Course on Momentum Transfer in Process Engineering
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Lecture 08
Module 2
Navier stokes equations utilization for solving problems

Now I hope you have tried with several other problems in solving different problems using the Navier Stokes Equations. So if you if you have any problem, any difficulty in understanding obviously you can come back to us we will try or our best to solve those, but as we said earlier that to to understand the utilities the the implications the the the places over it can be utilized, the areas where it can be utilized the equation of motions you need to find out you need to solve some more problems and that some more problems let us take one more problem today and that problem will let us come to that right so if we if we if we think that any problem which you can do or which you can that you bring to us and we will solve them, ok.

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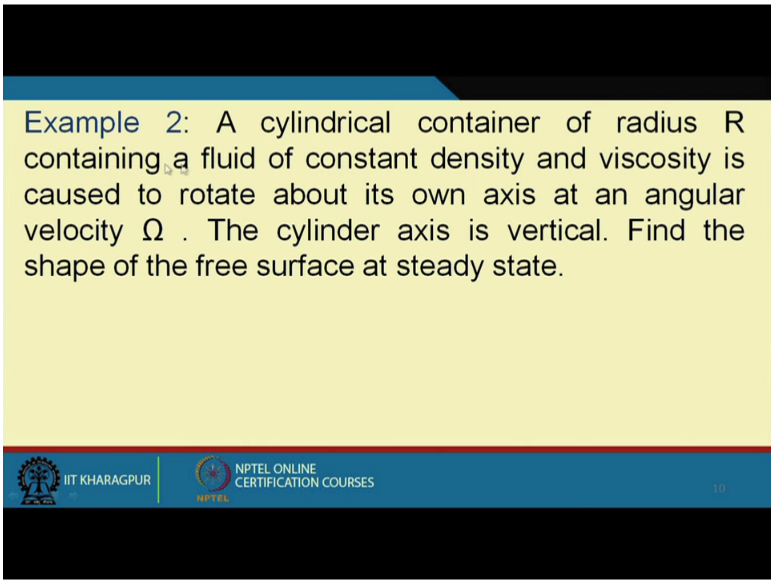

So let us look into the problem second that is a cylindrical container of radius R containing a fluid of constant density and viscosity is caused to rotate about its own axis at an angular velocity omega. The the cylinder axis is vertical. Find the shape of the free surface at steady state, right.

I hope in secondary we have done a locus of equation or locus of some surface etc those things we have done in a secondary. So here also a similar problem we will do in understanding the problem here you you just imagine I do not know whether during your childhood you had played it or you had that option or playing with that is also true because nowadays it is being so much what should I say urbanization that those openness is also not available all the places.

So if you if you think if you if you if you go back to your child ages and their if you had ever a bucket full of water right and you take that full of water and start making a round right, start rotating like this right bucket full of water and you are rotating on around you, right with that bucket. If you had remember if you remember that the surface of the bucket water that looks like this right that looks like this that the surface is like that when you are rotating with that bucket, right.

If you remember and know we will find out this surface of the bucket which is like this and what is the locus of that surface this you can easily do with the help of Navier Stokes Equations. So this is another kind of of application of the Navier Stokes to solve your time and again different problems, ok. So let us look into that.

So we say again this problem a cylindrical container of radius R containing a fluid of constant density and viscosity is caused to rotate about its own axis at an angular velocity omega. The cylinder axis is vertical. Find the shape of the free surface at steady state, right.

