

Fundamentals of Food Process Engineering
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Lecture – 10
Rheological Properties of Viscoelastic Food (Contd.)

Hello everyone, welcome to the NPTEL online certification course on Fundamentals of Food Process Engineering, we will continue with the viscoelastic properties of food today. So Rheological Properties of Viscoelastic Food that we have started in the last class, we will continue in that.

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Combined mechanical Models

Kelvin-Voigt models: Simple Creep behaviour is exhibited by the strain beginning to increase rapidly but the rate of increase diminishes over time in the form of an exponential decay.

- Spring and a dashpot connected in parallel
- All strains are equal to each other
- $\gamma = \gamma_{spring} = \gamma_{dashpot}$
- Total shear stress caused by the deformation is sum of the individual stresses
- $\tau = \tau_{spring} + \tau_{dashpot}$
- $\tau = G\gamma + \mu\dot{\gamma}$

Kelvin-Voigt models

The slide also features a diagram of a Kelvin-Voigt model, which consists of a spring with modulus G and a dashpot with viscosity μ connected in parallel. A force F is applied downwards to the bottom of the parallel combination, which is suspended from a fixed point. The slide footer includes the IIT Kharagpur logo, NPTEL Online Certification Courses logo, and the name 'Jayeeta Mitra, AFSE Dept.' along with a small video inset of the professor.

So, again we will see combined mechanical model that can express the behaviour of creep. So, this is the spring and dashpot attached in parallel network just the opposite case of Maxwell, where we have connected them in series. So, this is showing the simple creep behaviour.

So, what happened here is the strain beginning to increase rapidly, but the rate of increase diminishes overtime in the form of an exponential decay. So, we can see that since any force we will apply here, immediately the viscous material will respond and try to flow. Now since the strain or the flow will observe here, it will try to the strain will try to have similar effect here in the spring as well; so, both will try to expand or strain in similar fashion.


Now because of this linear relation of this spring, it has a linear relation with stress and strain, what happens that it will try to take away some amount of force through it. As a result, this dashpot will show the diminishing rate of the increase in the flow behaviour. So, eventually the exponential decay will be seen.

So, here the strains of all springs are equal to each other, when they are connected in parallel. So, total strain will be strain in the spring that is equal to strain in the dashpot as I said, because of this pressure F applied on it force F and total shear stress caused by the deformation is sum of the individual stresses. So, τ will be τ spring plus τ dashpot; so, τ spring can be signified by spring constant G into strain γ ; however, in the dashpot the stress will be μ into $\dot{\gamma}$.

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Combined mechanical Models

- ✓ **Burger model:** More suitable to biological materials.
- ✓ it will initially show a sudden elastic strain response before beginning the exponential retarding increase that will usually be followed by a perpetual linear increase over time.
- ✓ Kelvin model sandwiched between an external spring and an external dashpot



Burger model

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So, another advancement over the Kelvin model we will see here, what happened that the phenomena of creep has been explained by the material by the model mechanical model Kelvin Voigt model, but sometime it has been seen that the proper behaviour may not be fully expressed by the Kelvin model therefore, Burger model is introduced where some advancement over the earlier model is been done in this form.

So, this is the Burger model, where a spring and a dashpot in they are in series has been combined with spring and dashpot in parallel. So, it will show sudden initial elastic behaviour, when applied to a constant stress σ . Then it will show exponential retarding that will increase because of this parallel effect of spring and dashpot, and then

a perpetual linear increase in the flow will be observed. So, here Kelvin model sandwiched between an external spring and an external dashpot.

So, E_0 and E_R is the spring constant of the initial and here in this combined model, η_0 is the coefficient of viscosity coefficient of the external dashpot.

