

Fundamentals of Food Process Engineering
Prof. Jayeeta Mitra
Department of Agricultural and Food Engineering
Indian Institute of Technology, Kharagpur

Lecture - 24
Heat Exchangers (Contd.)

Hello everyone, welcome to NPTEL online certification course on Fundamentals of Food Process Engineering; we are discussing now Heat Exchanger ok.

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- Definition
- Types of heat exchanger
- Analysis of heat exchanger
 - ✓ LMTD (both for parallel and counter flow)
 - ✓ Overall heat transfer coefficient
 - ✓ Fouling factor
 - ✓ Correction factor
 - ✓ Effectiveness of heat exchanger (both for parallel and counter flow)
- Application of heat exchanger in food industry
- Numerical problems

$$U = \frac{1}{\frac{1}{h_i} + \frac{dx}{K} + \frac{1}{h_o} + \frac{1}{h_{s1}} + \frac{1}{h_{s2}}}$$

The slide also features a small video inset of Dr. Jayeeta Mitra in the bottom right corner. The footer includes the IIT Kharagpur logo, NPTEL Online Certification Courses text, and Dr. Jayeeta Mitra, AGEE Dept.

So; we have already covered few topics including the definition, types of heat exchanger, then we have discussed the analysis of heat exchanger. In the analysis we have discussed Log Mean Temperature Difference: LMTD for both parallel flow and counter flow. So, if you remember the log mean temperature difference we generally, you know express this as θ_m or ΔT_m or some book you may find this as ΔT_L . So, its value is $\frac{\theta_2 - \theta_1}{\ln \frac{\theta_2}{\theta_1}}$ right and you just have to remember that what will be the value of θ_1 and θ_2 because if it is you know the parallel flow

ok.

So, across the area if you are; if you are drawing the temperature profile of both the fluid hot and cold; so θ_1 will be this one and θ_2 will be at the end, whatever gap we are getting temperature difference we are getting. And if it is parallel flow this is for the parallel flow and if we are discussing counter flow then so this is the direction of the hot

fluid and this is the direction of the cold fluid and if we are getting θ_1 here and θ_2 here. So, this will be $t_{h1} - t_{c1}$ and this is $t_{c2} - t_{h2}$ right where the here in the first case parallel flow, this is $t_{c1} - t_{c2}$ and this is $t_{h1} - t_{h2}$ right. So, in the counter flow case θ_1 will be $t_{h1} - t_{c2}$ and θ_2 will be $t_{h2} - t_{c1}$. So, this we have discussed in our previous classes on heat transfer, we have also discussed the overall heat transfer coefficient. So, if you remember the overall heat transfer coefficient that is U .

So, U has been derived by this expression that is when area is same in that case we can express this by this fashion, if it is for a plate otherwise we need to consider the equation for a cylinder if it is a tube that is $\ln(r_o/r_i)$ divided by $2\pi K l$, where l is the length of the tube and h_i is the inner side heat transfer coefficient convective heat transfer coefficient and h_o is the outside convective heat transfer coefficient right. Now when we discuss the fouling factor then with this the factors of fouling resistance of fouling for the inside and outside that will come so $1/h_{si}$ plus $1/h_{so}$.

So, this has been done when area is same, if area is not same we need to consider area in this equation that we have already discussed. And then we have also discussed the correction factor that depends on the various configuration of shell and tube heat exchanger that is number of tube passes and this for this we have calculated the temperature ratio and capacity ratio, temperature factor and capacity ratio and from those two using the chart we have we have determined the values of correction factor.

Now, today we will going to learn a very important concept in case of heat exchanger that is effectiveness, again we will discuss for both parallel flow and counter flow. After that we will discuss the application of heat exchanger in food industry, that is what are the various kinds of various types of heat exchanger that are commonly used. And we will solve a numerical problem ok.

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Analysis of heat exchanger

✓ Heat exchanger effectiveness and number of transfer units (NTU):

✓ To determine the inlet and outlet temperatures for a particular heat exchanger, the analysis done more easily by using a method based on heat exchanger effectiveness and number of transfer units (NTU).

