

Momentum Transfer in Process Engineering
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Module 11
Lecture No 53
Fluidized bed flow

So we have seen that what is the minimum fluidised velocity or the condition at which the particles start moving corresponding to that velocity which is the minimum fluidisation velocity and the height is the minimum fluidisation velocity condition height and velocity is already said and of course before the fluidisation starts, the packing was compact, now when the fluidisation has started packing will be little if it was closed pack, now it will be little loose packed and that that void corresponding to that is called minimum fluidisation void, void under minimum fluidisation condition. So let us now find out what is the height, what is the velocity all this let us now find out okay.

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Porosity ϵ , velocity V' , length or height L , pressure drop of cross section area (CSA) = A = uniform. $\epsilon=0$

Volume of the bed = $LA(1-\epsilon)$
 under minimum fluidized condition ϵ_{mf} , velocity V'_{mf} , length L_{mf}
 Volume of the bed under minimum fluidized condition = $L_{mf}A(1-\epsilon_{mf})$
 Volume of the particle having no porosity = LA

$\therefore LA = L_{mf}A(1-\epsilon_{mf}) \therefore L_{mf} = \frac{LA}{A(1-\epsilon_{mf})} = \frac{L}{(1-\epsilon_{mf})}$

$L_1A(1-\epsilon_1) = L_2A(1-\epsilon_2) \dots$ or, $\frac{L_1}{L_2} = \frac{1-\epsilon_2}{1-\epsilon_1}$ $\frac{L_{mf}}{L} = \frac{1}{(1-\epsilon_{mf})}$

$L_{mf} = \frac{L}{1-\epsilon_{mf}}$... ① ✓ Length of the bed under min. fluidized condition.

Pressure drop under minimum fluidized cond:
 force balance: $\text{Pressure drop} \times \text{cross sectional area} = \text{Gravitational force exerted by the mass of the particles} - \text{buoyant force of the displaced fluid.}$

So let us see that what it is it, it is that we can determine this length and height of this. Now I will say that before onset of fluidisation if the porosity is Epsilon, velocity is V prime and the length is length or height whatever we call, this corresponding to the this we said and here we had particles were like this so this was the height or L whatever we call. Normally in most of the books it is said L, can also use nomenclature as H it does not matter as long as defined. Length or height that L and Epsilon V prime and pressure drop Delta P, so this is before onset of fluidisation.

Now the cross-sectional area of this is CSA is equal to A and which is uniform. And we can say the volume of the bed we can say that volume of the bed that is equal to L into A into $1 - \epsilon$, if ϵ is void the remaining solid is $1 - \epsilon$ so area is uniform so A , L is the height so volume is that $L A$ into $1 - \epsilon$. Now this under minimum fluidised condition this under minimum fluidised condition if we say corresponding to ϵ it is ϵ_{mf} , velocity is V_{mf} , length is L_{mf} , area remains same area of the particle. In that case we can say that the volume of the bed under minimum fluidised condition, this can be said equals to L_{mf} into A into $1 - \epsilon_{mf}$.

Now if we assume that there is no porosity, so here we have that containers and the particles are like that it is highly compact, no porosity ϵ is 0 then it becomes the full solid particle. In that case we can say the volume of the particle then you can say that volume of the particle having no porosity that should be equals to L into A , A is the cross-sectional area and L is the bed height so that is the volume of the particles which is containing in that container. So then we can say that this L into A volume of the particle that must be equals to L_{mf} times A times $1 - \epsilon_{mf}$ because same particle is there, this is the height under fluidised condition, particle sectional area remains same.

ϵ_{mf} got changed, earlier here it was 0 and now it is new ϵ_{mf} , so the particles remaining is $1 - \epsilon_{mf}$ into A into L_{mf} that is equal to $L A$. Therefore, we can write L_{mf} that must be equal to L times A over A times $1 - \epsilon_{mf}$, so this we can say is nothing but L over $1 - \epsilon_{mf}$. So we can easily then that $L_{mf} A (1 - \epsilon_{mf})$ must be equal to $L A (1 - \epsilon_{mf})$ etc etc dot dot dot. That means we can say that $L_{mf} / L = 1 / (1 - \epsilon_{mf})$ by $1 - \epsilon_{mf}$, same is true here that is $L_{mf} / L = 1 / (1 - \epsilon_{mf})$, here it was no ϵ no void so there is 1 by this.