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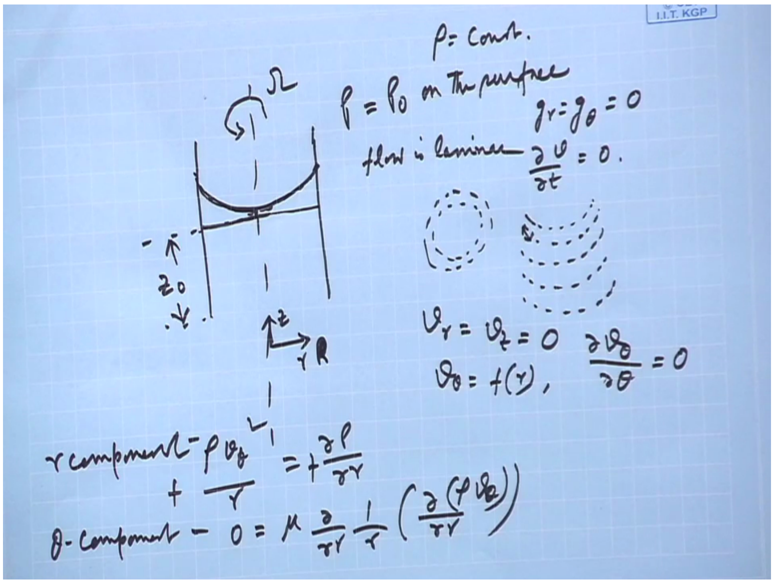

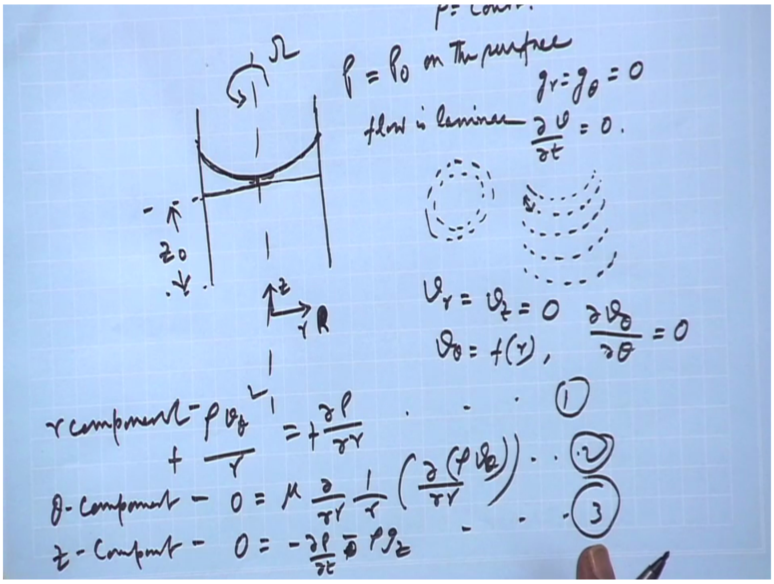

So let us let us look into this that we have a cylinder right and this cylinder has the axis like this ok and originally originally we had a a layer of liquid like this right. Now suddenly we started moving this with an omega in the axis in the own axis omega right and if we look at the this is the R or say let us write small r and this is z right and third one is the theta of course right. So initially if we say this is the height z0 initial height that is where we started with right and after some time when the steady state has arrived we will have a surface like this right we will have a surface like this.

Now we have to find out what is the locus of this surface right that is this equation of this surface that if we can solve then we can say that the understanding of Navier Stokes Equation has become little bit more easier or we have we have understood a little more application of the Navier Stokes Equation.

Here one more thing has to be said that P is equal to P0 on the surface right, P is equals to P0 on the surface if that be true again we bring the again we bring the condition the Physical understanding of the problem right. We said that we have a liquid in a container and this container is rotating on its own axis like this and with a velocity angular velocity of omega right.

So we have this axis at R and z these are the two initial height where we started with it was z0 and after some time when steady state arrived we got this surface with an angular velocity omega like this and P is equals to P0 obviously of that pressure on all the surface is same, right. So from the understanding of the problem we can say that since this is a vertical gr is equals to g theta is equals to 0. We also said flow is equals to constant, we also said at steady state, so del del t of any vr, vz, v theta all are is equals to 0.

Fourth thing which we have not said let us say again the flow or it is such a way that terminance is not occurring so flow is laminar right, again laminar means this layer, this layer they are all not mixing with each other, right a free layers is like that again when it became a surface like this the layers in the in the bucket were like this right. So that means there is no in mixing of the layers interaction of vr, vz v theta these were not there, right it is a streamline laminar flow, ok.

