Irrigation and Drainage
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Lecture - 48
Sub - Subsurface Drainage Design

Hi, this is lecture number 48 on Sub-surface Drainage Design. So, in this mainly we will be working on the unsteady state flow to drains. I mean the previous lecture we are talking about steady state conditions, now unsteady state conditions. So, let us see how the unsteady state conditions.

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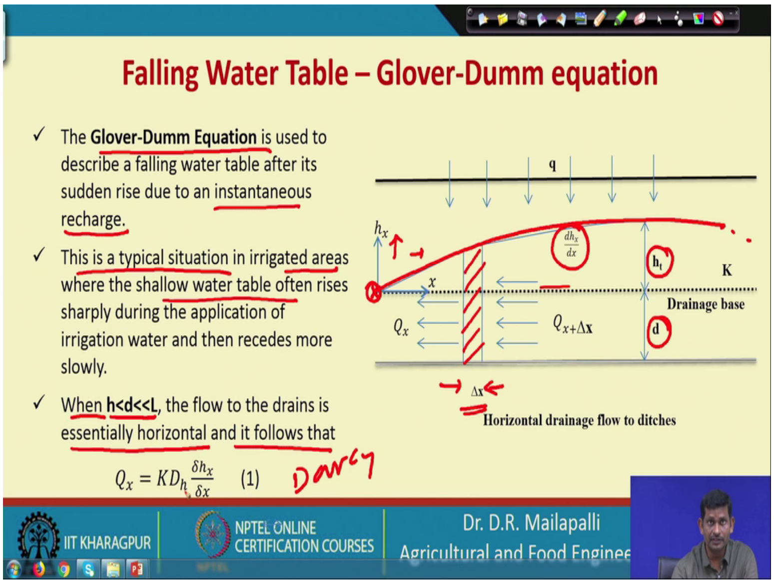

For example, here the falling water table. So, this is observed during you know the irrigation, when you are applying irrigation or the sudden it is the water table has fall down. So, Glover-Dumm equation will be used in order to simulate the falling water table in case of the drains. So, Glover-Dumm equation is used basically the falling water table after sudden rise due to an instantaneous recharge.

So, and the under equilibrium, so we in steady state condition we assume that the system is under equilibrium so that then the q which is recharge taking place will be equal to drainage out, but whereas, if there is a sudden instantaneous change in the recharge that definitely influence the drain discharge. So, this is the a typical situation irrigate in areas where you apply irrigation and the shallow water table goes down and then after irrigation it again falls down something like that.

So, basically when h less than small d less than less than L the flow to the drain is essentially horizontal and it follows that Q x for example, here. So, in this schematic if you see, so this is the drain I mean trial drain point, this is a half way of trial drain, ok. So, there is another trial drain somewhere other place. So, for example, this is the water table right this is the water table and at the midpoint this is h t, this is the head in the midpoint and this is the drain depth, and dh x by dx is the change in water table with distance. So, from here if you see x is going this direction and h the vertically upwards, ok.

So, we are going to see how the change in q is influencing the h, ok. So, the since h is smaller if h is very small compared to the drain spacing. So, mostly the horizontal flow is influencing and if you assumed a unit you know element in the flow media right in the soil media, the soil media that is that has a d del x, so Q x which is taking place will be equal to K in to D h into dou h x by d x. This is from Darcy's law. So, from Darcy's law Q x will be taking place through this will be equal to K into hydraulic conductivity into D h right. So, D h is the elemental you know the depth, and then for the unit width and this is the hydraulic gradient, ok. So, this is a one thing.

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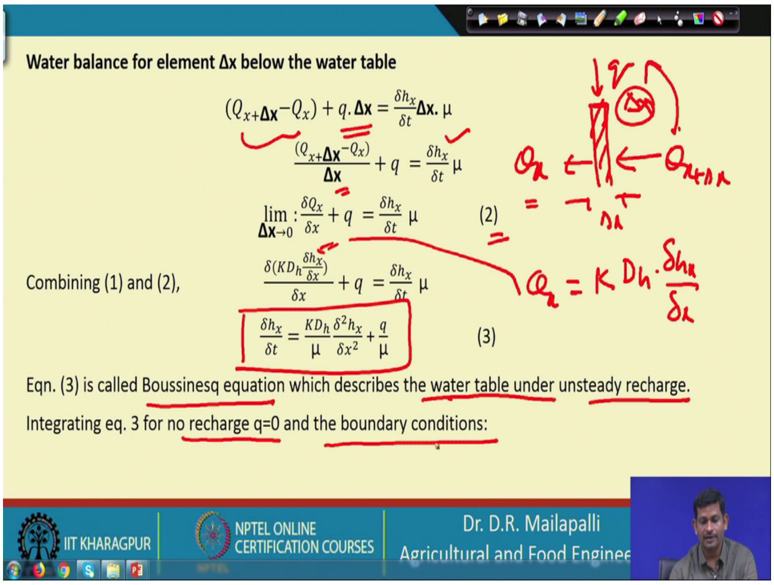

Then after that if you use the water balance I mean water balance for element delta x. Suppose, this is the element. So, you have seen this is the delta x, so here Q x right and at this point this is Q x plus delta x, ok. So and let us say this is q, ok. So, the water balance will be equal to. So, the change, so the Q x, so this is the difference input minus output, ok. So, here this is all input which is taking place q plus Q x plus dx and Q x, this is the output for example.

