## Fundamentals of Food Process Engineering Prof. Jayeeta Mitra Department of Agricultural and Food Engineering Indian Institute of Technology, Kharagpur

# Lecture - 22 Heat Exchangers (Contd.)

Hello everyone, welcome to NPTEL online certification course on Fundamentals of Food Process Engineering. We will continue with the topic of Heat Exchanger today.

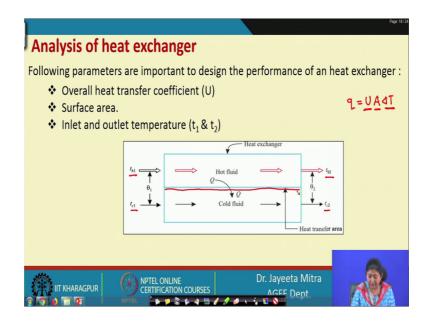
(Refer Slide Time: 00:31)

Page 10.2		
Contents:		
Definition		
Types of heat exchanger		
Analysis of heat exchanger		
$\checkmark$ LMTD (both for parallel and counter flow)		
✓ Overall heat transfer coefficient		
✓ Fouling factor		
✓ Correction factor		
$\checkmark$ Effectiveness of heat exchanger (both for parallel and counter flow)		
<ul> <li>Application of heat exchanger in food industry</li> </ul>		
Numerical problems		
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So, in the last class we have discussed few things the definition types of heat exchanger and today we will start analysis of heat exchanger ok.

So, as you know that heat exchangers are very important equipment and that is most commonly used many food processing application. So, we need to know that what are the basic parameters or important parameter, based on what thing we can design a heat exchanger what are the important parameters. So, today we will learn those things.

### (Refer Slide Time: 01:17)



So, analysis of heat exchanger if we want to do, the first thing is we need to know that the overall heat transfer coefficient this is very important factor. Another important is the surface area, then inlet and outlet temperature t 1 and t 2 of the 2 fluid right. So, if we want to analyze a heat exchanger, the main important parameters are what are the 2 fluids and what are the inlet and exit temperature that we want; that means, if we want to use the liquid the cold fluid which is our interest.

So, how far we need to increase the temperature from t c 1 to t c 2 and for that what is the amount of hot water or any hot fluid is needed, what will be the temperature then we need to know that the surface area the surface area available for heat transfer this is very important because if surface area changes then; obviously, the rate of heat transfer changes. Because normally the rate of heat transfer is expressed as q equal to U A delta T where U is the overall heat transfer coefficient A is the area and if A changes so, in that case because U is constant. So, then delta T will also vary right. So, that is why this is very important parameter. (Refer Slide Time: 03:14)

Page 10/2			
Analysis of heat exchanger			
Following parameters are important to design the performance of an heat exchanger :			
<ul> <li>Overall heat transfer coefficient (U)</li> </ul>			
<ul> <li>Surface area.</li> </ul>			
Inlet and outlet temperature ( $t_1 \& t_2$ )			
$ \begin{array}{c}                                     $			
Overall energy balance in a heat exchanger			
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So, heat exchange here happens via the heat exchange surface and when we try to analyze this thing, the first thing we need to know is we need to balance we need to see the overall energy balance or heat balance between the 2 fluid.

(Refer Slide Time: 03:34)

	Page 20128
Analysis of heat exchanger	try . A su
Heat given up by hot fluid,	$Q = \dot{m}_h c_{ph} (t_{h1} - t_{h2}) \frac{1}{b_{k_1}}$
Heat picked up by cold fluid,	$Q = \dot{m}_{c} c_{pc} (t_{c2} - t_{c1})$ $Q = \dot{m}_{c} c_{pc} (t_{c2} - t_{c1})$
Total heat transfer rate in heat exchan	
	Q=UA4T =4t
$\dot{m}$ = mass flow rate	
$C_p$ = specific heat of a fluid	
$\Delta t$ = temperature rise or drop across	
$\theta_m$ = log mean temperature different	nce
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So, now considering the figure that we have seen just now, that the hot fluid is entering and the cold fluid is also entering and those 2 have a similar direction. So, they are in the parallel flow direction. So, if we want to analyze now the heat given up by the hot fluid ok so, that will be Q into m h that is the mass flow rate of the hot fluid C p h specific heat of the hot fluid and t h 1 minus t h 2 t h one is greater than t h 2.