So this indicates that vr is equals to vz is equals to 0, right and v theta is also a function of r right and since it is laminar we can also say del v theta del theta is equals to 0, right. That means this layer is moving right this layer is moving but there is no interaction between this v theta and this v theta, right. So otherwise there would have been a rippling there would have been a mixing of the layers which is not there.

So del v theta del theta is also 0, right then we can say that r component r component of the velocity that is equals to rho v theta square by r this is del P del r, right you remember in the previous thing we have shown it was minus minus versus it is on the both site so we can write rho v theta square r is equals to del P del r. Similarly theta component we can write for theta component we can write that this was 0 equals to mu into del del r of 1 by r into del del r of rho v theta if you remember the same thing we have written.

And z component we can write this is 0 is equals to minus del P del z plus or here it was minus minus rho gz since it is there. So rho g vertical thing is there so rho gz is also acting or gz is also there, that these are the three equations we have three equations from the Navier Stokes Equation which originally started with and putting the boundaries putting the putting the understanding of this solution understanding of the problem we can we have identified different individual components and said that yes this is the equations which come after putting the after putting the right situations involved in the problem, right.

So the terms which are which are which can be neglected or which can be written as 0 is already shown also in the previous class right so we are not repeating their otherwise again and again the same thing we will be doing, right. So we understand that again if we have any any any understanding problem you can come back to ask and we will I will definitely try to solve your doubt if there be any, ok.

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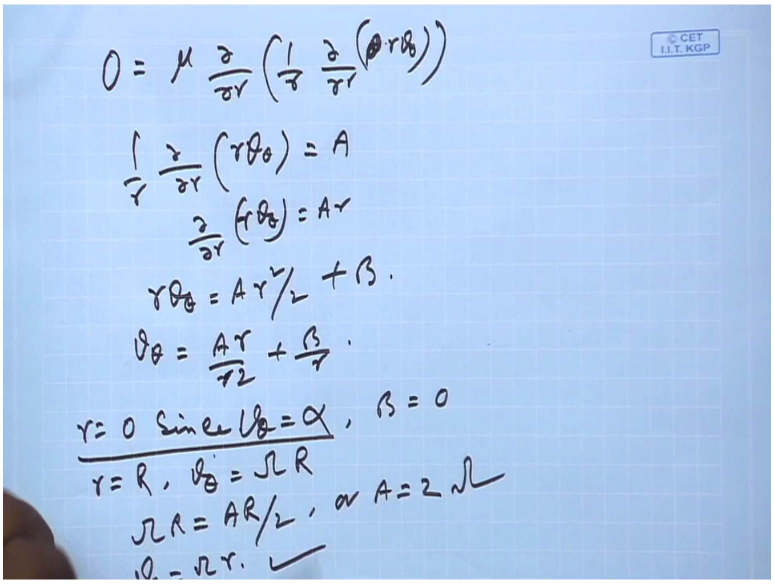

So if this is these are the three equations then we can say that the integrating the theta component that is 0 is equals to theta component towards 0 is equals to mu del del r of 1 by r del del r of 0 sorry r rv theta rv theta right. So if this is there this was that on first integration we can write del del r 1 by r del del r of rv theta is equals to A right on simplification we can write del del r of rv theta is equals to Ar right.

So this was first integration of with constant A and the second integration we can say rv theta is equals to Ar square by 2 plus B, where B is the second integration constant, right which we have also seen earlier. So we can write v theta is equals to A by A by sorry A by 2 into r Ar by 2, so v theta is equals to Ar by 2 r r so it goes out plus B by r right.

Now at r is equals to 0 since at r is equals to 0 since v theta is not equals to infinity at r is equals to 0 since v theta is not equals to infinity we can write B to be equals to 0, otherwise if r is 0 v theta has to be infinity then only you will have some value otherwise B is equals to 0, since this is applicable we can write B to be equals to 0 right the second (())(15:58) at r is equals to capital R, v theta is equals to capital omega R, right.