So, Q x plus dx plus q into delta x right q into delta x, delta x is the width the thickness sorry width of the element. So, that is q in to delta x is taking place and Q x plus delta x and Q x is the difference that will be equal to change in the storage. So, change in storage Q x plus delta x minus Q x by delta x will be equal to plus q is equal to delta Q x plus delta t into mu, ok.

So, this is the delta x is divided both sides, ok. So, limit delta x when delta x tends to 0, delta d delta a Q x by d dx plus q which is equal to delta h x by delta t into mu, ok. So, combining 1 and 2, so this is two and the previous equation Darcy's equation if you combine right. So, q of, so you are substituting for Q x. So, Q x is equal to K into D h into delta h x by delta x, right. So, you can substitute this here, substitute this here and arrange the terms. So, you get delta h x by d t, t is equal to K into D h by mu delta square h x by delta x square plus q by mu. So, where this equation is called Boussinesq equation, ok. So, this describes the water table and unsteady recharge.

So, now, you are going to integrate for no recharge q equal to 0 and the boundary conditions initial boundary conditions we are going to find out the solution. So, for example, for the 0 recharge, so what would be the solution for this equation.

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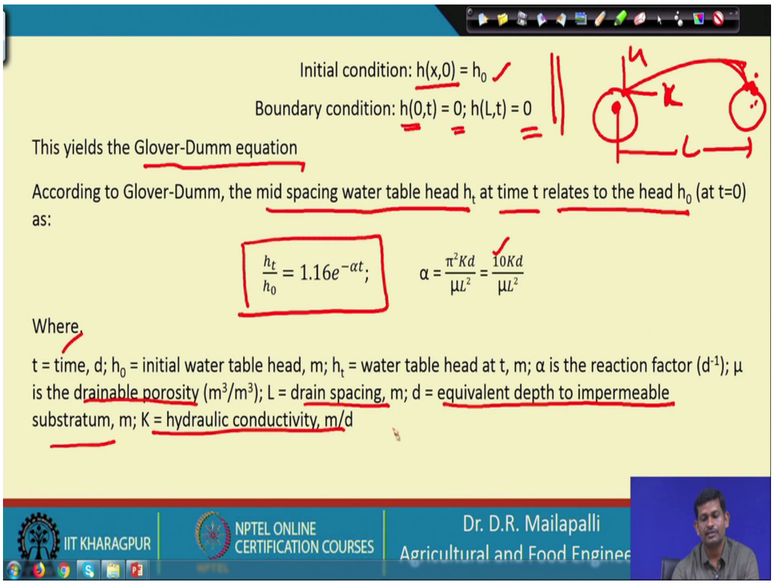

So, that will be the initial conditions part h x 0 is equal to h naught right that is the initial water table, and the boundary conditions are when x equal to 0 right and so when x equal to 0 and any time which is equal to 0 at the drain, drains at the drain. So, suppose here you have this is x and h, ok.

So, when x equal to 0, so h is 0 and suppose you have another drain right that is at L length l. So, h is equal to h of L in to t. So, here also h is equal to 0, ok. In both cases drain number 1 and drain number 2 the h will be 0, that is the meaning. So, this yields Glover-Dumm equation. So, according to Glover-Dumm the mid spacing water table head h t at time t relates to the head h naught, ok, h naught is the highest water level, ok, highest. So, that is h naught initial water table, you can say initial water table.

So, h t by h naught, so using this initial boundary conditions and then the equation. So, if you solve it then it leads to h t by h naught is equal to 1.16 e power minus alpha t, where alpha is equal to 5 square Kd by mu L square, where pi square can be replaced with 10 Kd by mu L square, ok.

So, and the terms are defined here t is the time, h naught that is in days h naught is initial water table h t is the water table at time t, alpha is a reaction factor which is d power minus 1, and mu is the drainable porosity L is drain spacing and d is equivalent depth of impermeable stratum and K is hydraulic conductivity, ok. So, this equation will give the water table at any time t, right.

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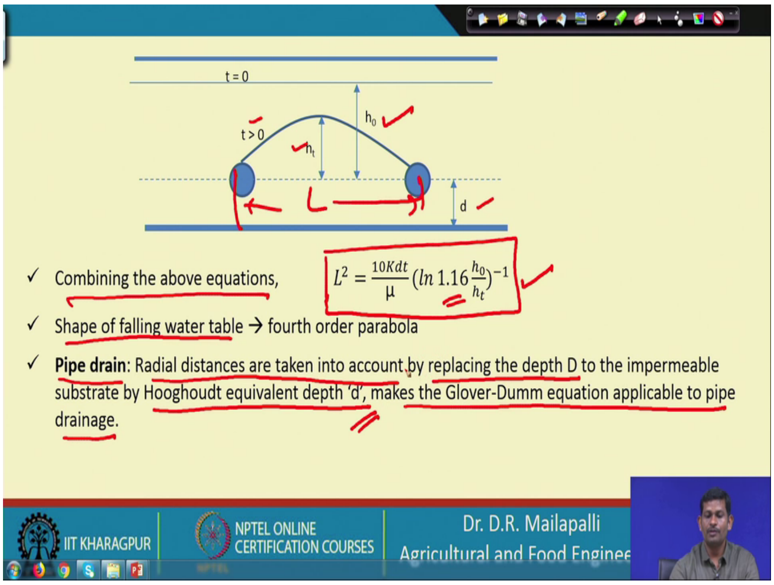

So, the next is, so the same thing if you see. So, at time t equal to 0 that is h naught, at time and greater than t t greater than 0 that is h t, ok. So, d is the small d. And the combining above equations, so you can find out the drain spacing L between the two drains is equal to 10 K d t by mu into ln of 1.16 into h naught by h t or minus 1, and the previous equation you can find out the drain spacing formula using Glover-Dumm equation for on steady state conditions.

