

Fundamentals of Food Process Engineering
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Lecture – 12
Thermal Processing and Kinetics of Microbial Death (Contd.)

Hello everyone. We will continue the last class where we have left. We will continue from there. So, today's topic will be Thermal Processing and Kinetics of Microbial Death.

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Fourier law of conduction

- ✓ Solving one dimensional unsteady state equation by variable separable method $Y = e^{(-a^2 \alpha t)} (A \cos ax + B \sin ax)$
- ✓ where A and B are constants and a is a parameter. By Substituting the boundary conditions $Y = \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \exp\left(-\frac{(2n+1)^2 \pi^2 \alpha t}{L^2}\right) \sin\left(\frac{(2n+1)\pi x}{L}\right)$
- ✓ On expansion it gives $Y = \frac{4}{\pi} \left[\exp\left(-\frac{\pi^2 \alpha t}{L^2}\right) \sin\left(\frac{\pi x}{L}\right) + \frac{1}{3} \exp\left(-\frac{3^2 \pi^2 \alpha t}{L^2}\right) \sin\left(\frac{3\pi x}{L}\right) + \dots \right]$

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So, we have stopped in the last class, in the calculation of the unsteady state conduction equation which is the one dimensional Fourier's law of heat conduction. So, the solution was this and using the boundary condition, we have found that Y is equal to 4 by pi exponential minus pi square alpha t by l square into sine pi x by L.

So, let us see the terms here. We have seen that only first term of this equation can be taken for calculation of the temperature where this parameter Y equal to t minus t 1, which is the temperature of the sink divided by t 0 minus t 1. And alpha is the thermal diffusivity of a material, l is the characteristic length of the body or the food that we are considering and x is the dimension which is related to the distance from the centre to the surface.

So, we will see that this analytical solution that we have solved now has certain problem for using in actual case because these analytical solution will depends on the different geometry and often this is very tedious to solve ok. And also sometime it happens that we consider the parameter the alpha thermal diffusivity as a constant parameter. So, also the thermal properties like alpha depends on k by rho c p. This these are also varies because of the composition, because of the composition of the material c p will also vary.

So, there are different cases ok, different conditions for this. The lump parameter approach that we have taken earlier has been changed to this but again the difficulty of solving Fourier's law of heat conduction for the actual cases and actual geometries, we will further move on to a graphical solution.

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Graphical solution

- ✓ **Gurney–Lurie Charts :**
- ✓ This is a plot of dimensionless temperature Y against Fourier number F_0 as a function of two further variables m and n .
- ✓ m is the reciprocal of Biot number, where for a cylinder or sphere the characteristic length is defined by the radius, and
- ✓ n is a dimensionless measure of the position within the object defined by $n = x/L$.

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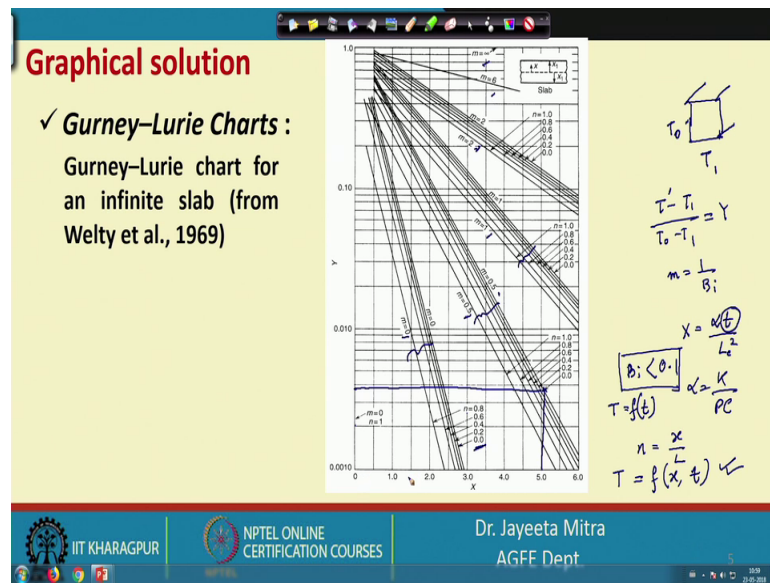
So, how we can solve in this graphical method is that graphical solution has been developed based on certain dimensionless parameter. So, first Gurney and Lurie developed this kind of a chart; so, based on the dimensionless temperature parameter Y against the Fourier number F_0 as a function of two other variables that is m and n .