So this omega R is the second equation so we can write omega R is equals to AR by 2 or A is equals to 2 omega, right. Therefore v theta is equals to omega r right so this is v theta is equals to v is 0 right and A is equals to 2 omega, so A is 2 omega so v theta is equals to omega r this is one situation from the solution of one component that is theta component.

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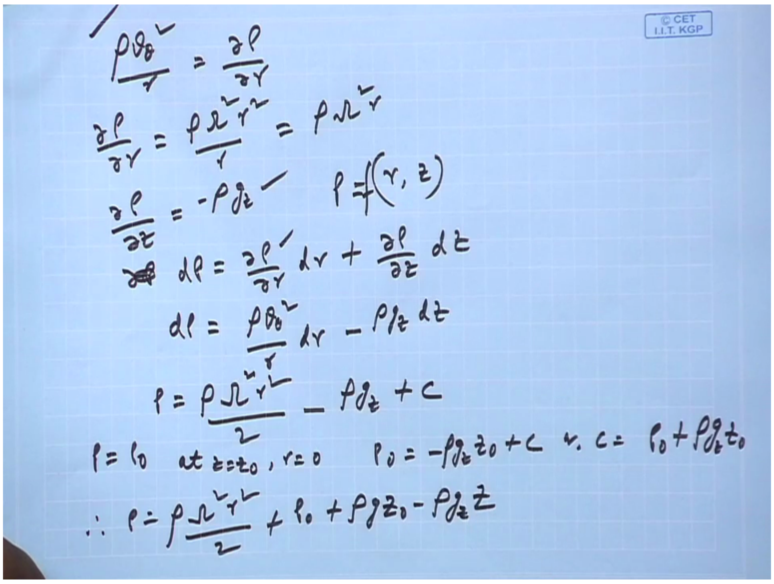

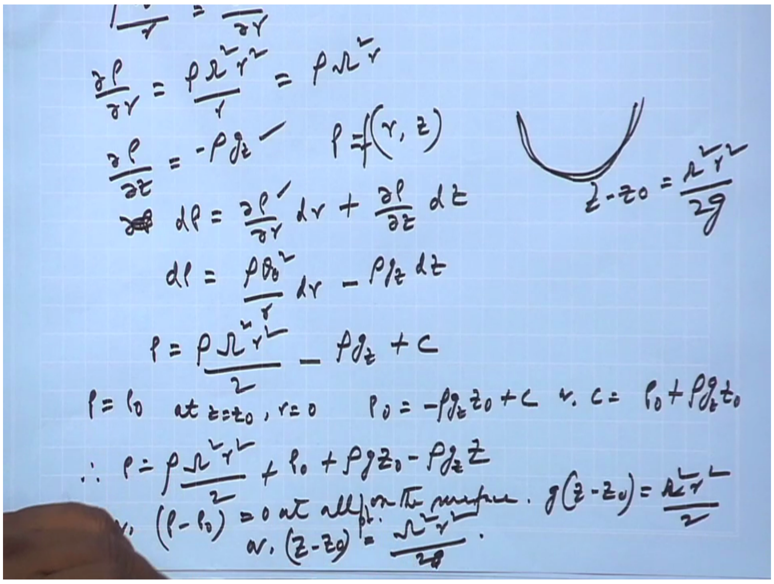

Now if we see the other one rho v theta square by r is equals to del P del r, rho v theta square by r this was from the r component right we did. So this on this on simplification we can also right del P del r is equals to already we have found out what is the value of v theta so that is rho omega square by r this is omega r square right r into r square omega r it was (A is) v theta was omega r so this is equals to this so that means rho omega square r is that del P del r, right.