The shape of the falling water table it is the 4th order, because 1.16 I mean if you can rearrange the terms. So, you get the shape in the 4th order parabolic term pipe drain in case of pipe drain radial distance are taken into account by replacing the depth d to the impermeable substrate by Hooghoudt equivalent depth D, makes the Glover-Dumm equation applicable to pipe drainage.

So, Glover-Dumm equation I mean this is equivalent to pipe drainage when you use the equal depth concept here like there is a two conditions. So, if you use this equation along with the two conditions d is the function of you know capital D, small d function of capital D. So, then the formula can be used for the trial drainage systems, ok.

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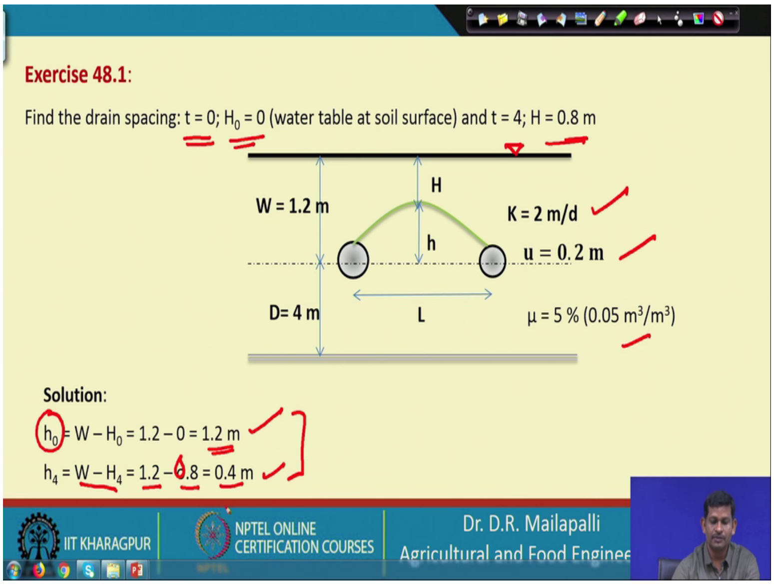

So, here is the exercise first find the drain spacing L for when t equal to 0, H naught equal to 0, water table soil surface exactly the water table is at soil surface, ok. So, here initial and t equal to 4, H is equal to 0.8. So, when t equal to 4 H that that means, the H is increase that mean the water table went down that is 0.8. So, the solution here is h naught is equal to W minus H naught. So, W is 1.2, so minus 0 that is the 1 point exactly at the soil surface; h 4 there is the W minus H 4, so 1.2 minus 0.8 so that will be 0.4, ok.

So, from 1.2 to 0.4 the water table is dropped and knowing the K values and u values and mu values and with this water table fall. So, find out the drain spacing. So, that is the question for this.

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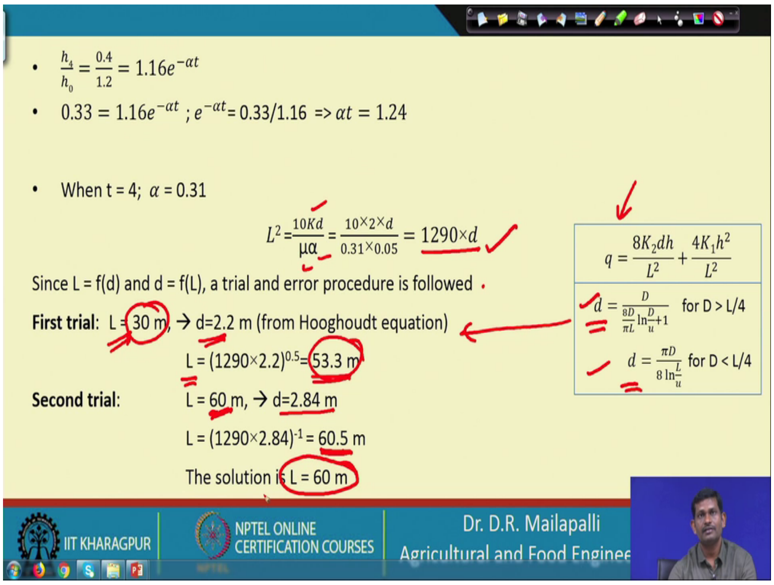

So, now from the Glover-Dumm equation; so, h 4 minus h naught you see h 4 means t is equal to 4, 4 days and t equal to 0.4 by 1.2 which will be 1.16 in e power minus alpha t, and. So, this is 0.33, 1.16 e power minus alpha t and e power minus alpha t is equal to and finally, alpha t will be 1.24 and when t is equal to 4 alpha is equal to 0.31.

So, now L square use the Glover-Dumm equation L square 10 Kd by mu alpha, right. So, 10 2 d, so all values are given, right. So, this is not given of course. So, K is given, mu is given, alpha we estimated and finally, L square is equal to 1290 into d, ok. So, now and since L is equal to function of d and d is the function of L a trial error function procedure is to be followed. So, that is what we explained. So, the Glover-Dumm equation can be applied to trial drainage by considering the equivalent depth concept.

So, here the equivalent depth concept if you bring from Hooghoudt equation the small d is a function of capital D, ok. So, here, so what we do? We initially take L equal to 30 meter that initial guess and from this equation knowing the capital D right L by 4, now find out L by 4 and see whether you know which equation is falling under and the find out small d, ok. Now, substitute it in this equation and find out capital L, now this is 53.3 meter. So, for 30 meter initial guess you get 53.3 meter. So, this is not a good, guess. So, now, what we have to do? You have to go for another guess L is equal to 60.