So, what is m and n ? m is the reciprocal of the Biot number. Biot number that is, hL_c/k , so, $1/hL_c/k$ by Biot number is represented as m ok. So, for a cylinder or sphere, the characteristic length will be the radius. And n is a dimensionless measure of the position within the object defined by $n = x/L$. So, we

have seen that x is the distance from the centre. So, that distance divided by L which is the thickness or characteristic length, characteristic dimension of the body.

So, x by L signify the value of n ; that means, if we say n equal to 0; that means, it is at the centre that we want to measure and n equal to 1 means x is L . So, L by L is 1; that means, at the surface we want to measure the temperature ok. So, graphical solution if we see the Gurney Lurie Chart.

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So, this chart is represented here. This is basically for the infinite slab. So, as we have seen that the geometry for different geometry, the characteristic dimension or characteristic length changes because this is the volume, the surface area. So, this is characteristic dimension is the volume by surface area. So, therefore, it is changing for different geometry. So, this chart is specifically for the infinite slab ok. So, in the x direction the here, x is represented by the Fourier's number that is αt by characteristic dimension L c square ok, y that is in the ordinate. So, this is the dimensionless parameter relating the temperature and this m is nothing but inverse of Biot number and so m is varying here from 0 to 0.5, 1, 2 and up to m infinity.

So, here, so, m is 0, then m is 0.5, then m is 1, 2, 6 and varies to infinity; however, the parameter in this is varying from 0. We have seen that n is equal to x by L . So, x is the distance from the centre. So, n equal to 0 means, this is at from the centre. Then gradually it coming it is coming to the surface; so, 0.2, 0.4 and 0.6 and 8 and n equal to

1, where m equal to 0; that is signified by this line, the ordinate. So, this is happening for all the different ratio of the 1 by Biot number.

So, from this chart we can take the value of the parameter, while let us say if we want to analyze that when a particular block having a known geometry or ok. So, this the temperature of this block, initial temperature T_0 is known to us and then we are dipping it into a temperature T_1 and then we are calculating this y value which is $T - T_1$ by $T_0 - T_1$. So, let us say we want to achieve a final temperature of T_{dash} . So, when we reach the t_{dash} , our process will be stopped. So, we put here T_{dash} , calculate the parameter of this, the value of this y and we also check the value of Biot number, so, m that is equal to 1 by Biot number.

And also the distance from the centre, where we want to measure the temperature; so, let us take if you want to measure at the centre. So, n will be 0 and based on this Biot number block because this m is a set of equation, m is the set of equation sorry set of lines here, set of lines, the whole line representing a particular m that is 1 by Biot number. So, with value required we will see n equal to 0 and y value. So, let us say, we are concerned for m equals to 0.5. So, this is the 0 value and here is the y value. So, will connect them and then see what is the x value is coming ok.

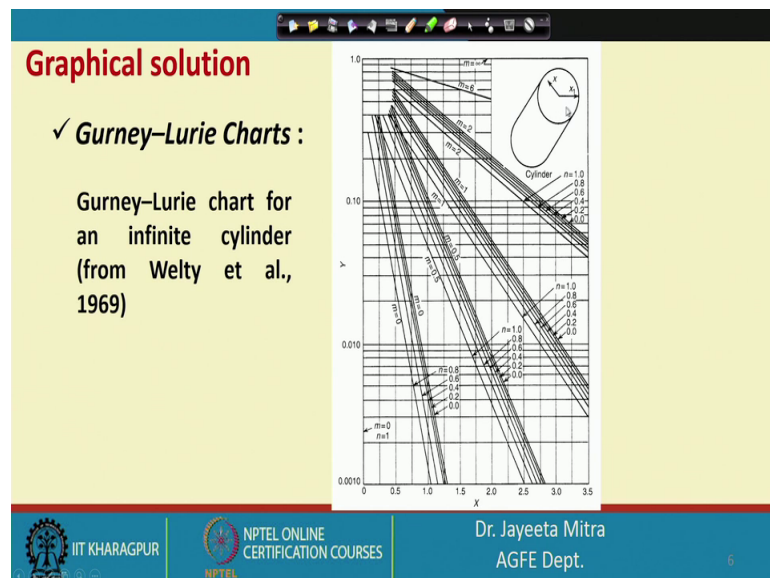
So, x is αt by L^2 , the characteristic dimension. So, α is again K by ρC . So, K is the thermal conductivity. So, the if the material is known to us the thermal conductivity, we can measure density and specific heat is also known to us. So, then putting these value and characteristic dimension, we can measure the time required, so that the centre of this food will attend the temperature T_{dash} which is required for the process. So, this is how we use the Gurney Lurie Chart for getting the temperature profile at different time and different position. So, let us just once again think about the process we have considering now; for the unsteady state process and that will be eventually helpful in analyzing the heat treated preservation techniques ok.