And the third equation was del P del z this is equals to minus rho gz right if you remember that it was 0 is equals to del 2 (())(18:09) del z minus rho gz so we can write del P del z del P del z minus rho gz so no it was not minus it was plus del 2 del z minus rho plus rho gz so that on simplification it was it comes del P del z is equals to minus rho gz right.

So now if that be true we can write since P is a function of r and z right since P is a function of r and z right we can write that del P or dP is equals to del P del r dr plus del P del z dz. So dP is equals to del P del r dr plus del P del z dz. This we can write right. So now we have already found out the value of del P del r this is rho v theta square by r and also we have found out del P del z is rho gz.

So we can write that dP is equals to this is del P del r is 0 v theta square by r right. So dr (plus) minus so this is minus this plus goes out rho g del P del z is we have found out minus rho gz so minus rho gz dz, right. So this is then dP is equals to rho v theta square by r dr minus rho gz dz, right.

So on integration of this we can write P is equals to rho right v theta we have already found out that was equals to omega r so we can we we we can write this is omega square r square right by rho v theta square by r is del P del r ok. So this is P is equals to rho omega square by 2 because this dr this becomes 1 by r dr then that becomes r square by 2 and on integration minus rho gz, right this plus C.

So dP become P 1 by r dr this becomes rho omega square is v theta was omega square r square right. So omega square by r square that became r so on r dr integration r square by 2 rho gz plus C, right so this is that and from the given boundary we know P is equals to P0 right that means at z is equals to z0, r is equals to 0, this is true at P is equals to 0 when this is at z is equals to z0 and r is equals to 0 right. Therefore we can write P0 is equals to minus rho minus rho gz right or or minus rho gz, z0 plus C right or C is equals to P0 plus rho gz z0, right.

Therefore we can write P is equals to rho omega square r square by 2 plus P0 plus rho gz0 minus rho gz z right or we can write P minus P0 P minus P0 this is equals to 0 at all the points on the all the points on the surface, right. Therefore we can write g into z minus z0 is equals to omega square r square by 2 this is omega square r square by 2 right or we can write that z minus z0 this is equals to omega square r square by 2g, right.

So the locus of this equation right locus of this curve we can write this is nothing but z minus z0 is equals to omega square r square by 2g, right. So we have solved this problem by using the equation of motion or by using the Navier Stokes Equation, obviously we have not done some of the things which we have done earlier that showing initially that how the conditions r applicable right.

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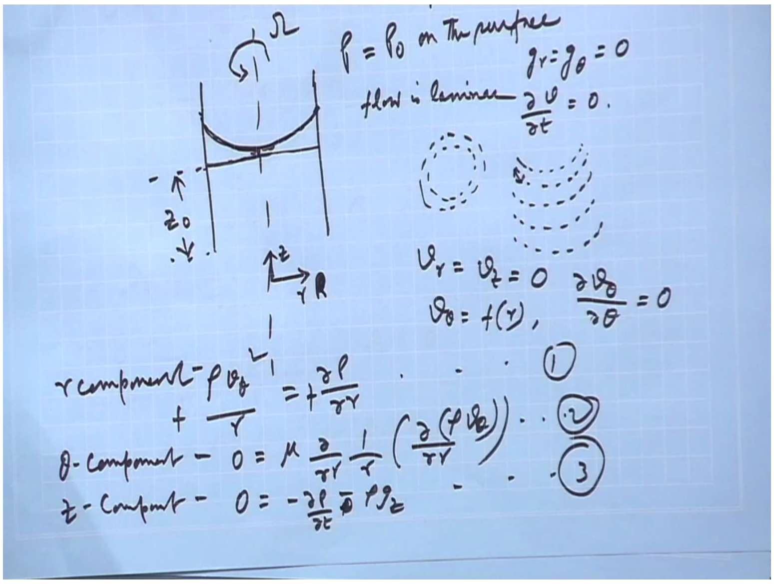

Now since we have little time so what we can do we can what we can do oh this was our problem right we can go back a little and show that from the original equations we have how arrived. The problem was given that we have a bucket and that bucket was rotating that I gave the example we have a bucket which we were rotating like this but in this case say we have some bucket which is rotating at its own axis with an angular velocity of omega right and now we were said what is the steady state flow this curve there will be there will be there will be a circular this thing so that will be how where is the locus of this right this we were asked that what is the equation right.