So, always go with the last I mean I mean the determined value as initial guess. So, that it converges faster. So, so the L is equal to 60 meter and d is equal to 2.84 if you follow the same procedure. So, then, so the new solution will be 60.5. So, when it is 60, you are getting 60.5. So, L is equal to 60 meter is the right solution, spacing solution, ok. So, this way you are going to estimate the spacing using Glover-Dumm equation by recalling the equivalent depth concept, ok.

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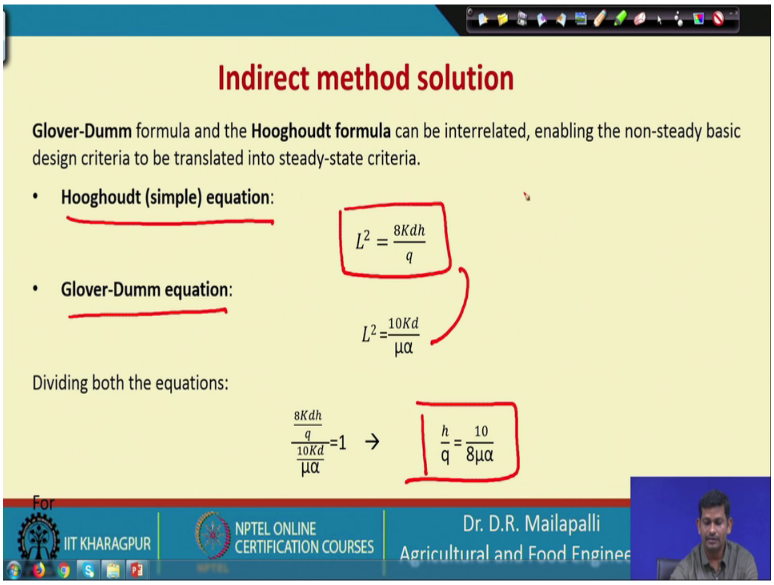

So, there is a indirect method of solution. So, Hooghoudt simple equation by eliminating the you know the the flow the horizontal flow above the drain space. So, then that is L square 8 K D h by q because we are only considering the flow below the drain space and then Glover-Dumm equation L square 10 Kd by mu alpha. So, combining these two you get h q, h by q is equal to 10 by 8 mu right alpha, ok. So, this is and the same equation, ok; so, the same equation; so h by q into 10 by 8 mu alpha, ok.

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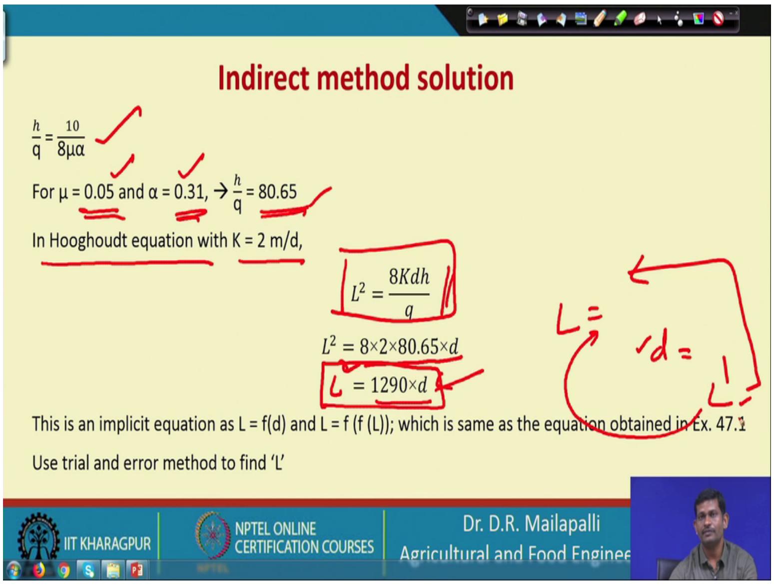

For mu equal to 0.05 and alpha is equal to 0.31 h by q is equal to 80.65, ok. So, and so the these values are taken from the previous exercise, previous example then h by q is equal to 80.65, in Hooghoudt equation with k equal to 2 meter per day right. So, L square is equal to 8 K D h by q, and L square is equal to 8, 2 into 80 point d and finally, 1290 into d. So, you are going to get the; so I mean this is from Hooghoudt simple equation Hooghoudt simple equation and then L square into 8, 2. So, find out I mean put the values here h by q value, h by q is this and then other values and finally, L square is equal to 1290 into d. So, this is same as the previous example Glover-Dumm, actually Glover-Dumm equation when you use.

So, this is the final you know the equation will be like this. And then after that you can find out make the initial guess of L and find out d, right and find out the new L. Now, you just check these two L's whether they are equal or not. If they are not equal again this value as initial guess and find out the and find new value and see again whether they are equal or not. So, we will follow the same procedure here. So, this is the simple I mean this is indirect way of giving solution when you combine this simple I mean simple Hooghoudt equation with the Glover-Dumm equation.