So, first we have seen the lumped parameter method. So, lumped parameter method we consider where the system geometry follows the restriction of the Biot number that is Bi less than 0.1 and also it signifies that the internal conductive resistance is very low, conductivity is very high. Also in this case, we assume that α is constant where in actual case, α varies and also the measurement of the thermal property will also vary

and this use of lump parameter can give us the temperature distribution with respect to time but not with respect to position. So, then we have come to the analysis using the unsteady state one dimensional heat conduction equation that is from the Fourier's law and then we have derived the final form of the equation where the dimensionless parameter Y can be related with the distance x from the centre.

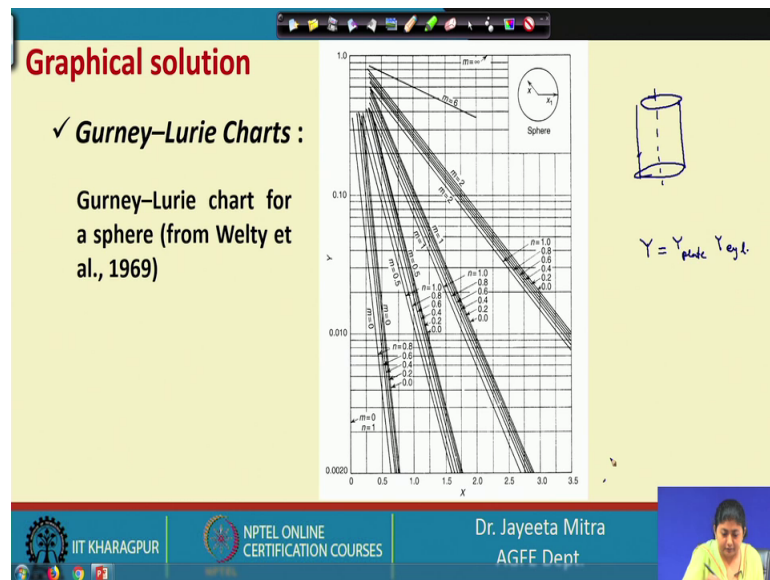
So, in this pattern, we can relate the temperature variation inside the food with respect to position and time right. So, then from the same analysis, we can, we can perform by using this chart as well just to simplify the tedious process of the analytical method. So, from this, we can again move on to the next chart which is the, which is the chart for infinite cylinder.

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So, this is the chart for infinite cylinder in the similar way as we have analyzed for the infinite plate. So, here is the value of x which is Fourier's number and here the Y value, here the m value which is 1 by Biot number and m value and n value which is x by L . So, the x is signifying the radial direction distance of the cylinder.