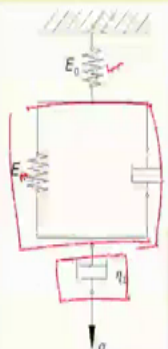
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Combined mechanical Models

✓ **Burger model:**

□ External spring allows the initial elastic deformation to occur in response to the applied stress, while the external dashpot allows for the perpetual linear increase in strain over time so long as the applied stress remains.

Burger model used to predict strain as a function of time by

$$\epsilon(t) = \frac{\sigma_0}{E_0} + \frac{\sigma_0}{E_R} \cdot \left(1 - e^{-\frac{t}{\tau}}\right) + \frac{\sigma_0 \cdot t}{\eta_0}$$


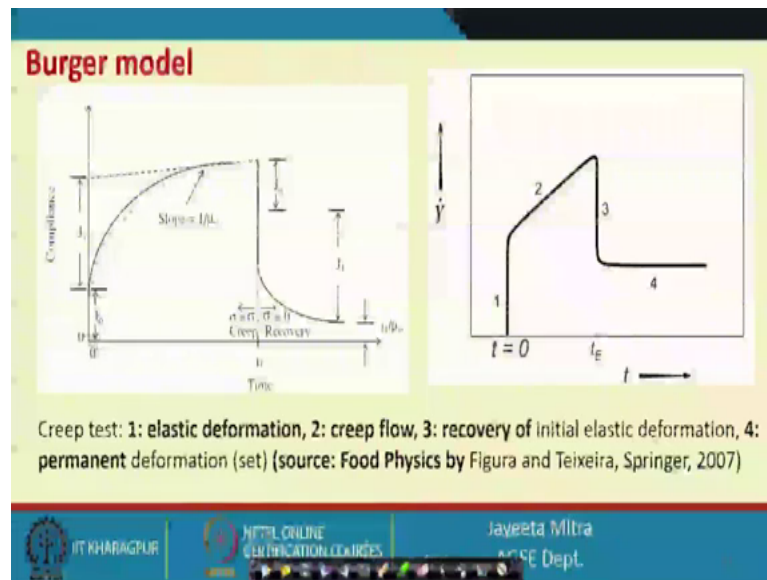
Burger model

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So, in the Burger model, what happened that initial external spring shows the initial elastic deformation to occur in response to the applied stress σ while the external dashpot allows for the perpetual linear increase in strain overtime so, as long as the applied stress remain. Now Burger model used to predict the strain as a function of time by this equation, strain $\epsilon(t)$ that will be equal to $\frac{\sigma_0}{E_0}$ plus $\frac{\sigma_0}{E_R} \cdot (1 - e^{-\frac{t}{\tau}})$ plus $\frac{\sigma_0 \cdot t}{\eta_0}$.

So, this strain is because of the first elastic component that is the spring, this part this exponential decay part is because of this combined model of spring and dashpot, and the last part is because of this dashpot. So, we have discussed it this that when we call modulus of elasticity that is stress versus strain, if we try to have the reverse function that is strain versus stress that is called the compliance.

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So, here j is been used to measure the creep compliance, that is the strain versus stress for the creep test that we are getting. So, here J_0 that is the initial elastic compliance that we are getting J_1 which is increasing exponentially that is one by μ_0 ; and then after time t_1 it is decreasing initially because of the elastic component J_0 and then exponentially because of J_1 .

So, this is the recovery some part is instance recovery because of the elastic nature and because of the Kelvin model the elastic after the elastic material, after the elastic nature we are getting some exponential decay in the recovery and some part which is because of the dashpot, which is lost completely. So, this will be the compliance overtime behaviour.

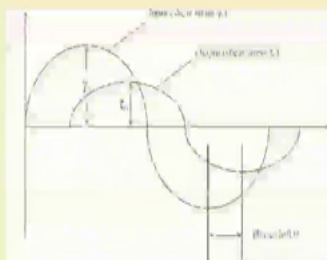
Here also if we plot the $\dot{\gamma}$ that is shear rate shear rate over time. So, t equal to 0 part one which is showing the elastic deformation, and then two path 2 or curve 2 is showing the creep flow so; that means, the flow is increasing continuously, then recovery of initial elastic deformation that is 3 and 4 shows the permanent deformation or set.