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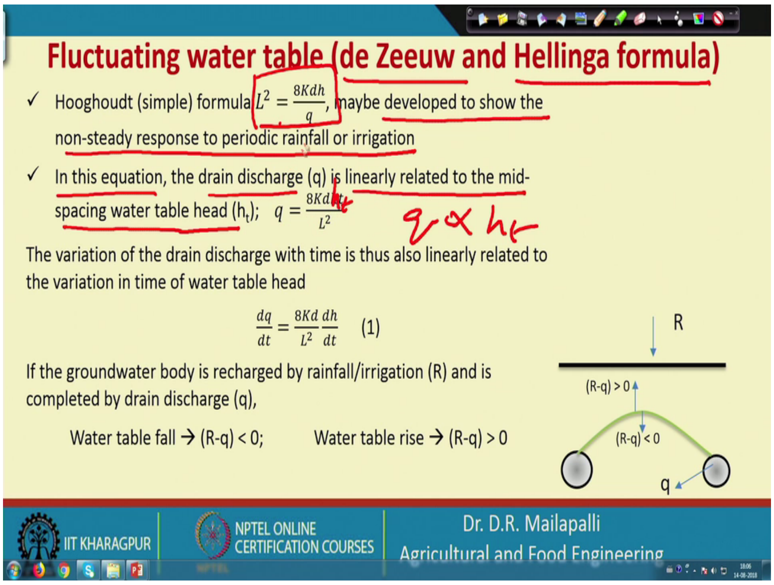

And then other one is the fluctuating water table. One is falling water table that we have seen using Glover-Dumm equation, the other one is fluctuating water table. So, this is basically using the de Zeeuw and Hellinga formula. So, in this, so main thing is the Hooghoudt simple formula will be using this one and then so this may be developed to show the non-steady response to periodic rainfall and irrigation. So, you will be using the Hooghoudt simple formula in the following you know steps to see to determine the fluctuating water table situation.

So, in this equation the drain discharge q is linearly related to mid spacing water table head h t. So, if you see here q is equal to 8 K dh t by L square, this is from Glover-Dumm. So, h t is a h and small t, ok. So, so q and h t are linearly, so proportionally linear related, ok. So, now, so from this is simple Hooghoudt equation, so just rearrange the terms here and q is equal to and q is equal to 8 q d h t by L square.

So, now differentiate this dq by d, 8 q d L square dh by dt here right dh t by t basically, ok. If the ground water body is recharged by a rainfall or irrigation for example, here so this is the recharge irrigation or I mean and q is taking place here. So, if R minus q is greater than 0. So, R minus q, so suppose this is q is taking place. So, R minus q is greater than 0 so that means, the water table I mean water is adding right water is adding. So, water table is go into go up, whereas, if R minus q less than q so that means, the water table is going below, ok. So, that that means, the water I mean the drain is drain is taking place right the drainage is taking place to the drain and crop is taking this water as crop water uptake, ok.

So, with this concept and with this formula we are going to develop the fluctuating water table I mean the equation for finding out the fluctuating water table. So, water table fall is happens when R minus q less than 0 and water table rise happens, when R minus q greater than 0, ok.

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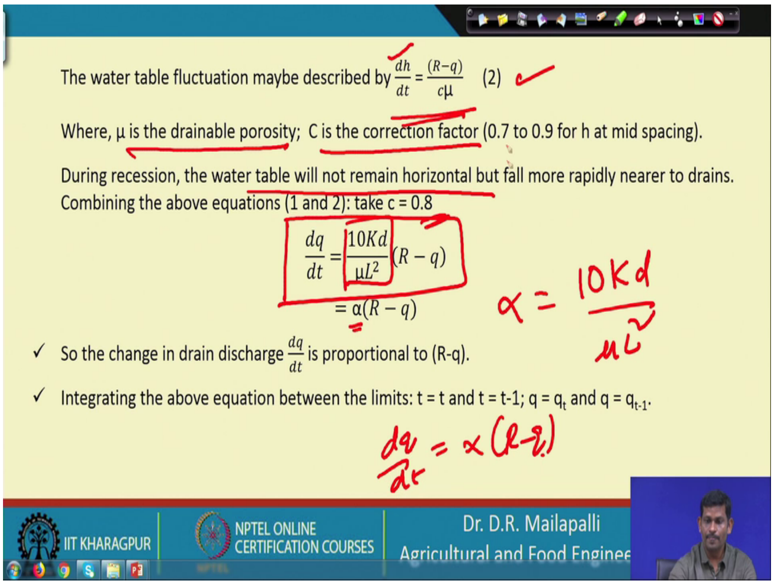

With these two conditions we are going to develop equation like this. So, the water table fluctuation may be described by dh by dt which is R minus q by c q, dh by dt is equal to R minus q by c q, where c is the correction factor and mu is the drainable porosity, ok. So, during the recession the water table will not remain horizontal, but fall more rapidly near to the drains, ok. So, now, combine 1 and 2 equations by taking 0.8 as c. So, dq by dt is equal to 10 Kd by mu L square into R minus q.

So, this is the equation when you combine the dh by dt the change in the water table that will be equal to R minus q the the effective recharge you can say c by q. So, which is equal to, so this can be written as alpha if you recollect alpha is equal to in the previous I mean slides Kd into mu L square, ok. So, now you can put this value as alpha and alpha into R minus q. So, finally, what we got is dq by dt is equal to alpha into R minus q, ok. This is the equation we got.

So, the change in drain discharge dq by dt is proportional to R minus q. So, this is true. And integrating the above equation between t when t equal to t between t and t minus 1 that is q 2 and q t minus 1, ok. So, if you can integrate that. So, you are going to get integrate in dq by R minus q divided by alpha into dt. So, R minus q t by R minus q t minus 1 this is exponential of minus alpha into dt, ok.