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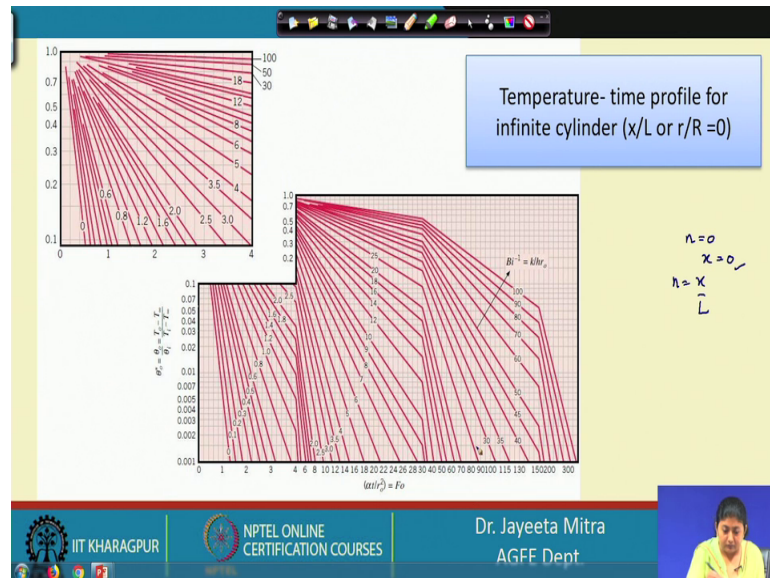
Also for this sphere, we can see this chart. So, this is for a sphere and x is again represented in the towards x axis, in the y axis we are representing y which is dimensionless temperature parameter; m and n are as representing the 1 by Biot number and characteristic or the reference position of the temperature measurement as it was for the earlier two cases. Now, the difference of this chart from the earlier two chart is that here we are measuring of a getting data for a finite dimension for a sphere; however, in the previous two cases for infinite slab or infinite cylinder, we are getting one dimension is in finite measurement, the others are infinite long.

So, there is a problem for analyzing this chart for using this chart for actual calculation because what happened that in actual food process, most of the cases, the dimensions are finite and in that if we use this kind of you know, if we use this chart, so, some error we may get. Because if we are taking an cylinder, if we can take a cylinder, so, what happened that, we need to calculate the parameters in between the infinite cylinder and infinite slab. So, in that case, the Y that is the parameter we actually need to take, we actually need to consider will come as Y_{plate} into $Y_{cylinder}$ and then we relate this with other Fourier's number and the dimensions m and n . So, there is a problem and most of the cases in the food sample when we go for preservation using the sterilization.

So, we need to have the centre point temperature because that is very important because the assurance that no contamination will be there or no microbial growth will be there, is

very important for preservation of food and to ascertain that, we need to maintain the centre temperature at a set level or we can we can perform the process in such a manner, so that the centre temperature should attend the sterilizing temperature. So, for that the centre temperature is very important. So, that is why further you know advancement in the chart has been done by Heisler.

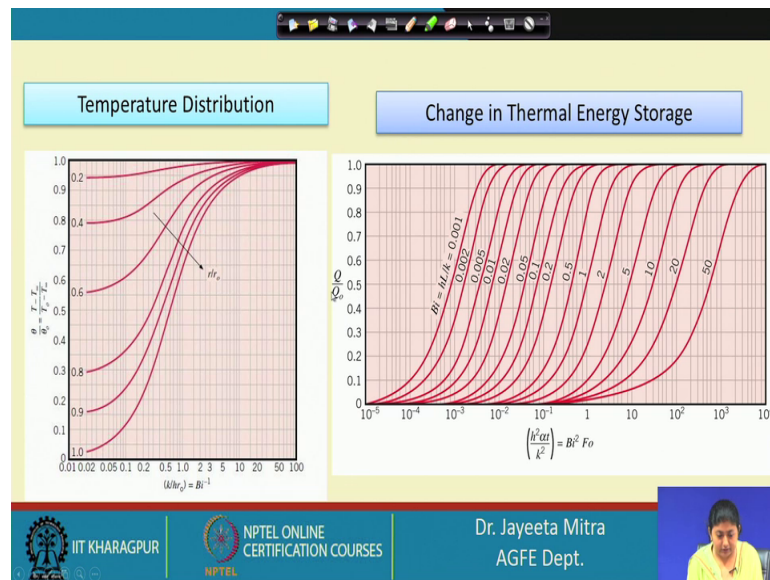
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And , we will see now, the other chart which is called the Heisler chart where again the Fourier's number is represented here in the x axis alpha t by r 0 or you can term it has again L c which is the which is your characteristic dimension square of that. And in the y axis, we are having dimensionless parameter y or theta like T temperature T 0 minus T infinity by T i minus T infinity.