So, heat given by this will be this quantity whereas, the similar amount of heat will be picked up by the cold fluid, if we assume that there is no heat loss right. So, the heat gain by the cold fluid will be m c dot that is the flow rate of this into C p c the specific heat into t c 2 minus t c 1 ok so, because it is increasing the temperature from t c one low temperature to the hard temperature t c 2 right.

So, total heat transfer rate in the heat exchanger is equal to UA theta m. So, this total heat transfer is UA theta m because this whatever heat it is given the hot fluid is giving and the cold fluid is taking, that is happening across a surface area surface area A, and theta m this is actually signifying the difference between the hot and cold fluid. So, theta m we can write in this way t h minus t c right.

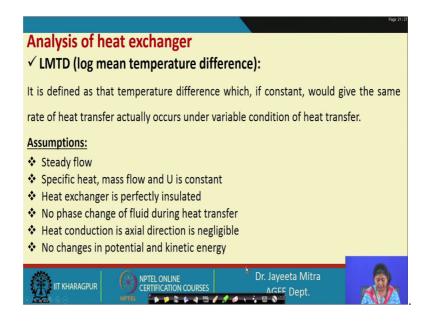
So, m dot is the mass flow rate m h dot indicates for the hot fluid mass flow rate and m c dot is the mass flow rate of the cold fluid. C p again the specific heat as I mentioned for the hot and cold fluid, delta t is the temperature rise or drop across the heat exchanger. So, theta m can be termed as theta m this can be termed as delta t right. So, any terminology you can use either delta t or theta m both are same ok.

So, now theta m is log mean temperature difference ok. So, why we are telling it as log mean temperature difference? I mean delta t which is the delta t which is the gradient delta t which is the gradient between the 2 fluid ok. So, this is actually the temperature rise or drop across the heat exchanger ok.

So, let us say if since this 2 are parallel flow. So, this way the heat flow is going this is the t h 1, t h 2, t c 2, t c 1 right. So, delta t this difference can be can be calculated in many ways right. So, one such way is by calculating the log mean temperature difference, and we will see that why log mean temperature difference is so important ok.

So, we can write this Q that is equal to UA delta T signifying that delta T is the pressure drop and we also can signify it as UA delta t m where delta where theta m is log mean temperature difference between these 2 fluid stream.

#### (Refer Slide Time: 08:01)



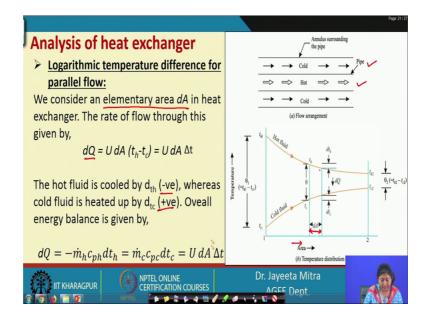
So, what is log mean temperature difference right? So, it is defined as the temperature difference which if constant would give the same rate of heat transfer that actually occurs under variable condition of heat transfer. So, let us check this by calculating the value of LMTD.

So, most of the cases in case of heat exchanger we consider the steady flow, because in heat exchanger it normally allows to run for a longer time ok. So, generally the heat transfer in case of heat exchanger follows the steady state. So, we take sudden assumption here to calculate the log mean temperature difference, first is the steady flow will be occurring in the heat exchanger steady flow of both the stream then the next is specific heat mass flow and overall heat transfer coefficient is constant ok.

So, we will discuss the overall heat transfer coefficient concept in detail and may be some of you have learned this in your heat transfer course; however, we will discuss that. For that for this moment you understand that this is the overall heat transfer coefficient specific heat and mass flow will also be constant.

Then heat exchanger is preferably insulated; that means we consider that there is no heat loss to the ambience, whatever heat the cold fluid is getting that is coming directly from the hot fluid no phase change of the fluid during heat transfer. So, the heat that is transferring is causing sensible change of both the fluid, no latent heat or no phase change is involved and the last is heat conduction in axial direction is negligible ok. However, one more assumption you might take that no change in the potential and kinetic energy will be there.

(Refer Slide Time: 10:12)



So, with this assumption we will see that logarithmic temperature difference for parallel flow right. So, parallel flow we know the configuration the hot fluid and the cold fluid are moving in a similar direction and temperature profile across the area if you draw if you plot it, you are getting this kind of a plot which is the difference between these 2 that is delta t or if you take theta. So, that theta is gradually decreasing theta 1 t h 1 minus t c 1 and theta 2 is t h 2 minus t c 2.