✓ The heat exchanger effectiveness (ϵ) is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

$$\epsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{\max}}$$

The actual heat transfer of heat exchanger is given by:

$$Q = \dot{m}_h c_{ph} (t_{h1} - t_{h2}) = \dot{m}_c c_{pc} (t_{c2} - t_{c1})$$



So, now heat exchanger effectiveness and number of transfer units. So, effectiveness is very important; this in the general terms if you analyze the effectiveness; that means, that what is the maximum possible effect or heat transfer that we can get out of the heat exchanger ok. So, in this case we need to look that what is the actual heat transfer happening and what the maximum heat transfer can be done and the ratio of these will be termed as effectiveness.

So, determine the inlet and outlet temperatures for a particular heat exchanger, the analysis done more easily by using a method based on heat exchanger effectiveness and number of transfer unit. So, the effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer. So, actual heat transfer means the heat transfer from the hot fluid to the cold fluid that we are getting; that means, the $m c p \Delta t$ for the hot fluid or $m c p \Delta t$ for the cold fluid ok. So, effectiveness epsilon that is equal to actual heat transfer by maximum possible heat transfer so Q by Q_{\max} .

Now, the actual heat transfer of heat exchanger is given by as I said flow rate hot fluid $\dot{m}_h c_{ph}$ specific heat of hot fluid; so together this constitute the heat capacity of the hot fluid into temperature drop that is $t_{h1} - t_{h2}$ and this heat is taken by the cold fluid, so mass flow rate $\dot{m}_c c_{pc}$ specific heat of the cold fluid and temperature it has increase on t_{c1} to t_{c2} . So, $t_{c2} - t_{c1}$ this will be the value here for the change of temperature so, this is the actual amount of heat transfer happened.

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Analysis of heat exchanger

The maximum possible heat transfer will occur if the outlet temperature of the fluid with smaller values of heat capacity (C_h or C_c) is equal to the inlet temperature of other fluid. So we can say,

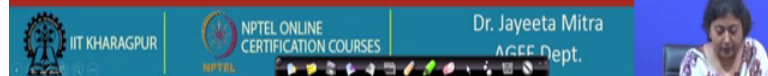
$$Q_{\max} = C_h (t_{h1} - t_{c1}) \text{ or } C_c (t_{h1} - t_{c1})$$

Q_{\max} will be minimum among these two values.

$$\epsilon = \frac{C_{\min} (t_{h1} - t_{c1})}{C_h (t_{h1} - t_{c2})} = \frac{C_c (t_{c2} - t_{c1})}{C_{\min} (t_{h1} - t_{c1})}$$

So heat transfer is given by the following equation,

$$Q = \epsilon C_{\min} (t_{h1} - t_{c1})$$



Now, the maximum possible heat transfer will occur if the outlet temperature of the fluid with smaller values of heat capacity that is either C_h or C_c is equal to the inlet temperature of other fluid. So, it means that heat capacity the term C_h or C_c right now so C_h or C_c which is respectively the heat capacity of hot fluid and cold fluid. So, this is equal to $m_h \dot{C}_p h$ and this is equal to $m_c \dot{C}_p c$ right, now if the flow rate of this fluid the hot fluid is less or if the specific heat that is heat required to increase per unit temperature a for that particular fluid is lower so this will be lower and similar is the case of cold fluid right. So, if this is low either C_h or C_c ; that means, the maximum heat transfer $t_{h1} - t_{c1}$ into $t_{h1} - t_{c2}$ ok.

So, Q is equal to $C_h (t_{h1} - t_{h2})$ or $C_c (t_{c2} - t_{c1})$. So, if this is small or this is small so this difference will be very high and if it happen that is the maximum heat transfer happens; so what will be there the outlet temperature of the fluid for which the heat capacity is minimum the outlet temperature will be inlet temperature of the other fluid ok. So, that is the maximum amount of heat transfer that may takes place right.

So, for example, if the if C_c if the cold fluid heat capacity of the cold fluid is the minimum in this case ok, so outlet temperature of the fluid with smaller values of heat capacity. So, then it is equal to the inlet temperature of other fluid right so in that case if this is minimum this is minimum t_{c2} will be equal to t_{h1} and if C_h is minimum. So, t_{h2} will be equal to t_{c1} .