So, initial temperature is of the body is noted by T i here, T infinity is the temperature of the surrounding and T o is the temperature it attained at any temperature any time. And what is being done here is, the value of 1 by Biot number that is very important. So, that has been varied from 0 to 0 to 100. So, this number has been varied from 0 to 100 because here we are considering the cases where the conductive resistance is very high. And also, all this lines all this lines has been drawn considering n equal to 0. So, all such line has been drawn at n equal to 0; that means, x is equal to 0 because n is equal to x by L. So, this is the centre temperature.

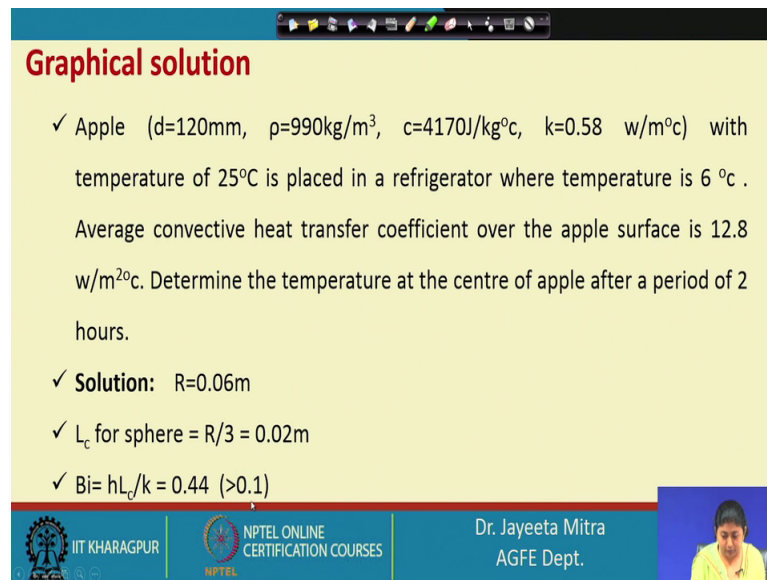
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So, we can also get from this chart, if we plot Biot number, inverse of Biot number in the x direction, x axis and theta by theta 0 in the y direction and r by r 0 will be represented by this. So, we can calculate based on this, what will be the temperature distribution inside the food.

So, change in thermal energy storage. So, change in thermal energy storage can also be seen from this data where the again Biot number is plotted and q by q 0 that is the heat content, change in the heat content over the initial heat content q 0. And this is Biot number square into Fourier number ok. So, from this chart we can get the thermal energy storage, the idea of the thermal energy storage.

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Graphical solution

- ✓ Apple ($d=120\text{mm}$, $\rho=990\text{kg/m}^3$, $c=4170\text{J/kg}^\circ\text{C}$, $k=0.58\text{ W/m}^\circ\text{C}$) with temperature of 25°C is placed in a refrigerator where temperature is 6°C . Average convective heat transfer coefficient over the apple surface is $12.8\text{ W/m}^2\text{C}$. Determine the temperature at the centre of apple after a period of 2 hours.
- ✓ **Solution:** $R=0.06\text{m}$
- ✓ L_c for sphere = $R/3 = 0.02\text{m}$
- ✓ $Bi = hL_c/k = 0.44 (>0.1)$

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Now, this as I said that, this kind of analysis will be varied for both if we consider the sterilization or else if we consider the freezing; So, one problem of the similar kind has been taken here where apple of whose the diameter is 120 mm, density is 990 kg per meter cube, specific heat 4170 Joules per kg degree Celsius, thermal conductivity 0.58 watt per meter degree Celsius with temperature of 25 degree Celsius is placed in a refrigerator where temperature is 6 degree. So, that is our surrounding is at lower temperature. So, heat will come out of the material, average convective heat transfer coefficient over the apple is 12.8 watt per meter square degree Celsius. Determine the temperature at the centre of the apple after a period of 2 hours.

So, let us see the solution. First, will have to find out that if diameter is 120 mm, radius is 0.06 mm, so characteristic dimension will be $R/3$ ok. So, this is volume of the sphere by surface area. So, $R/3$, so, that is 0.02 meter. Again, we will calculate the Biot number that is h into characteristic dimension L_c by k thermal conductivity. So, that is coming 0.44. So, this is greater than 0.1; that means, we should not use the lump parameter approach.