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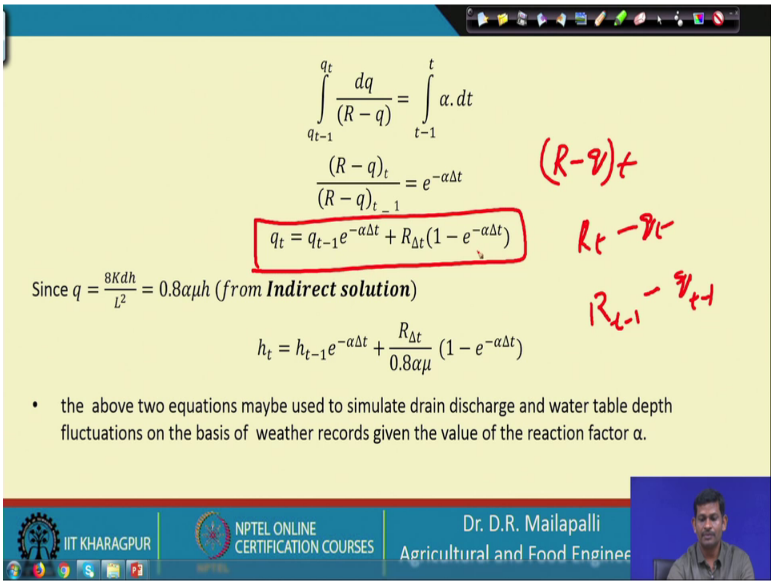

So, then qt is equal to qt minus 1, e power minus alpha into dt plus R delta t 1 minus e power minus alpha into dt, ok. So, this once you get this you have to use the I mean this is the all arranging the terms basically. So, delta t is delta t will be same, but R minus q into R minus q into t, so that will be what R t you know minus qt, ok. Similarly R minus q, R t minus 1 minus q t minus 1, and if you can you know rearrange this terms you are going to get the final equation q t is equal q t minus 1 right, e power minus alpha into delta t plus R delta t into 1 minus e power minus alpha into d t, ok.

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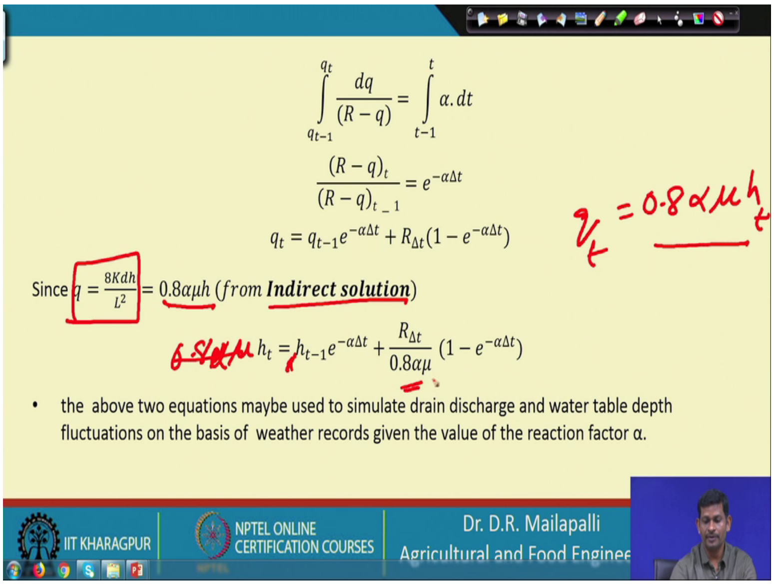

So, then since q is equal to 8 K dh by L square which is equal to 0.8 into alpha into mu h from this is from indirect solution if you recollect indirect solution. So, then, so then substitute for, so q is equal to here 0.8 into alpha into mu into h. So, q t means alpha into h t, right. So, substitute q t as 0.8 into alpha into mu h t, right. So, here 0.8 mu alpha mu h t the; similarly here 0.8 alpha into mu h t, ok; so, then you take common. So, this gets cancels out and finally, 0.8 mu alpha will be in you know denominator, ok.

So, this way, so not only discharge I mean the recharge. So, h t the water table height also can be I mean the water table position at a particular time can also estimated. So, the above two equation may be used to simulate the drain discharge and water table depth fluctuations on the basis of weather records given the value in the reaction factor alpha. So, based on alpha you can estimate q t as well as h t.

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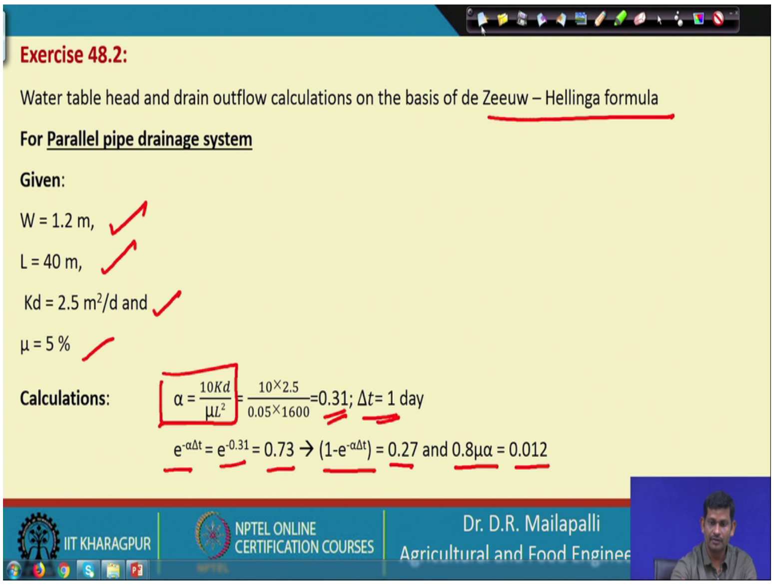

So, here is an exercise and the example water table head and drain outflow calculations on the basis of Zeeuw Hellinga formula. So, here W is given, L is 40 meter, Kd is given, mu is given, ok. So, calculations here alpha you have to estimate using this formula so that will be 0.31 and delta t is let us let us say time interval like 1 day. And e power minus alpha delta t that is e power minus 0.31. So, which will be 0.73 and another term we have to find out 1 minus e power minus alpha delta t that is 0.27 and finally, 0.8 into mu alpha is equal 0.012. So, we are going to use these values in the equations. So, so this is we are going to see those equations in the next slide, ok.