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Dynamic Test (Oscillatory Test)

With an oscillating mode, both viscous and elastic properties of a material can be measured simultaneously.

- Rate controlled - stress is measured at a constant strain or
- Stress controlled - deformation is measured at a constant stress amplitude



Usually, a sinusoidal strain is applied to the sample, causing some level of stress to be transmitted through the material. Then, the transmitted shear stress in the sample is measured

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Now coming to another important behaviour that is dynamic test or oscillatory test; we have seen so far that all the method of viscosity in measurement we have seen. So, in that rotational viscosity rotational viscosity measurement device we have seen where a constant angular speed or constant angular motion we are providing.

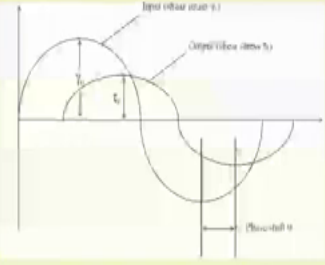
Now instead of putting angular motion, if we provide some oscillatory motion and then we can measure the both elastic as well as the viscous, property of the viscoelastic material. So, those tests are comes under the dynamic test or oscillatory test. So, with an oscillating mode both viscous and elastic properties of the material can be measured simultaneously. So, what we do is in a in a sequential pattern or following a Sinusoidal wave, we apply the stress and then we try to measure the response of the stress. So, it can be rate controlled where stress is measured at a constant strain or stress controlled, where deformation is measured at a constant stress amplitude.

So, normally we apply a Sinusoidal strain to the sample, causing some level of stress to be transmitted through the material and then the transmitted shear stress in the sample is measured.

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Dynamic Test (Oscillatory Test)

Sinusoidal oscillation $\gamma = \gamma_0 \sin \omega t$
cosine function for the resulting shear rate.

$$\dot{\gamma} = \gamma_0 \cdot \omega \cdot \cos \omega t = \dot{\gamma}_0 \cos \omega t$$
$$\dot{\gamma} = \dot{\gamma}_0 \sin(\omega t + 90^\circ)$$


- shear rate function follows the deformation function with a phase shift of 90° .
- elastic material will show a shear stress which is proportional to the shear deformation (strain), and
- a viscous material will show a shear stress which is proportional to the shear rate.

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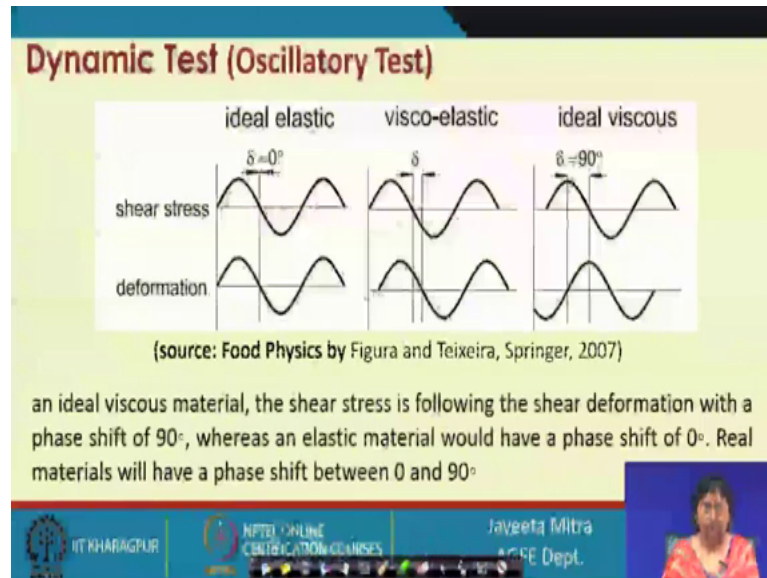
So, let us see what happened when we provide a Sinusoidal oscillation that is $\gamma = \gamma_0 \sin \omega t$. So, we will plot it $\gamma = \gamma_0 \sin \omega t$. So, γ_0 is the magnitude the maximum point γ_0 , and with respect to the different ω it is being changed from following the Sinusoidal pattern.