So this he can say what is the minimum height when we have got minimum fluidisation or we have got fluidisation that height corresponding to the bed height is the minimum fluidised bed height, or height of the bed under minimum fluidisation condition height of the bed under minimum fluidisation condition that is L_{mf} . So L_{mf} we can find out if we know the L that is original height of the particles when there is no void and in that condition we have seen that the relation between the void as on the fluidised condition and the initial length or height of the bed that is written as $L_{mf} / L = 1 / (1 - \epsilon_{mf})$, where ϵ_{mf} is the void under minimum fluidised condition. So what we can write is that L_{mf} is $L / (1 - \epsilon_{mf})$ say we note it to be 1.

Next comes this is the length of the bed under minimum fluidised condition, so we can write length of the bed under minimum fluidised condition L_{mf} . So next comes what is the pressure under fluidised condition, so pressure drop under minimum fluidised condition what is that. Now in that case we have to do the force obtained from the pressure drop times sectional area must be equal to the gravitational force as we have said extended by the mass of the particles that – the Buoyancy force of the displaced fluid.

So if we say that if we do the force balance by doing force balance we can write that the pressure drop times sectional area it is a cross-sectional area okay, let us also write cross-sectional area, this must be equals to the ohh gravitational force exerted by the mass of the particles by the mass of the particles by the mass of the particles. And we can also say this – the Buoyancy force of the displaced fluid.

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Volume of the bed = $LA(1-\epsilon)$
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 Volume of the particle having no porosity = LA
 $\therefore LA = L_{mf}A(1-\epsilon_{mf}) \therefore L_{mf} = \frac{LA}{A(1-\epsilon_{mf})} = \frac{L}{(1-\epsilon_{mf})}$
 $L_1A(1-\epsilon_1) = L_2A(1-\epsilon_2) \dots$ or, $\frac{L_1}{L_2} = \frac{1-\epsilon_2}{1-\epsilon_1}$ $\frac{L_{mf}}{L} = \frac{1}{(1-\epsilon_{mf})}$
 $L_{mf} = \frac{L}{1-\epsilon_{mf}}$... ① ✓ Length of the bed under min. fluidized condition.
 Pressure drop under minimum fluidized cond:
 force balance:
 Pressure drop \times sectional area = Gravitational force exerted by the mass of the particles – buoyant force of the displaced fluid.
 $\Delta P A = L_{mf} A (1-\epsilon_{mf}) (\rho_p - \rho) g$
 or, $\Delta P = \frac{L_{mf}}{(1-\epsilon_{mf})} (\rho_p - \rho) g$... ②
 where, ρ_p = density of the particles
 ρ = density of the fluid

So this if we write mathematically we can write $\Delta P A$ that is equal to $L_{mf} A$ into $1 - \epsilon_{mf}$ times $\rho_p - \rho$ into g , where ρ_p = density of the particle and ρ = density of the gas or fluid. So we got that $\Delta P A = L_{mf} A$ into $1 - \epsilon_{mf}$ into $\rho_p - \rho$ into g , this is equal to we can write that ΔP over L_{mf} this is equal to $1 - \epsilon_{mf}$ by or $1 - \epsilon_{mf}$ into $\rho_p - \rho$ this into g ΔP by L_{mf} is A goes out uniform cross-sectional area, so ΔP by $L_{mf} = 1 - \epsilon_{mf}$ into $\rho_p - \rho$ into g , so this we say to be equation 2.

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The image shows handwritten mathematical derivations on a blue grid background. At the top right, there is a small logo for 'CET IIT KGP'. The first equation is Ergun's equation:
$$\frac{\Delta P}{L} = \frac{150 \mu V'}{\phi_s^2 D_p^2} \frac{(1-\epsilon)^2}{\epsilon^3} + \frac{1.75 \rho (V')^2}{\phi_s D_p} \frac{(1-\epsilon)}{\epsilon^3}$$
 Below this, it says 'At minimum fluidization condition'. The second equation is the minimum fluidization condition:
$$\frac{\Delta P_{mf}}{L_{mf}} = \frac{150 \mu V'_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3}$$
 The third equation is the force balance at minimum fluidization:
$$\omega_s (1-\epsilon_{mf}) (\rho_p - \rho) g = \frac{150 \mu V'_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3} + \frac{1.75 \rho (V'_{mf})^2}{\phi_s D_p} \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3}$$
 The fourth equation is the simplified force balance:
$$\omega_s (\rho_p - \rho) g = \frac{150 \mu V'_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3} + \frac{1.75 \rho (V'_{mf})^2}{\phi_s D_p} \frac{1}{\epsilon_{mf}^3} \cdot \frac{D_p^2 \rho}{\phi_s \mu}$$

Now we can write Ergun's equation as ΔP by L that is equal to $150 \mu V'$ over $\phi_s^2 D_p^2$ into $1 - \epsilon$ whole square over ϵ cube + $1.75 \rho V'$ prime square over $\phi_s D_p$ into $1 - \epsilon$ over ϵ cube this is the Ergun's equation. So if this is the Ergun's equation so at minimum fluidisation condition we can say that minimum fluidisation condition we can say that this Ergun's equation we can rewrite as ΔP_{mf} also at minimum fluidised condition over L_{mf} , this is equal to $150 \mu V'$ prime mf divided by ϕ_s^2 into D_p^2 because particle size will not change because we said uniform cross-sectional area into $1 - \epsilon$ mf whole square by ϵ mf cube.