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Graphical solution

- ✓ So, use heissler charts to solve given problem

$$1/B_i = 2.267$$
$$F_0 = \alpha\tau/L_c^2 = 0.281$$

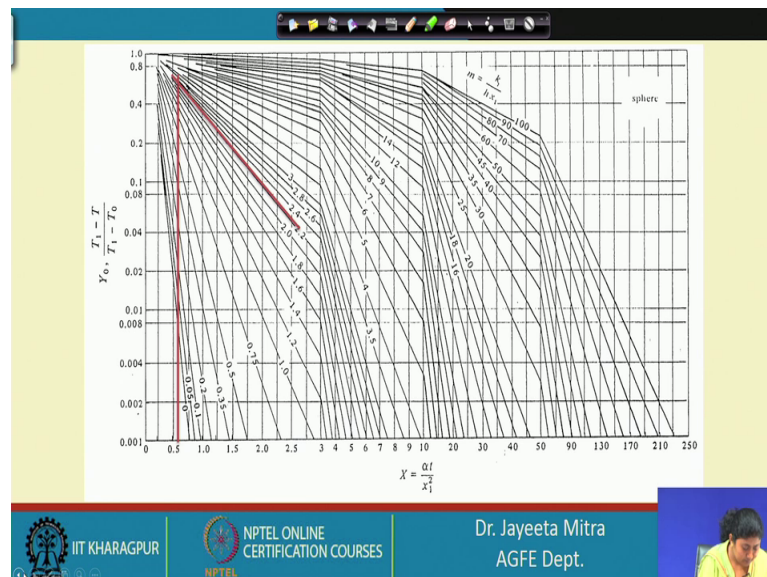
- ✓ Temperature at centre, x/L or $r/R=0$
- ✓ From Heissler chart:

$$\frac{T - T_1}{T_0 - T_1} = 0.75$$
$$\frac{T - 6}{25 - 6} = 0.75$$
$$T_0 = 20.25^\circ\text{C}$$

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So, will move on to the graphical solution using the Heisler chart; So, 1 by B_i is 2.267 Fourier's number f_0 that is equal to αT by L_c square that is equal to 0.281 ; Temperature at the centre x by L or r by R 0 that is equal to 0 . So, from the Heisler chart we will calculate the parameter T minus T_1 by T_0 minus T_1 .

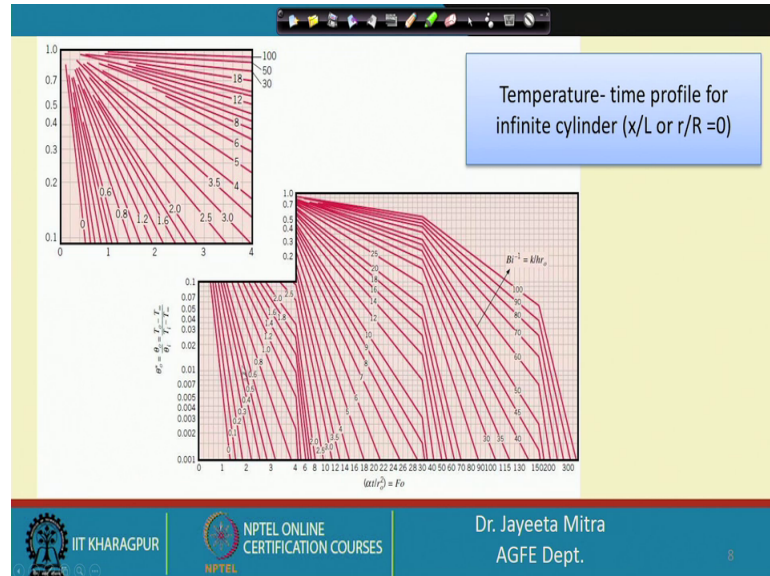
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So, we know all the parameter that is 1 by B_i and Fourier's number and also the n is equal to 0 . So, using these three from the Heisler chart, we can calculate the parameter of

Y ok. Now, once we calculate Y, we know we need to know the temperature of the product, we know the so.