So, this is the shear strain that we applied and then what is the stress will be to maintain that strain, that will be analysed. So, cosine function for the resulting shear rate will be there that is $\dot{\gamma} = \gamma_0 \omega \cos \omega t$. So, that is $\dot{\gamma}_0 \cos \omega t$. So $\dot{\gamma}_0$ that will be equal to $\gamma_0 \omega \sin \omega t + 90^\circ$ that means we can observe a 90 degree shift in the shear rate ok. So, shear rate function follows the deformation function shear rate function $\dot{\gamma}$, follows the deformation function which is γ with a phase shift of 90 degree ok. From this equation we can see that and elastic material will show a shear stress which is proportional to the shear deformation or strain.

So, what we want to measure is that, we told that we are providing an oscillatory motion ok. So, and the material is viscoelastic so, some part of the elasticity and some part of the viscous nature we need to analyze. So, elasticity is in terms of the force we know that in the in case of elastic material, the stress will be proportional to the deformation and in case of the viscous material stress will be proportional to the rate of deformation right. So, here we are trying to relate them elastic material will show a shear

stress which is proportional to shear deformation or strain which is this, and also viscous material will show a shear stress which is proportional to shear rate.

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So, we will see now the ideal elastic, viscoelastic and ideal viscous. So, this 3 kind of material when exposed to oscillatory test what will be their shear stress and deformation behaviour. So, shear stress is showing in this plot, for the ideal elastic materials since stress will be proportional to strain so, it will follow by similar kind of plot so, delta equal to 0 so, delta signifies the phase shift that is equal to 0.

Now for the viscoelastic, there will be a phase shift so, that delta we can observe; however, for the ideal viscous nature the delta will be 90 degree ok. So, 90 degree phase shift will be observed in case of ideal viscous material whereas, ideal elastic will show delta equal to 0 and in between them any value can be observed in case of the viscoelastic material ok. So, the material will have a phase shift between 0 and 90 degrees will be taken as the biological material or real material.

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Dynamic Test (Oscillatory Test)

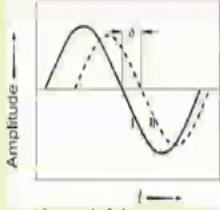
□ **Complex shear modulus** : characterize both the viscous and the elastic properties of a material by measuring the phase shift δ only.

□ **Complex shear modulus is** $G^* = G' + G''$

□ G' for the elastic component, and

□ G'' for the viscous component

□ The phase shift then simply is $\tan \delta = \frac{G''}{G'}$



Amplitude

t

Phase shift between deformation (I) and shear stress (II)

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So, coming to the complex shear modulus, so it is characterized both the various both the viscous and the elastic properties of a material by measuring the phase shift δ only. So, why this complex shear modulus is that, in case of viscoelastic material we do not have a fix modulus like modulus of elasticity, we have in case of the elastic material ideal elastic material. So, we introduce a complex shear modulus that is G^* equal to G' plus G'' .

As we have seen that the deformation which is curve 1 and the shear stress observed that is curve 2 has a phase shift of δ ok. Now G' stands for the elastic component, we know that G' it is we may consider it as a spring constant ok. So, this is for the elastic component and G'' for the viscous component.

. So, the phase shift simply is $\tan \delta$ that is equals to G'' by G' . So, in any oscillatory test if we can understand the value of $\tan \delta$, we can find out what will be the component of G' and G'' in that ok. So, if G'' is that is for the viscous component so, that is 0. So; that means, $\tan \delta$ is coming 0. So, δ is 0 so; that means, it is pure elastic material. So, phase shift does not observed there and if we are getting the higher value so, that shows that G'' has a significant effect.

