
- ✓ Rittingers law: $E = K_R\left(\frac{1}{Xp} - \frac{1}{Xf}\right)$

So, here we will take one example where sugar crystals were ground from an average Sauter diameter of 500 micrometre to powder with an average Sauter diameter of 100 micrometre ok. The net energy consumption was 0.5 kilowatt hour per ton what would be the net energy consumption for grinding the crystal to 50 micrometre powder? First is, according to the Rittinger's law and second according to the Kick's law, so, X f will be 500 micrometre the feed size. For that, we need to make 100 micrometre and another is we require 50 micrometre.

So, energy net energy consumption in the first case what 0.5 kilowatt hour per ton we need to find E 2. Now, applying the Rittinger's law we can apply this equation E equal to K R into 1 by X p minus 1 by X f.

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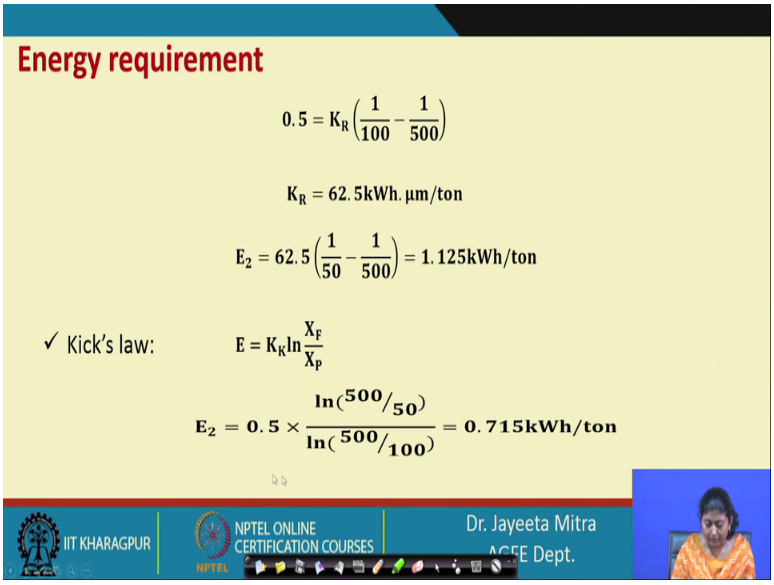

Energy requirement

$$0.5 = K_R\left(\frac{1}{100}-\frac{1}{500}\right)$$

$$K_R = 62.5\,\mathrm{kWh.\mu m/ton}$$

$$E_2 = 62.5\left(\frac{1}{50}-\frac{1}{500}\right) = 1.125\,\mathrm{kWh/ton}$$

✓ Kick's law: $$E = K_K \ln\frac{X_F}{X_P}$$

$$E_2 = 0.5 \times \frac{\ln(500/50)}{\ln(500/100)} = 0.715\,\mathrm{kWh/ton}$$

So, 0.5 will be equal to K R into 1 by X p is 1 by 100 minus 1 by X f is 500. So, from that, we can get the Rittinger's constant K R that is 62.5 kilowatt hour micrometre per ton. Now, using that we will calculate that what will be the you know energy requirement for Rittinger's law by making the particle to smaller size of 50 50 micron.

So, here we are getting 1.125 kilo watt hour per ton. Coming to the Kick's law, it will be again the ratio of K k of ln of X f by X p divided by X f by X p were that the changes both the time. Now, for that we can get this expression. So, ln of 500 by 50 divided by ln of 500 by 100 into 0.5 so, this we are getting 0.715 kilowatt hour per ton. We will stop here and will continue in the next class.

Thank you.