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Here we are getting all these for $n = 0$. We can straight away take the value of B_i , Biot number that is coming and then αT by R_0 square that is Fourier number and from this, we can calculate straight away the value of the parameter theta.

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Graphical solution

- ✓ So, use heissler charts to solve given problem

$$1/B_i = 2.267$$

$$F_0 = \alpha t / L_c^2 = 0.281$$

- ✓ Temperature at centre, x/L or $r/R = 0$
- ✓ From Heissler chart:

$$\frac{T - T_1}{T_0 - T_1} = 0.75$$

$$\frac{T - 6}{25 - 6} = 0.75$$

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So, using that, here we are getting the temperature dimensionless temperature parameter as 0.75 ok. So, putting all the value, we can find the T_0 that is 20.25 degree Celsius right.

So, this was our Fourier number, 0.281 and 1 by Biot number that was 2.267. So, we have taken here 2.267, somewhere here this line for all n equal to 0 this is and we have taken the Fourier's number data here and find the point where these two intersect and calculate the y value which is the dimensionless parameter. So, this is how we can use the Heisler chart for calculation of temperature with respect to different centre points for different time in case of the unsteady state ok.

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Graphical solution

- ✓ A meat slice 10 mm thick, and initially at a temperature of 80°C, is to be cooled by blowing cold air at 2°C. Estimate the time for the centre of a slice to reach 20°C ($k = 0.375 \text{ W m}^{-1} \text{ K}^{-1}$, $\rho = 1250 \text{ kg m}^{-3}$ and $c = 2130 \text{ J kg}^{-1} \text{ K}^{-1}$, $h = 150 \text{ W m}^{-2} \text{ K}^{-1}$).
- ✓ Solution: $L_c = 0.01/2 = 0.005 \text{ m}$, $Bi = hL_c/k = 2 (>0.1)$
- ✓ $m = 1/B_i = 0.5$
- ✓ $\alpha = k/\rho c = 1.41 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$
- ✓ $Y = (20-2)/(80-2) = 0.231$

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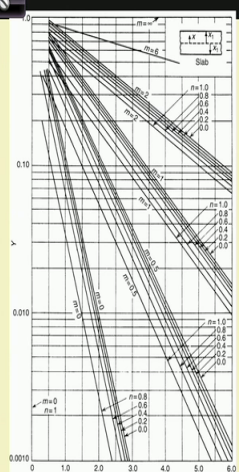
So, now another problem based on the Gurney Lurie chart, we will see where a meat slice 10 mm thick and initially at a temperature of 80 degree Celsius is to be cooled by blowing colds air at 2 degree Celsius. Estimate the time for the centre of a slice to reach 20 degree Celsius and the properties of the material is given here k equal to 0.375 Watt per meter Kelvin, ρ is the density of the meat that is 1250 kg per meter cube and c which is the specific heat that is 2130 joule per kg Kelvin.

So, so, then we will see that solution again will find the characteristic length and the Biot number of this. So, m will be 1 by Biot number that is 0.5 here it is it is it is a slice. So, α equal to k by ρc that we will calculate, y the dimensionless parameter that we will calculate because we know here the final temperature and the initial temperature.

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Graphical solution

- ✓ $Y = (20-2)/(80-2) = 0.231$
- ✓ At centre, $n=0$
- ✓ F_0 is obtained from Gurney–Lurie chart
- ✓ Read the value of x axis for ($m = 0.5, Y=0.231, n=0$)
- ✓ X or $F_0 = 1.4$
- ✓ $F_0 = \alpha\tau/L_c^2 = 1.4$
- ✓ So, $\tau = 248s$



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And after getting this, F_0 is obtained from this chart. So, this is for an infinite slab. So, here putting all the value like putting the values of Y at 0.231 and at the centre where n equal to 0 and for the we can find the Fourier number ok.

So, Fourier number is coming 1.4 using these value and Fourier number is αT by L_c square. So, it is 1.4. So, from here we can calculate the time that is 248 second. So, this is how using the Heissler chart and Gurney Lurie chart, we can calculate the temperature profile in case of different food material when this foods are you know exposed to the higher or lower temperature surroundings. So, the principal, we have discussed today, we will be using this one for the thermal preservation techniques that we will follow in our next class; so till then.

Thank you.