So, then we can say that Q_{max} will be $C_h (t_{h1} - t_{c1})$ or $C_c (t_{h1} - t_{c2})$, this happens because $t_{h1} - t_{h2}$ now since, C_h is minimum so t_{h2} will be transferred by t_{c1} outlet will be inlet of the other fluid so, it is t_{c1} or C_c if it is minimum then $t_{c2} - t_{c1}$ so, outlet of cold fluid will be equal to inlet of the other fluid t_{h1} right. So, maximum heat transfer will be this one that is a maximum extent of transfer of heat we can achieve. So, Q_{max} will be minimum among these 2 values ok. So, now Q_{max} that is C_{min} whatever it may be either C_h or C_c both the case; what we are getting that $C_{min} (t_{h1} - t_{c1})$ or $C_{min} (t_{h1} - t_{c1})$ here also $t_{h1} - t_{c1}$ only thing either C_h is minimum or C_c is minimum right.

So, we can write Q_{max} equal to $C_{min} (t_{h1} - t_{c1})$ so, effectiveness will be the actual transfer in case of hot fluid that we are getting that is $C_h (t_{h1} - t_{h2})$ by C_{min} . So, if C_h is C_h becomes minimum in that case outlet will be t_{h2} will change by t_{c1} so we can write $C_{min} (t_{h1} - t_{c1})$ ok. Similarly, if we consider cold fluid for finding the effectiveness we can write actual heat transfer that is $C_c (t_{c2} - t_{c1})$ divided by $C_{min} (t_{h1} - t_{c1})$. So, in this case C_h is minimum and in this case C_c is minimum ok. So, this will be effectiveness.

So, heat transfer is given by the following equation that is Q is equal to effectiveness so Q will be equal to effectiveness into $C_{min} (t_{h1} - t_{c1})$ because, actual heat transfer, actual heat transfer is this one or this one ok. So, this is equal to Q_{actual} heat transfer either this or this, now actual heat transfer will be in effectiveness into this value $C_{min} (t_{h1} - t_{c1})$.

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Analysis of heat exchanger

Number of transfer unit method (NTU):
Heat exchanger effectiveness is a function of several variables. But it is inconvenient to combine them in a graphical form. Heat exchanger effectiveness can be expressed as a function of three non dimensional parameters, which is known as NTU method.

Dimension less parameters:

- ❖ $NTU = (UA/C_{min})$
- ❖ $R = C_{min} / C_{max}$
- ❖ Flow arrangement.

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Now, what is number of transfer unit method ok, NTU method it is very common in case of calculating the heat exchanger effectiveness in terms of NTU we generally calculate and it just given in the specification of heat exchanger as well. So, heat exchanger effectiveness is a function of several variables, but it is inconvenient to combine them in a graphical form.

Heat exchanger effectiveness can be expressed as a function of three non dimensional parameter, which is known as NTU method so, what are those parameters, first is NTU the value is $U A$ by C_{min} U is the overall heat transfer coefficient watt per meter square degree Celsius or meter square degree Kelvin, A is the area meter square by C_{min} that is $m \cdot \text{mass flow rate into specific heat}$. So, this is one parameter the other is R capacity ratio C_{min} by C_{max} and then the other factor that we need to consider is flow arrangement.

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Analysis of heat exchanger
Effectiveness of parallel flow heat exchanger

The heat exchange dQ through an area dA is given by:

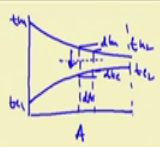
$$dQ = U \cdot dA (t_h - t_c)$$

$$= -\dot{m}_h c_{ph} \cdot dt_h = \dot{m}_c \cdot c_{pc} \cdot dt_c$$

$$= -C_h \cdot dt_h = C_c \cdot dt_c$$

$$dt_h = \frac{-dQ}{C_h} \quad \text{and} \quad dt_c = \frac{dQ}{C_c}$$

$$d(t_h - t_c) = -dQ \left[\frac{1}{C_h} + \frac{1}{C_c} \right]$$

$$\frac{d(t_h - t_c)}{(t_h - t_c)} = -U \cdot dA \left[\frac{1}{C_h} + \frac{1}{C_c} \right]$$


