

Course on Momentum Transfer in Process Engineering
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Lecture 59
Module 12
Problem and solution (Continued)

Okay, now we have been doing problems on different aspects, now another problem which is very very useful that for Newtonian and Non-Newtonian fluids, right? We have done flow through Newtonian, flow through pipes, flow through slits for Newtonian fluids we have also done flow through pipes, flow through slits for Non-Newtonian fluids no and in the Non-Newtonian fluid it flows the power law equation and there we have seen that the flow behavior and consistency coefficient or index indices they are having tremendous effect on the flow characteristics, right?

But we also need to know the what could be the values which are normally we handle with, for example if is Newtonian fluid then the value of n is 1 and the viscosity is the k that is the consistency coefficient we call it to be that viscosity, but if it is not Newtonian if it is Non-Newtonian then both the consistency coefficient and the flow behavior index or indices this we must have some idea about the values, right? So let us do also some problem to be solved or problem solution such that we have some idea on that, okay.

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Problem: A fluid is flowing through a tube having an inside diameter of 0.008 m and a length of 0.3 m at a flow rate of $5 \times 10^{-5} \text{ m}^3 / \text{s}$ and thereby induces a pressure drop of 1000 Pa.

(1) Calculate the apparent viscosity, defined as the viscosity of a Newtonian fluid which exhibit the same pressure drop as this fluid at the rate of flow through this tube.



(2) If the flow rate of the fluid is $1 \times 10^{-4} \text{ m}^3 / \text{s}$ and the length of the tube is reduced to 0.2 m keeping the same diameter induces a pressure drop of 2000 Pa, what will be the flow behavior and consistency indices of the fluid?

Solution: From Hagen Poiseuille equation, we get,

$$V_{av} = \frac{(P_{in} - P_{out}) R^2}{8 \mu L} = \frac{\Delta P R^2}{8 \mu L} \quad \text{..... (1)}$$



So let us look into this problem that, A fluid is flowing through a tube having an inside diameter 0.008 meter and a length of 0.3 meter at a flow rate of $5 \times 10^{-5} \text{ m}^3 / \text{s}$ and thereby induces a pressure drop of 1000 Pascal. Calculate the apparent viscosity as defined as the viscosity of a Newtonian fluid which exhibit the same pressure drop as this fluid at the rate of flow through this tube, this is 1.

Then, if the flow rate of the fluid is $1 \times 10^{-4} \text{ m}^3 / \text{s}$ and the length of the tube is reduced to 0.2 meter keeping the same diameter induces a pressure drop of 2000 Pascal. What will be the flow behavior and consistency indices of the fluid, right? So if we

reread the problem is like this, A fluid is flowing through a tube having an inside diameter of 0.008 meter and a length of 0.3 meter.

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$D = 0.008 \text{ m} = 2R$
 $L = 0.3 \text{ m}$
 $Q = 5 \times 10^{-3} \text{ m}^3/\text{s}$
 $\Delta P = 1000 \text{ Pa}$

$Q = 1 \times 10^{-4} \text{ m}^3/\text{s}$
 $L = 0.2 \text{ m}$
 $D = 0.008 \text{ m}$
 $\Delta P = 2000 \text{ Pa}$

Hagen-Poiseuille eqn.
 $\Delta P = \frac{32 \mu v_{av} L}{D^2}$; $v_{av} = \frac{\Delta P D^2}{32 \mu L} = \frac{\Delta P R^2}{8 \mu L}$... ①

$\mu_{app} = \frac{\Delta P R^2}{8 v_{av} L} = \frac{1000 \times (0.004)^2}{8 \times \pi (0.004)^2 \times 0.3}$

$v_{av} = \frac{Q}{A} = \frac{5 \times 10^{-3}}{\pi (0.004)^2} \text{ m/s}$

$= \frac{1000 \times (0.004)^2 \times \pi (0.004)^2}{8 \times 5 \times 10^{-3} \times 0.3} = 6.7 \times 10^{-5} \text{ Pa}\cdot\text{s}$ ✓

② $v_{av} = \frac{1 \times 10^{-4}}{\pi (0.004)^2} = 1.99 \text{ m/s}$

So inside diameter D is equals to 0.008 meter and length L is equals to 0.3 meter, at a flow rate of 5 into 10 to the power, so Q is equals to 5 into 10 to the power minus 3 meter cube per second, thereby induces a pressure drop so delta P is equals to 1000 Pascal. Calculate the apparent viscosity as defined as the viscosity of a Newtonian fluid which exhibit the same pressure drop as this fluid at the rate of flow through this tube.