So, if we consider an elementary area d A here in the heat exchanger, the rate of flow through this elementary area will be d Q let us say. So, that is U d A into difference between 2 streams t h minus t c or U into d A into delta t as of now we are not considering that this is LMTD, just we are taking that delta t is the temperature gradient between hot and cold stream. So, the hot fluid is cooled ok.

So, the d t h; that means, the lowering of the temperature is happen. So, we are considering this as negative whereas, d t c if you consider that is the change, the differential change in the cold stream in the temperature that is positive right. So, d Q will be equal to minus m h dot, that is mass flow rate into C p h specific heat into d t h we are putting minus sign because of that and this q will be m c dot C p c into d t c. So, there is a positive change in the direction of the area. So, that is U A delta t.

(Refer Slide Time: 12:27)

	Page: 22 / 28
Analysis of heat exchanger	
$dt_h = -\frac{dQ}{\dot{m}_h c_{ph}} = \frac{dQ}{C_h}$	
$dt_c = rac{dQ}{\dot{m}_c c_{pc}} = rac{dQ}{C_c}$	
$C_h$ = heat capacity or water equivalent of hot fluid	
$C_c$ = heat capacity or water equivalent of cold fluid	
$\dot{m_h}$ and $\dot{m_c}$ are mass flow rate of hot and cold fluid	
Subtracting above two equations we get,	
$dt_h - dt_c = -dQ[\frac{1}{C_h} + \frac{1}{C_c}]$	
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So, then we can write d t h is equal to minus d Q by m h dot C p h or d Q by C h where C h is the heat capacity or water equivalent of hot fluid. And d t c can be written as d Q by m c dot C p c or d Q by C c where C c is the heat capacity or water equivalent of the cold fluid. So, heat capacity of the cold fluid.

Now, subtracting these 2 d t h minus d t c because this is what this is actually the gradient of temperature between those 2 stream across the area d A. So, d t h minus d t c will be we can take this minus common; so, 1 by C h plus 1 by C c. So, minus d Q from here and here we have taken common. So, 1 by C h plus 1 by C c remains.

(Refer Slide Time: 13:40)

		Page: 23 / 29
Analysis of heat exchanger	dQ=UdASt	
$d\mathbf{b}_{\mathbf{k}} - d\mathbf{k} \cdot \mathbf{d} \theta = -dQ[\frac{1}{c_{\mathbf{k}}} + \frac{1}{c_{\mathbf{k}}}]$		
$db_{\mathbf{h}} - d\mathbf{k} \cdot d\theta = -dQ[\frac{1}{C_h} + \frac{1}{C_c}]$ $d\theta = -U  dA  (t_h - t_c)[\frac{1}{C_h} + \frac{1}{C_c}]$ $d\theta = -U  dA  \theta[\frac{1}{C_h} + \frac{1}{C_c}]$		
$\int_{1}^{2} \frac{d\theta}{\theta} = -U \int_{A=0}^{A=A} \frac{dA}{dA} \left[ \frac{1}{C_{h}} + \frac{1}{C_{c}} \right]$		
$\ln(\theta_2/\theta_1) = -U  dA  \left[ \frac{1}{c_h} + \frac{1}{c_c} \right]$		
$Q = C_h(t_{h1}-t_{h2}) = C_c(t_{C2}-t_{C1})$		
$\frac{1}{C_h} = \frac{(\mathbf{t}_{h1} - \mathbf{t}_{h2})}{Q} \checkmark$		
$\frac{1}{C_c} = \frac{(\mathbf{t}_{c2} - \mathbf{t}_{c1})}{Q}  \checkmark$		
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Then what we can write is d theta that is nothing, but d t h minus d t c ok. So, d theta that is equal to minus d Q 1 by C h plus 1 by C c ok.

So, again d theta equal to minus U because this Q can be written as U d A into theta the temperature difference. So, that is t h minus t c delta t if we consider that d Q this is equal to U d A delta t. So, we can write delta t as t h minus t c into 1 by C h plus 1 by C c. So, this t h minus t c is termed as theta and theta should U d A theta into 1 by C h plus 1 by C c. Now we need to integrate this because d theta is the differential change in the temperature difference of the 2 sides or 2 fluids from 1 to 2 section ok. So, A equal to 0 to A equal to A if we integrate initially a equal to 0 to A equal to A; that means, the total area.