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So, first we will see the effectiveness of parallel flow heat exchanger so for parallel flow heat exchanger in the in the previous class we have derived this for the log mean temperature difference that: how we can relate the differential change in the temperature for a differential area and then we have equated the change for hot and cold fluid to find the log mean temperature difference. So, here also we will do the same thing so just to have the idea of the heat transfer I am drawing the profile temperature profile. So, this is the hot fluid and this is the cold fluid and I am taking a small area dA . So, this is t_{h1} t_{c1} t_{h2} t_{c2} and this one is dt_h dt_c right. So, heat is coming from the hot fluid to the cold fluid.

The heat exchange dQ through the area dA so; heat exchange that will be U into dA into t_h minus t_c . So, U is the overall heat transfer coefficient as I mentioned that there will be a surface, there will be a surface through which these two fluids are exchanging heat because always heat exchange happens through happens through a surface in case of heat exchanger. So, overall heat transfer coefficient U based on the conductivity of the surface and both the side hot fluid and cold fluid side convective heat transfer coefficient or fouling whatever may be there we will calculate U we have already discussed how overall convective heat transfer coefficient can be determined right.

So, U into dA that is the elemental area into temperature difference between the hot and cold fluid so this difference is the driving force for moving dQ amount of heat through the dA amount of area from the hot fluid to the cold fluid. So, for the hot fluid we can write; \dot{m}_h into C_{ph} specific heat into dt_h the temperature drop in the hot fluid and

we are putting minus sign, because it is reducing with increasing A that is equal to m c dot cold fluid flow rate into C p c into d t c so this is increasing towards increasing A.

So, minus C h into d t h that is equal to C c into d t c this is equal to d Q. So, now, we know that we have already seen that we need to find d t h minus d t c. So, we can write that d Q by this C h and here d Q by C c so what we will do now see d t h equal to minus d Q by C h and d t c is d Q by C c. So, d of t h minus t c, that is the temperature difference between the small area d A that is equal to minus d Q 1 by C h plus 1 by C c. So, C h and C c heat capacity of the hot and cold fluid respectively.

So, then again this d Q that we can write in this form that is U into d A into here t h minus t c was there I have taken this t h minus t c to the left side so d of t h minus t c by t h minus t c. So, the similar form like d theta by theta that we have got in the previous class so, ln theta that will be the term and then we can if we integrate this and here also d A will be; integrated and the value of A will come so finally, the equation that we will get ok.

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Analysis of heat exchanger

Integrating we get,

$$\ln \left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})} \right] = -UA \left[\frac{1}{C_h} + \frac{1}{C_c} \right]$$

$$\ln \left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})} \right] = -\frac{UA}{C_h} \left(1 + \frac{C_h}{C_c} \right)$$

$$\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})} \right] = \exp \left[- (UA/C_h) \{1 + (C_h/C_c)\} \right]$$

We know,

$$\epsilon = \frac{C_h (t_{h1} - t_{h2})}{C_{min} (t_{h1} - t_{c1})} = \frac{C_c (t_{c2} - t_{c1})}{C_{min} (t_{h1} - t_{c1})}$$

$$t_{h2} = t_{h1} - \frac{\epsilon C_{min} (t_{h1} - t_{c1})}{C_h}$$

$$t_{c2} = t_{c1} + \frac{\epsilon C_{min} (t_{h1} - t_{c1})}{C_c}$$

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So after integration we are getting this equation, we are integrating from 1-2 so ln of t h 2 minus t c 2 t h 2 minus t c 2 divided by t h 1 minus t c 1 and that is equal to minus U A 1 by C h plus 1 by C c. So, then here what we can do is minus U A by C h we can take 1 by C h common so this will be 1 1 plus C h by C c and this term left hand side we can write t h 2 minus t c 2 divided by t h 1 minus t c 1 that is equal to e to the power minus U

$A = C_h (T_{h1} - T_{h2})$ into $1 + C_h (T_{h1} - T_{h2}) / C_c (T_{c2} - T_{c1})$ ok. So, then what we can do is; we will relate with effectiveness, effectiveness what was actual heat transfer divided by the maximum possible heat transfer. So, effectiveness for the hot fluid was $C_h (T_{h1} - T_{h2}) / C_{\min} (T_{h1} - T_{c1})$ and for the cold fluid it was $C_c (T_{c2} - T_{c1}) / C_{\min} (T_{h1} - T_{c1})$ ok. So, from this equation and from this equation we need to calculate T_{h2} and T_{c2} ok.