Second question, if the flow rate of the fluid is 1 into 10 to the power minus 4 meter cube per second and the length of the tube is reduced, so L is 0.2 meter the same diameter (induce) keeping the same diameter that is D is equals to 0.008 meter and induces a pressure drop so delta P is equals to 2000 Pascal, right? What will be the flow behavior indices and the consistency induces of the fluid, right?

So this we have to do, so let us solve it now for solution I think you may have a little problem in this let me try, now from the Hagen Poiseuille's equation which we know from the Hagen Poiseuille's equation which we know that delta P is equals to 32 mu v average L by D square, right? So this is the delta P, okay therefore from there we can write that v average is equals to delta P D square by 32 mu L that can be written as delta P R square by 8 mu L, right? So if we call it to be equation number 1, right?

Now this equation can be used to determine the viscosity of Non-Newtonian fluid if the viscosity obtain will be apparent your viscosity this will be apparent viscosity and using a single rate of flow and in that case we can say $\mu_{\text{apparent}} = \frac{\Delta P R^2}{8 v_{\text{average}} L}$, right?

So given D is so much so R by 2 rather D is so much so is equals to $2R$ should not be R by 2 is equals to $2R$, right? So R is equals to 0.004 , right? So if we write that value we can say that ΔP given is 1000 , R is 0.004 square 8 , right? Now v_{average} that can be found out from v_{average} is equals to flow rate has been given that is Q over area cross sectional area, right? So if we say that this is equals to flow rate has been given 5×10^{-3} meter cube per second and area is πR^2 $\pi 0.004$ square, right?

So this is we get so much meter per second, so if we apply that $8 v_{\text{average}}$ $8 v_{\text{average}}$ is that so 8 into π into 0.004 square, right? L , L given is 0.3 meter, right? So we can write that this is into 0.3 and this is already done, this is so and here π into did we do wrong, yes we had done wrong 1000 into 0.004 square by $8 v_{\text{average}}$ is that so into 5×10^{-3} in the denominator, right? Into L is 0.3 into numerator is 5 and this is 0.004 square, okay.

So this comes to equal to let us look into that 1000 into 0.004 square this much into π into 0.004 square this much divided by 8 divided by 5×10^{-3} divided by 0.3 is equals to 6.7×10^{-5} so much Pascal second, right? 6.7×10^{-5} Pascal second, why it is 10^{-3} is coming here 10^{-3} it is $\pi 0.004$ square 5×10^{-3} meter cube per second this is meter square, so $1000 \Delta P 0.004$ square π into 0.004 square, fine by $8 v_{\text{average}}$ is this 5×10^{-3} and L is 0.3 .

So somewhere again we have done little wrong let us look into 0.004 square into 0.004 square is equals to this much into π into 1000 this much divided by 8 divided by 5×10^{-3} divided by 0.3 divided by 10^{-3} , yeah it is coming 6.7×10^{-5} , right? So perhaps we have done something wrong, v_{average} is coming 0.995 let us see once whether v_{average} is coming 5×10^{-3} , yeah divided by π divided by 0.004 square this is so 99.47 , so that was something wrong it was done here is not 5 so there it is comes it is correct, okay.

So mu apparent is so much, then next we what we have to find out is second case, first case we have been asked that what is the mu apparent, right? Apparent viscosity we have found out, now second case we have to do that v average second case we have to do v average this is equals to 10 to the power minus 4 Q divided by pi into 0.004 square, right? This is equals to 1 into 10 to the power minus 4 this much divided by pi this much divided by 0.004 square this so 1.989 so 1.99, right? Meter per second.

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$$\mu_{app} = \frac{2000 \times (0.004)^2}{8 \times 1.99 \times 0.2} = 0.01005 \text{ Pa.s.} = 1.005 \times 10^{-2} \text{ Pa.s.}$$

Now, shear rate at the wall ($r=R$) for a Newtonian fluid is

$$-\frac{dv}{dr}|_W = \frac{4v_{av}}{R} \quad \text{Where as for non-Newtonian fluid}$$

$$\mu_{app} = K(\gamma)^{n-1} \quad \text{and}$$

$$-\frac{dv}{dr}|_W = v_{av} \left(\frac{3n+1}{n}\right) \frac{1}{R} = \frac{4v_{av}}{R} \left(\frac{3}{4} + \frac{1}{4n}\right)$$

$$\omega, \gamma_W = \frac{4v_{av}}{R} (\phi) \quad \text{Where, } \phi = 0.75 + \frac{0.25}{n}$$

$$\therefore \mu_{app} = K \left(\frac{4\phi}{R}\right)^{n-1} v_{av}^{n-1}$$

So if it is so 1.99 meter per second, then we can write mu apparent, right? So mu apparent how much it is coming for 2000 delta P into R square 0.004 square divided by 8 into v average, v average is 1.99 into L, L is 0.2 so this comes equal to 2000 into 0.004 square that is this much divided by 8 divided by 1.99 divided by 0.2, 0.0100, right? 0.01005, right? So much Pascal second so that can be said 1.005 into 10 to the power minus 2 Pascal second so mu apparent, right?