So, theta will vary from theta 1 to theta 2 theta 1 is the difference of those 2 stream in the inlet condition and theta 2 is the difference of those 2 stream in the exit condition. So, integrating this d theta by theta from 1 to 2 that is equal to minus U A equal to 0 to A equal to A d A 1 by C h plus 1 by C c. And we are finally, getting this equation 1 n theta 2 by theta 1 that is equal to minus U d A 1 by C h plus 1 by C c ok. So, heat capacity of the hot fluid plus 1 by heat capacity of the cold fluid.

Now, the heat Q that is the total heat Q has been given by the hot fluid that is C h into t h 1 minus t h 2 ok. So, if we want to write that heat balance. So, we are getting this one that is equal to C c into t C 2 minus t C 1 because h 1 to h 2 the change has happen the

heat it has given that is been taken by the cold fluid; so, t C 2 minus t C 1. So, from here we can write 1 by C h 1 by C h that is equal to t h 1 minus t h 2 by q and 1 by C c is t c 2 minus t c 1 by Q. Now this one and this one will going to put in this equation.

Analysis of h	eat exchanger	t te	Page: 24
Substituting,	$\ln(\theta_2/\theta_1) = -U  dA  \left[ \frac{\mathbf{t}_{h1} - \mathbf{t}_h}{Q} \right]$	$\frac{h^2}{Q} + \frac{t_{C2} - t_{C4}}{Q}$ $t_{m} - t$	$z = \theta_2 \checkmark$
	$\ln(\theta_2/\theta_1) = \frac{UA}{Q} (\theta_2 - \theta_1)$		
	$Q = \frac{UA(\theta_2 - \theta_1)}{\ln(\theta_2/\theta_1)}$		
Here,	$Q = UA \theta_m$	$\theta_m = \log \text{ mean temp}$ (LMTD).	erature
	$\theta_m = rac{( heta_2 -  heta_1)}{\ln( heta_2/ heta_1)}$	2	
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So, substituting the values of 1 by C h and 1 by C h and 1 by C c; so, we are getting that t h 1 minus t h 2 plus t c 2 minus t c 1. And if I take this minus inside I can write t h 2 minus t c 2 minus t h 1 minus t c 1 right. So, here t h 2 minus t c 2 this is equal to theta 2 and t h 1 minus t c 1 this is equal to theta 1 the temperature difference of the both fluid both fluid stream in the inlet and exit section. So, I can write U A by Q into theta 2 minus theta 1 or Q will be equal to U A theta 2 minus theta 1 divided by 1 n theta 2 by theta 1. So, that is again termed as U A theta m right.

So, theta m is this value theta 2 minus theta 1 divided by l n of theta 2 by theta 1. So, this is called log mean temperature difference LMTD. So, that is why we can calculate that why this log mean temperature difference is very important, and normally we use this one. Now this we have solved for parallel flow the similar case we can solve for counter flow as well.

(Refer Slide Time: 18:49)

	Page: 25/31
Analysis of heat exchanger	Annulus surrounding the pipe
LMTD for counter flow:	$\begin{array}{cccc} \leftarrow & \leftarrow & Cold & \leftarrow & \leftarrow & \checkmark \\ \hline \Rightarrow & \Rightarrow & Hot & \Rightarrow & \Rightarrow \\ \end{array}$
For counter flow heat exchanger energy	(a) Flow arrangement
balance equation is given as follows:	
$Q = -\dot{m}_h c_{ph} dt_h = -\dot{m}_c c_{pc} dt_c = U  dA  \Delta t$	$\begin{array}{c} \uparrow & \iota_{h} \\ \theta_{i} \\ \downarrow \\ t_{c} \end{array} \begin{array}{c} Cold flaid \\ flaid \\ \uparrow \\ \downarrow \end{array} \begin{array}{c} \iota_{h} \\ \downarrow \\ $
In counter flow heat exchanger, both the	$u_{hd} \rightarrow u_{dQ} \rightarrow u_{h2} \rightarrow u$
fluid temperature decrease along the length	
of the heat exchanger. Rest of the derivation approach is same as parallel flow.	
approach is suffic as parallel now.	Area
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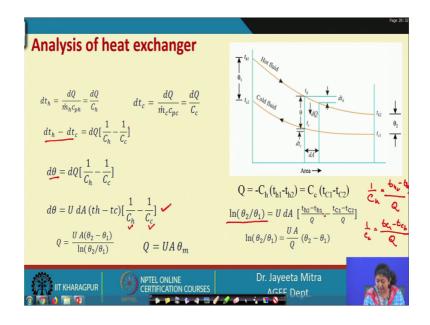
So, in the counter flow, in the counter flow what happen that the fluid 2 fluid streams are moving in opposite direction and the hot fluid and the cold fluid temperature distribution across the area will look like this way. Where theta 1 is t h 1 minus t c 2 and theta 2 is t h 2 minus t c 1 and these 2 temperature difference theta 2 and theta 1 are may be almost constant or there is not much variation between them.