So, the viscous component will be there so, that we can identify from the oscillatory test. And in the similar fashion we can calculate also the complex viscosity, where there will

be it it is kind of a real viscosity and imaginary viscosity; real viscosity for the viscous component and imaginary viscosity for the elastic component. So, can be G instead of G' we can also express in terms of the complex viscosity.

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Viscoelastic Behaviour

Dynamic mechanical analysis (DMA) :

- Instruments which perform oscillation in an uniaxial or linear direction.
- Closely related to texture testers.
- Assumption that temperature was always maintained constant and known.
- Certain types of experiments have been devised specifically for the purpose of studying the effect of increasing temperature on the rheological behaviour of materials.

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So, another thing is dynamic mechanical analysis we often do this test in various food material or two or different polymer material, to understand the behaviour the textural behaviour of that. So, what we do here is that, instead of the rotational movement that we have replaced by the oscillatory mechanism, here the uniaxial force is being done in the in the method of oscillation.

So in which the uniaxial, extension and compression is being done and the behaviour is observed on a material so, the force is applied in linear direction only. So, as I said that textural behaviour can be observed from that and assumption over all the analysis we are considering so, far is that temperature should remain constant, because temperature has a significant role in viscosity as well as to deform or breakdown the molecular structure of the material. So, that is why temperature should be taken constant and it should not be very high compared to the room temperature.

So, there are certain type of test can be devised to analyse the effect of increasing temperature on the rheological behaviour of materials.

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Viscoelastic behaviour

Problem: The creep analysis results were obtained for wheat flour dough by applying a constant stress of 50 Pa for 60 s on the sample. The data are given in Table E.2.9.1. Using creep compliance data, determine the viscoelastic parameters, G_0 , G_1 , μ_1 , and μ_0 of a Burger model.

The diagram shows a mechanical model of a Burger model, which consists of a spring and a dashpot in parallel, followed by another spring and dashpot in series. A constant stress $\sigma = 50 \text{ Pa}$ is applied to the model.

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We can think of a problem here the problem has been taken from taken from Shaheen book where [FL] ok.

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Viscoelastic behaviour

Table: Deformation of Wheat Flour Dough

Time (s)	Deformation
0	0.0060
5	0.0095
10	0.0140
15	0.0160
20	0.0180
25	0.0188
30	0.0195
35	0.0203
40	0.0210
45	0.0218
50	0.0225
55	0.0233
60	0.0240

Burger Model is

$$\gamma = \frac{\tau_0}{G_0} + \frac{\tau_0}{G_1} \left(1 - \exp\left(\frac{-t}{\lambda_{rel}}\right) \right) + \frac{\tau_0 t}{\mu_0}$$

The Burger model in terms of creep compliance is:

$$J(t) = J_0 + J_1 \left(1 - \exp\left(\frac{-t}{\lambda_{rel}}\right) \right) + \frac{t}{\mu_0}$$