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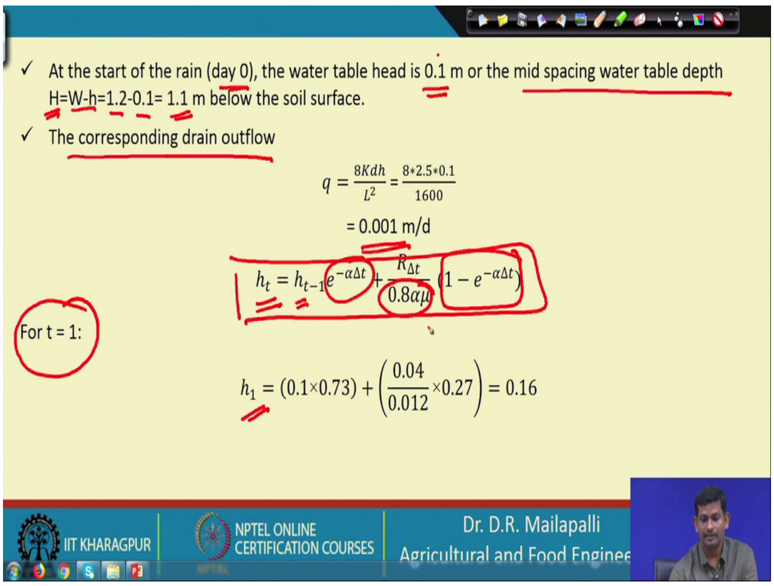

So, and even I would go the next slide, here at the start of the rain the 0th day the water table head is 0.1 right, 0.1 meter that is a given the value is given and the mid spacing water table depth. So, H, capital H is equal to W minus h 1.2 minus 0.1, 1.1 meter below the soil surface, ok. So, initially the water table head is 0.1, and that is at 1.1 below the soil surface in the corresponding drain outflow.

So, this h t is known right h t is known and the corresponding q will be estimated q Kd h by L square that is 0.001 and h t is the formula we know we know the formula. So, use this formula to find out the next time intervals you know h, but knowing the h t minus 1 we know h 0 right and then and the other values. So, we estimated these values like e power minus alpha delta t and this value also estimated and this value also estimated, and R delta t values will get from the table. So, we will go to the previous the slide.

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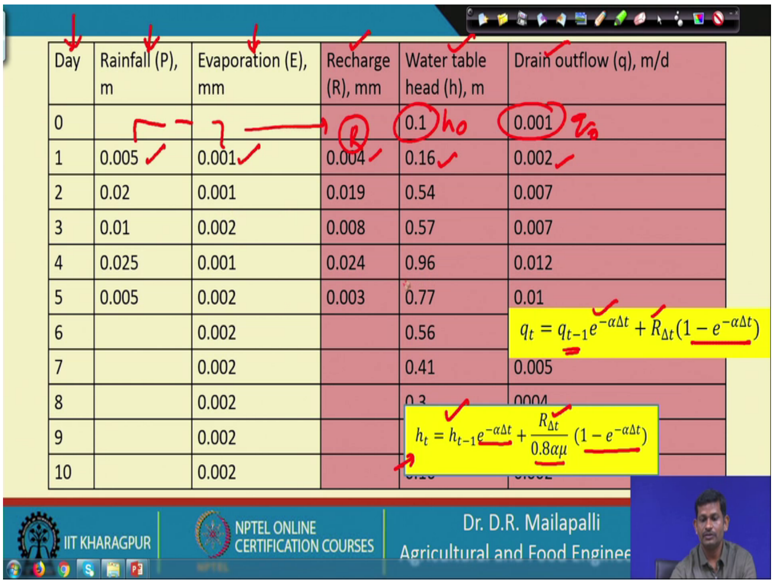

So, this is the slide we need to concentrate. So, here the day one days are on the first column and the rainfall on the second column, evaporation on the third column and the rest we need to estimate; what is the recharge, water table, head and drain outflow, these things will be estimated.

So, the given value 0 and 3 and the next day you have 0.005 rainfall and the evaporation is 0.001, and the recharge will be 0.004 that is the difference, difference of these minus these you get this one. And then and the just I mentioned point one we estimated the water table head based on the based on W and H, ok. So, that is the water table head we estimated and the corresponding I mean the inverse formula we estimated the 0.001 as the q, ok. So, this is h 0 and this is q 0.

So, knowing these h 0 q 0, so I mean the recharge R, right. So, here if you observe q, so for example h t so this one, h t; so this is known right, R delta is known and we have estimated this, and we have estimated this value, we have estimated this value. So, all values are known just substitute in and you get this one, ok. Similarly, here we have estimated this value, we have estimated, this value, this value and q, q naught and you can find out this q t. So, like that for each row you can estimate the recharge h t and q t values, ok. So, this table will gave the for a particular day what is the water table head and what is the drain outflow, ok. So, this is the advantage of this exercise.