So, considering this one we can write that effectiveness into $C_{\min} (T_{h1} - T_{c1})$, we can write effectiveness into $C_{\min} (T_{h1} - T_{c1})$ this is equal to $C_h (T_{h1} - T_{h2})$ right. So, I can take C_h this side right and we can also write that $T_{h2} - T_{h1} + T_{h1} - T_{h2}$ this is equal to $T_{h1} - \epsilon C_{\min} (T_{h1} - T_{c1}) / C_h$ ok. Similarly, if I consider this part I can write effectiveness into $C_{\min} (T_{h1} - T_{c1})$ this is equal to $C_c (T_{c2} - T_{c1})$ and here again I will take this C_c this side so, then my $T_{c2} - T_{c1}$ will be $T_{c1} + \text{effectiveness} \times C_{\min} (T_{h1} - T_{c1}) / C_c$ ok.

So, very clearly you can understand that see T_{h2} will be the lower temperature than T_{h1} so, $T_{h1} - \text{effectiveness} \times C_{\min} (T_{h1} - T_{c1}) / C_h$ this value, similarly T_{c2} will be higher than T_{c1} because, T_{c1} is going to increase to T_{c2} so, $T_{c1} + \text{effectiveness} \times C_{\min} (T_{h1} - T_{c1}) / C_c$ by this. So basically, this is kind of a; you know kind of the fraction that we are achieving by the ratio of C_{\min} by C_c so, if it is very close to this so, we can get effectiveness higher right.

So, because if effectiveness will be 1 in that case C_{\min} will be equal to C_c and this T_{c2} will be eventually T_{h1} so this is how we have found out T_{h2} and T_{c2} . Now, in the previous in this case that we are we are getting $T_{h2} - T_{c2}$ here we will substitute these two equation ok. So, from here $T_{h2} - T_{c2}$ we will put this two equation to remove this value T_{h2} and T_{c2} right.

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Analysis of heat exchanger

Eliminating t_{h2} and t_{c2} from the above equation we get,

$$\frac{1}{(t_{h1} - t_{c1})} \left[(t_{h1} - t_{c1}) - \epsilon C_{min} (t_{h1} - t_{c1}) \left(\frac{1}{C_h} + \frac{1}{C_c} \right) \right] = \exp[-(UA/C_h) (1 + C_h/C_c)]$$

$$1 - \epsilon C_{min} \left(\frac{1}{C_h} + \frac{1}{C_c} \right) = \exp[-(UA/C_h) (1 + C_h/C_c)]$$

$$\epsilon = \frac{1 - \exp[-(UA/C_h) (1 + C_h/C_c)]}{C_{min} \left(\frac{1}{C_h} + \frac{1}{C_c} \right)}$$

□ If $C_c > C_h$ then,

$$\epsilon = \frac{1 - \exp[-(UA/C_{min}) (1 + C_{min}/C_{max})]}{1 + (C_{min}/C_{max})}$$

□ If $C_c < C_h$ then

$$\epsilon = \frac{1 - \exp[-(UA/C_{max}) (1 + C_{max}/C_{min})]}{1 + (C_{min}/C_{max})}$$

Combining both the two equation we get,

$$\epsilon = \frac{1 - \exp[-(UA/C_{min}) (1 + C_{min}/C_{max})]}{1 + (C_{min}/C_{max})}$$

So, eliminating t_{h2} and t_{c2} from that equation what we are getting that we have put the values of t_{h2} and t_{c2} in terms of the other parameter that just we have derived so 1 by t_{h1} minus t_{c1} that is equal to t_{h1} minus t_{c1} minus effectiveness into C_{min} t_{h1} minus t_{c1} into 1 by C_h plus 1 by C_c this will be; the right side is same exponential minus UA by C_h into 1 plus C_h by C_c .