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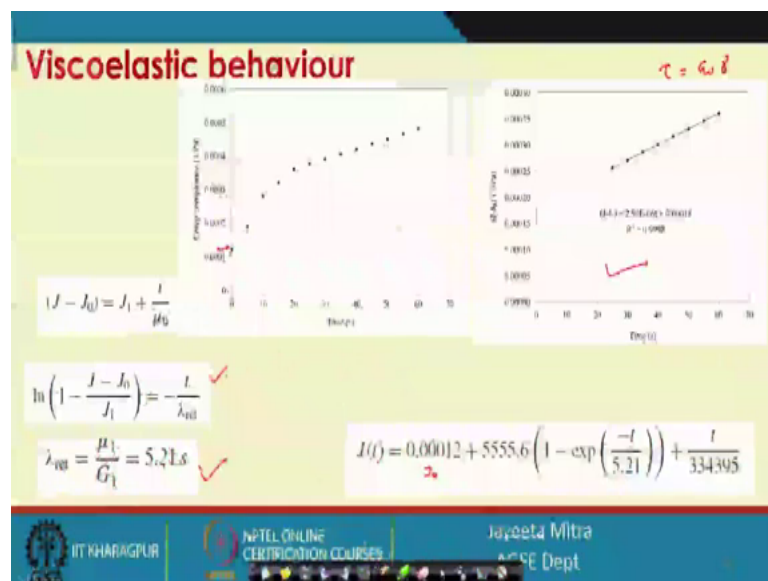
So, [FL] so we will just discussion short a problem that the creep analysis results were obtained for wheat flour dough by applying a constant stress of 50 Pascal for 60 second. So, the stress is given and initial time the time duration is given on the sample, the data are given in a table that will show using creep compliance data determine the viscoelastic parameter G_0 , G_1 , μ_1 and μ_0 of burger model.

So, Burger model and this problem is being taken from the Shaheen (Refer Time: 26:59) book and so, we know that the Burger model is like that, there is a spring then the we have and this kind of force is applied, the constant force that is constant stress sigma that is 50 Pascal right so, we will see how we will solve that lets see the table. So, this is the data table deformation of wheat flour dough with time up to 60 second, what is the deformation that is given here gamma the deformation. So, Burger model is gamma that is equal to tau 0 by G 0 this is because of the initial spring stress.

Because in case of spring we know that tau will equal to G 0 into gamma. So, gamma is equal to tau 0 by G 0 here; then because of the combined parallel system of the Kelvin model that is there. So, because of that the exponential decay term will be there tau 0 by G 1 into 1 minus exponential minus t by relaxation retardation time, here it will be retardation the creep will be retarding. So, lambda r e t plus tau 0 t by mu 0; so, in terms of creep compliance we can write this equation as J t, just we have taken the inverse that is the strain versus stress.

So, divided by tau 0 we are getting this J t equal to J 0 plus J 1, 1 minus exponential minus t by lambda retardation plus t by mu 0 and then what we will do? We will plot this.

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We will plot first the creep compliance with respect to time, and we can get that its initially there is a curvature exponential increase and then a little bit of state section. So,

from the from the state section data if we if we plot that in the section minus the initial stress that is that is J_0 , then we will get this equation by linear regression.

And from this data J minus J_0 with respect to time, we can get the intercept and slope value. So, we can get the values of J_1 and μ_0 again which is the non-linear part or the or that is the gradual increase that section, we will plot with respect to time so that the part of the Kelvin model, the parallel arrangement of the spring and the dashpot that section can be can be calculated from this chart. And lambda retardation will be given by μ_1 by G_1 ; μ_1 is the viscous viscosity coefficient of the dashpot 1 and this is the spring constant of the spring 2 that is attach in parallel dashpot.

So, we are getting the retardation time as 5.21 second. So, will just put all the values that we are get from here and here and also from the this section, that is that is from the curve section with respect to time. So, we will get finally, this plot ok. So, first this creep behaviour here that we are getting J_0 as we can see the equation see the equation J_0 , then was J_1 and λt and μ_0 .

So here we are getting J_0 from this plot, then once we get this value. So, J_1 will be calculated from here ok. So, so J_1 we are getting J_1 we are getting and we will we will calculate the J_1 we are getting and then from 1 minus exponential minus t by here the lambda retardation time. So, retardation time 5.21 second we will plot it here and finally, μ_0 which is coming from the inverse of that slope that we are getting.

So, using that we can find the equation of the creep compliance of the dough sample ok;so, this is how we solve the phenomena of the creep from the compliance versus time behaviour. So, we will stop here and as we are ending the chapters over here, that is the rheology of food. So, next we will in the next class we will start a new chapter.

Thank you all.