And we said which starts from the Navier Stokes Equation we have already shown you what is the what is the locus of this line that we have shown you but again for recapitulation or a reunderstanding that let us look into the r component, theta component and z component.

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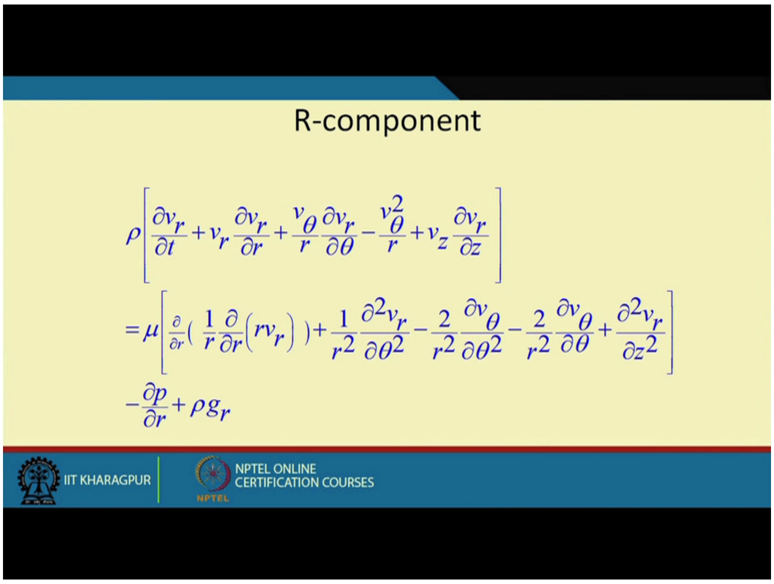

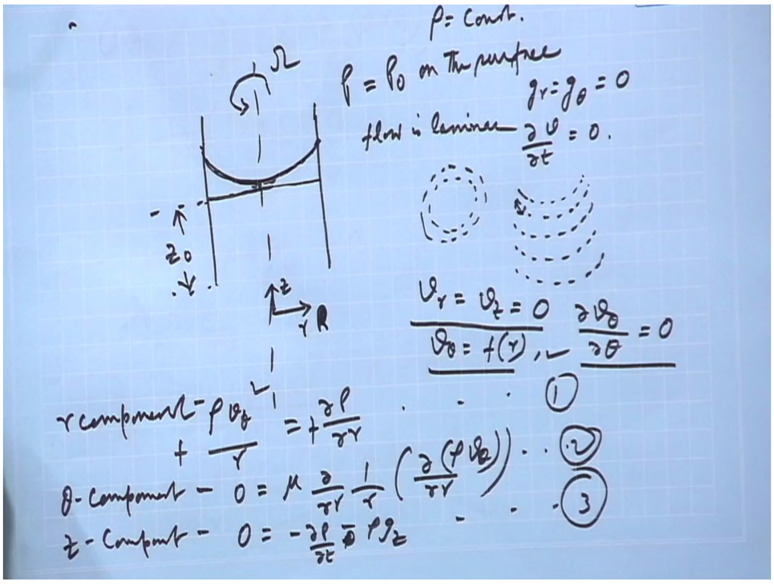

So here we have said that the given condition P is P0 on this surface, gr is g theta is equals to 0 in earlier we had earlier we also had vr is equals to v theta is equals to 0 but here we have vr is vz is equals to 0 and v theta is a function of r right and in this case no in that vr, vz was 0 and v theta was also a function of r here also v theta is a function of r and del v theta del theta is 0. These r the three from the (())(27:22) vr is v theta vz 0 because have there be any any r component right so there would have been a mixing, (())(27:34) there be any z component then there would have been a mixing, which is not there it is under laminar condition right.