Now, from this we can calculate if if we can take t_{h1} minus t_{c1} common so this will be cancelled so, 1 minus so we are we are taking this factor t_{h1} minus t_{c1} from here and here so this will be cancel so 1 minus effectiveness into C_{min} ok, 1 by C_h plus 1 by C_c this is equal to exponential of this term right. So, effectiveness can be termed as 1 minus exponential minus UA by C_h into 1 by C_h by C_c divided by C_{min} into 1 by C_h plus 1 by C_c .

Now the thing is which one is minimal C_h or C_c based on that we can define effectiveness, first case if C_c is greater than C_h ; that means, C_h is our C_{min} in that case here, see what will happen that 1 minus UA by C_h . So, C_h is C_{min} now in into 1 plus C_h that is C_{min} by C_{max} divided by C_{min} into C_h is minimum here.

So, we take this minimum out so or we take this C_{min} inside so we will getting 1 plus C_{min} by C_c that is C_{max} . And the 2nd case would be if C_c is less than C_h that is if C_c is the minimum, then the value will be 1 minus exponential minus UA by C_{max} into 1 plus C_{max} by C_{min} , because C_h by C_c so C_h is high now and C_c is the

minimum divided by 1 plus C minimum by C max ok, because C c is less than C h so C c is the minimum one. So, now we can write here this is minimum so we are getting 1 plus C min by C max here. So, combining both the equation we can get effectiveness is equal to 1 minus exponential minus U A by C min into 1 plus C min by C max divided by 1 plus C min by C max so this is the common equation that we are getting.

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Analysis of heat exchanger

The effectiveness of parallel flow heat exchanger is given by,

$$\epsilon = \frac{1 - \exp[-NTU(1+R)]}{1+R}$$

Effectiveness of counter flow heat exchanger

In this case the temperature of the both the fluid decreases along the length of heat exchange. So the heat flow equation is given as follows:

$$\begin{aligned} dQ &= U \cdot dA (t_h - t_c) \\ &= -\dot{m} c_{p,c} dt_c = -\dot{m} c_{p,h} dt_h \\ &= -C_c dt_c = -C_h dt_h \end{aligned}$$

Rest of the derivation approach is same as parallel flow case. So, the final derived equation is follows:

$$\epsilon = \frac{1 - \exp[-NTU(1-R)]}{1 - R \exp[-NTU(1-R)]}$$

The slide also includes a temperature profile diagram showing two curves (hot and cold) that decrease along the length of the heat exchanger. A small video inset shows Dr. Jayeeta Mitra, AGEE Dept.

So, the effectiveness of parallel flow heat exchanger is given by 1 minus exponential minus U A by C min that is replaced by NTU number of transfer unit into 1 plus C min by C max that is the capacity ratio R divided by 1 plus R. So, effectiveness for counter flow heat exchanger; this will be done in a similar fashion so, the temperature of both the fluid decreases along the length of heat exchange so the heat flow equation is given as follows: here because the framework will be the configuration of system is such that temperature profile if we draw it will be kind of this so this is t h 1 this is t h 2 and this is t c 1 t c 2 so across the small area d A.

So, if we look towards the small area change we are getting d t h that is negative and also if we move to this direction so temperature a temperature profile of the cold is also decreasing right. So, we can write here as d Q is equal to U d A t h minus t c minus m dot C p h d t h so similar fashion as we did for the earlier case so minus C h into d t h that is equal to minus C c into d t c.

So, rest of the derivation approach is similar as we did for the parallel flow, similarly we will just have $d t_h$ minus $d t_c$ in terms of $d Q C_h$ and C_c ok. So, the expression will come as effectiveness equal to $1 - e^{-NTU}$ into $1 - R$ divided by $1 - R$ that is again capacity ratios C_{\min} by C_{\max} into exponential minus NTU into $1 - R$ ok.

So, we will stop here we will continue in the next class.

Thank you.