So, theta; that means, that any instant if the difference between these 2 stream we consider we will term this as theta, and here also if we move on this direction I mean positive area direction. So, hot fluid is reducing temperature. So, d t h is again this is negative and d A while across the across the area if we go here also it is negative because the flow direction is now reverse right.

So, then we can write here Q is equal to minus m h dot that is mass flow rate of the hot fluid, C p h specific heat d t h the temperature difference across the small area d A, that is equal to minus m c dot C p c into d t c because again d t c also reducing because the flow direction of the cold fluid is reverse now right and this is equal to U d A delta t ok.

So, the similar approach that we have taken for the earlier section we will apply here.

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So, what we can do is again we will write d t h that is d Q by m h C p h. So, that is d Q by C h and d t c will be equal to d Q by C c. Now d t h minus d t d t c that is the temperature difference across the area d A that is d Q we will take common into 1 by C h minus 1 by C c and again we can term this as d theta right. So, d theta equal to d Q into 1 by C h minus 1 by C c, and d Q will be written as u into d a differential area into t h minus t c the temperature difference between 2 stream in the area d A right. Now from this what we can do t h minus t c will term as theta we will take d theta by theta and then we will integrate this and we get this term that is 1 n theta 2 by theta 1 in the left side and in the right what we can do? We can again put the values of 1 by C h and 1 by C c as we did in the earlier case.

Now, here what we can do is C h the hot fluid it is reducing t C h into t h 1 minus t h 2, and the cold fluid that is going from t c 1 minus t c 2 that direction is reverse now t c 1 if we move in this direction this is actually going from inside to outside, but since there is a opposite direction we have a minus and that minus we have introduced here ok.

So, we can write this now as c 1 by C h 1 by C h will be 1 by C h will be equal to t h 2 minus t h 1 by q and 1 by C c will be t c 1 1 by C c will be t c 1 minus t c 2 by Q ok. So, we will put it here t h 2 minus t h 1 by Q minus t c 1 minus t c 2 by q ok. So, eventually we will get this expression that 1 n theta 2 by theta 1 equal to U A by Q theta 2 minus theta 1.

So, Q will be U A theta 2 minus theta 1 by l n of theta 2 by theta 1. So, q is equal to U A theta m or log mean temperature difference is theta m; however, here as a it is mentioned that theta 1 will be difference of t h 1 and t c 2 not t h 1 and t c 1 and theta 2 will be difference between t h 2 and t c 1 because it is a counter flow problem.

(Refer Slide Time: 24:29)

100
Analysis of heat exchanger
Special case, when ( $\theta_1 = \theta_2 = \theta$ ) for counter flow: •The equation becomes, Q = U A $\theta$
•LMTD for a counter flow heat exchanger is always greater than parallel flow heat exchanger. But when, the temperature difference between and is small, in that case arithmetic mean temperature (AMTD) is used.
$\Theta = \frac{(\theta_1 + \theta_2)}{2}$
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Now there are few cases where theta 1 equal to theta 2 ok so theta. So, it is remain constant, it is theta which is from the inlet to the exit side remain constant.

So, the temperature difference between those 2 fluids are constants. So, in that case the equation will be simply Q equal to U A theta and log mean temperature difference for a counter flow heat exchanger is always greater than parallel flow heat exchanger log mean temperature difference for counter flow is always greater than the parallel flow heat exchanger. But when the temperature difference between them is small in that case the arithmetic mean temperature is used; that means, if theta 1. So, we will better write it as when the temperature difference between theta 1 and theta 2 is small ok. So, in that case arithmetic mean of the temperature is being used.

So, generally LMTD for counter flow for counter flow heat exchanger is greater; that means, if log mean temperature difference we are getting higher. So, area can be reduced right. So, therefore, for the counter flow we can require less area compare to the parallel flow for the same amount of I mean the same temperature distribution if we have. So, we will stop here and we will continue in the next class.

Thank you.