If that is true then vr is vz is equals to 0 and del v theta del theta is equals to 0 and also vr is a function of r right. So if we go back to that r component, right since vr is 0 and del vr del t that is it is steady state, rho is constant ok but this is 0 this component is 0, this component is

0, right but v theta square by r with a negative minus rho v theta square by r that remains vz 0, this is 0 right since vr is 0 this 0, vr is 0 this is 0 del v theta del theta is 0, so that del v theta del theta 0, del v theta del theta also becomes 0 or this should be del 2v theta del theta square so this is 0, this is 0 and vr is 0 this is 0 minus del p del r plus rho gr.

So we got minus rho v theta square by r minus is equals to minus del p del r plus rho gr, right. But here also gr is 0 so we get del p del r so this we said. And since there was two negatives we can make it positive. Now for that theta component we have said right the first v theta del v theta del theta del t is 0 which is steady state, vr is 0 this is out del v theta del theta is 0, vr is 0 this is out vz is 0 this is out right mu del del r of 1 by r del del r of rv theta this is not 0 this remains, but del 2 v theta del theta square is out then del vr del theta is out vr is 0, del v theta del z square del 2 v theta del z square del v theta del z is also 0.

So del 2 v theta del z square is also 0. So this is minus 1 by r del p del theta plus rho gr, right gr is also and this should be g theta, right. So we know g theta is we said that gr is equals to g theta is equals to 0 so gr g theta is 0. So this becomes 0, but del p del theta that is also we said this del p del theta cannot be there so that is also 0 because this p on this on this surface this p is same at every place. So got the second equation 0 is equals to mu del del r of 1 by r del del r of rho v theta.

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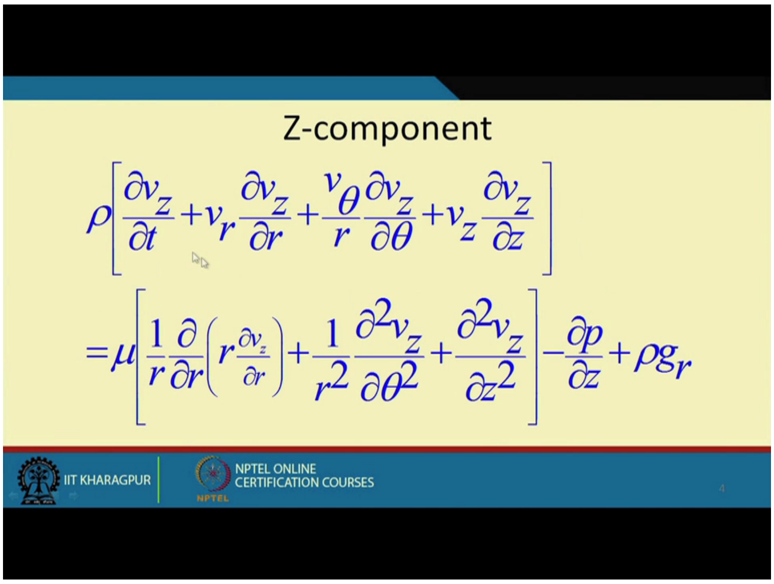

And the third z component we got this del vz del t is 0 vr is 0 this del vz del theta vz is 0 vz is 0 this all terms goes out then vz is 0 this comes out this vz is 0 this comes out del 2 vz del z square goes out what remains del p del z minus plus rho gz right.

So we wrote 0 equal to minus del p del z of course this minus rho gz, there was also a negative ok. Now this was rho gz that was since it is acting vertical so here we r saying that gravity is acting on that so means rho gz plus this has become minus and this on solution we have found out that ultimately we got this v theta was omega r and we got the locus of the equation as z minus z0 is equals to omega square r square by 2 right, z minus z0, omega square r square by 2g, this is the solution, right.

If you remember that we said that we will do some problems for the application of the Navier Stokes Equation, right so this we say that ok we have done some problem on Navier Stokes Equation, thank you.