Now we can say that shear rate at the wall that is at R is equals to R the Newtonian fluid is minus dv dr at the wall is equals to 4 v average over R, right? Whereas for a Non-Newtonian fluid whereas for a Non-Newtonian fluid mu apparent is equals to K into gamma to the power n minus 1 and minus dv dr at the wall is equals to v average into 3 n plus 1 over n into 1 by R that can be written as 4 v average divided by R into 3 by 4 plus 1 by 4 n, right?

Or we can say gamma at wall w is equal to 4 v average divided by R times shi so where we can say shi is equals to 3 by 4 means 0.75 plus 1 by 4 means 0.25 over n, right? So mu apparent we can write to be k into 4 shi divided by R to the power n minus 1 v average to the power n minus 1, right?

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1 & 2. Velocity & corresponding apparent viscosity
 at $5 \times 10^{-2} \text{ m/s}$ & $1 \times 10^{-4} \text{ m/s}$

$$\frac{\mu_{app1}}{\mu_{app2}} = \frac{k \left(\frac{4\phi}{R}\right)^{n-1} v_{av1}^{n-1}}{k \left(\frac{4\phi}{R}\right)^{n-1} v_{av2}^{n-1}} = \left(\frac{v_{av1}}{v_{av2}}\right)^{n-1}$$

$$\log\left(\frac{\mu_{app1}}{\mu_{app2}}\right) = \log\left(\frac{v_{av1}}{v_{av2}}\right)^{n-1}$$

$$\therefore \log\left(\frac{6.7 \times 10^{-5}}{1.005 \times 10^{-2}}\right) = (n-1) \log\left(\frac{99.42}{1.99}\right)$$

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$$\therefore \log\left(\frac{6.7 \times 10^{-5}}{1.005 \times 10^{-2}}\right) = (n-1) \log\left(\frac{99.42}{1.99}\right)$$

$$\therefore \log(0.0066) = (n-1) \log(49.92)$$

$$(n-1) = \frac{\log(0.0066)}{\log(49.92)} = -2.18$$

$$\therefore n = \boxed{-1.18}$$

Now if we use now if we use 1 and 2 for subscript for velocity and corresponding apparent viscosity then this is at viscosity means at 5 10 to the power minus 5 meter cube per second and 1 10 to the power minus 4 meter cube per second, right?

So for these two flow if we use 1 and 2 as its subscripts, then we can write that $\mu_{apparent 1}$ over $\mu_{apparent 2}$ this can be written as $K \cdot \Delta \phi$ by R to the power $n - 1$ $v_{average 1}$ to the power $n - 1$ divided by $K \cdot \Delta \phi$ divided by R to the power $n - 1$ $v_{average 2}$ to the power $n - 1$, right? So this is nothing but $v_{average 1}$ by $v_{average 2}$ to the power $n - 1$, right? So if we take log in both the sides so we can say log of $\mu_{apparent 1}$ over $\mu_{apparent 2}$ this is equals to log of $v_{average 1}$ over $v_{average 2}$ to the power $n - 1$ or we can write substitute the values log of $6.7 \cdot 10$ to the power minus 3 divided by it is was 6.7 it was $6.7 \cdot 10$ to the power minus 5 not minus 3, $6.7 \cdot 10$ to the power minus 5 and second one we got to be $\mu_{apparent 1.005}$ into 10 to the power minus 2, right?

This is equals to $n - 1$ log of $v_{average}$ is equals to that we have found out average v this is in one case we have gotten average v to be 1.99 in one case and in other case we have seen here it to be $5 \cdot 10$ to the power minus 5 by $5 \cdot 0.004$ square, right? So let us look into that $5 \cdot 10$ to the power minus 3 square 3 plus minus is equal to divided by π divided by 0.004 square. So that is 99.47, right? 99.47 meter per second, right? So that if we substitute here that log $v_{average 1}$ was 99.47 over 1.99, right?

So we can say then $6.7 \cdot 10$ to the power minus 5, $6.7 \cdot 10$ to the power minus 5 is equal to this divided by $1.005 \cdot 10$ x to the power minus 2 this is equal to this, right? So log of 0.0066 is equals to $n - 1$ into log of 99.47 by 1.99 is equals to 49.98, right? So $n - 1$ is equals to log of that means let us put that 49.98 log is this much divided by 0.0066 log is equal to minus 2.18, right? So n is equals to this plus 1 minus 1.18 just check whether this value is correct or not, okay we will continue in the next class, thank you.