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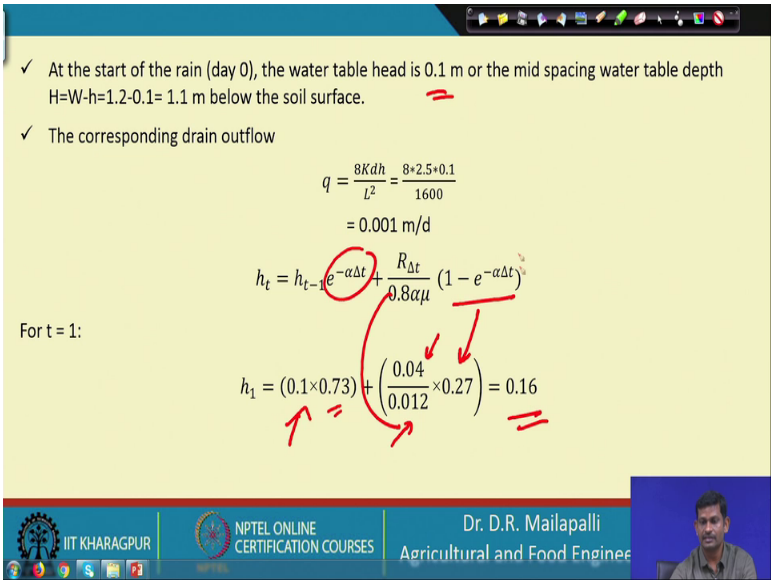

So, this is all calculations you can go through h 1 is estimated by knowing the by knowing the h naught right this is h naught. And then other values like e alpha naught and this is the recharge and this is the value you get from here and this is the value from here, so finally, 0.16. And using h 1 you can find out h 2, ok; and h 3, h 6 all are similarly the q 1, q 2, q 3, q 4 values, right, alright.

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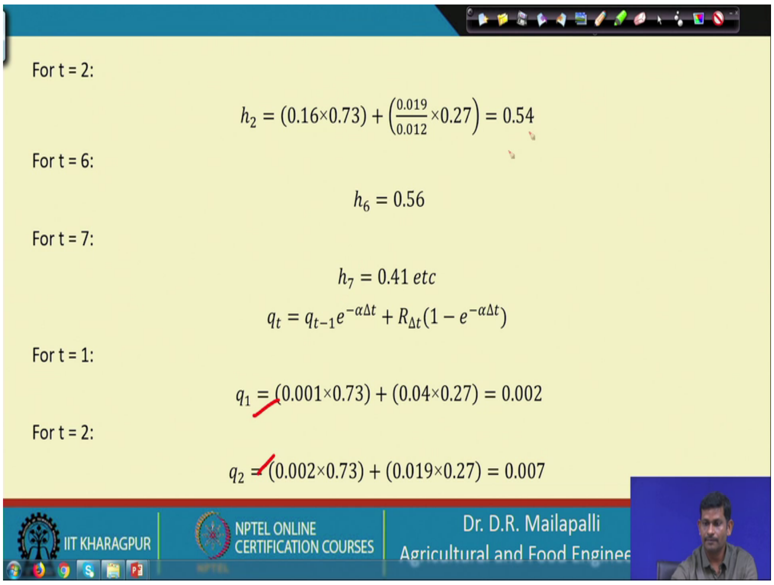

So, at the end of tenth day, so q is equal to 0.002 which is the evaporation may be expected to I mean start expected to start evaporation.

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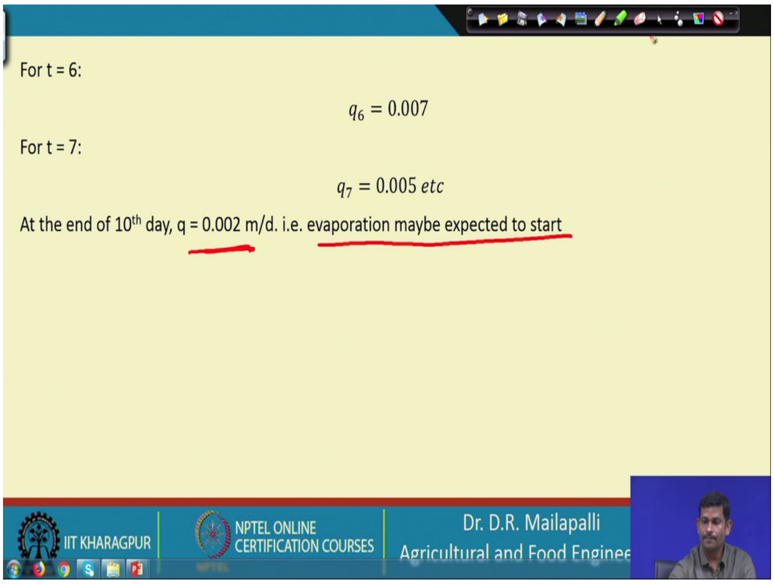

So, then after that evaporation going to start here on the tenth day because, this is equivalent to evaporation rate, ok; so, this is all about the lecture. So, in this the basically we focused on the water table a situation in case of unsteady state condition. So, first we found the Glover-Dumm equation in case of falling head you know water table falling water table condition. So, this is happen when the irrigation is taking place, right. So, that is the thing. Then other one is Hellinga formula for you know sudden like fluctuating water table. So, for both the cases we derived the equations for you know spacing in case of Hellinga formula. So, we will be finding out both water table fluctuations as well as the drainage water fluctuations and we have shown that fluctuations through I mean by you know by solving problem, ok, yeah.

Thank you so much.