So this we can rearrange as $1 - \epsilon$ mf times $\rho P - \rho$ into g this must be equal to 150ρ sorry V' mf prime over ϕ_s^2 into D_p^2 whole into $1 - \epsilon$ mf square over ϵ mf cube just rewriting that, this + $1.75 \rho V'$ mf prime over $\phi_s D_p$ into $1 - \epsilon$ mf over ϵ mf cube, so 1.75 into $\rho V'$ mf prime by $\phi_s D_p$ into $1 - \epsilon$ mf over ϵ mf cube, so this is the Ergun's equation as required by you, in this case we have said that so $1 - \epsilon$ mf $\rho P - \rho$ into g this is that Ergun's equation okay fine.

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$$\frac{L}{\phi_s^2 D_p^2} = \frac{1}{\epsilon^3} + \frac{1.75 V_{mf}}{\phi_s D_p} \frac{(1-\epsilon)}{\epsilon^3}$$
 Ergun's eqn.
 At minimum fluidization condition

$$\frac{\Delta P_{mf}}{L_{mf}} = \frac{150 \mu V_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3}$$

$$\rho_s (1-\epsilon_{mf})(\rho_s - \rho) g = \frac{150 \mu V_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})^2}{\epsilon_{mf}^3} + \frac{1.75 \rho (V_{mf})^2 (1-\epsilon_{mf})}{\phi_s^2 D_p^2 \epsilon_{mf}^3}$$

$$\rho_s (\rho_s - \rho) g = \frac{150 \mu V_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3} + \frac{1.75 \rho (V_{mf})^2}{\phi_s^2 D_p^2 \epsilon_{mf}^3}$$

$$\rho_s \frac{D_p^3 \rho}{\mu^2} (\rho_s - \rho) g = \frac{150 \mu V_{mf}}{\phi_s^2 D_p^2} \frac{(1-\epsilon_{mf})}{\epsilon_{mf}^3} + \frac{D_p^3 \rho}{\mu^2} \frac{1.75 \rho (V_{mf})^2}{\phi_s^2 D_p^2 \epsilon_{mf}^3}$$

So if this is true then we can also write $\rho_s - \rho$ into g this is equal to $150 \mu V_{mf} / \phi_s^2 D_p^2 (1 - \epsilon_{mf}) / \epsilon_{mf}^3 + 1.75 \rho V_{mf}^2 / \phi_s^2 D_p^2 \epsilon_{mf}^3$. So this is what was $D_p^3 \rho / \mu^2 (\rho_s - \rho) g = 150 \mu V_{mf} / \phi_s^2 D_p^2 (1 - \epsilon_{mf}) / \epsilon_{mf}^3 + 1.75 \rho V_{mf}^2 / \phi_s^2 D_p^2 \epsilon_{mf}^3$. So this is no more required.

So now we can rewrite our $D_p^3 \rho / \mu^2 (\rho_s - \rho) g$ this is equal to $150 \mu V_{mf} / \phi_s^2 D_p^2 (1 - \epsilon_{mf}) / \epsilon_{mf}^3 + 1.75 \rho V_{mf}^2 / \phi_s^2 D_p^2 \epsilon_{mf}^3$. We multiply both the sides with this $D_p^3 \rho / \mu^2 (\rho_s - \rho) g$ into $D_p^3 \rho / \mu^2 (\rho_s - \rho) g$.

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If this is the Ergun's rearranged condition, we can further rewrite that $D_p^3 \rho / \mu^2 (\rho_s - \rho) g$ this is equal to $150 \mu V_{mf} / \phi_s^2 D_p^2 (1 - \epsilon_{mf}) / \epsilon_{mf}^3 + 1.75 \rho V_{mf}^2 / \phi_s^2 D_p^2 \epsilon_{mf}^3$. So this on rearrangement we can write $1.75 \rho V_{mf}^2 / \phi_s^2 D_p^2 \epsilon_{mf}^3$.

square into $1 - \epsilon \mu^2$ over $\epsilon \mu^3 N Re^2 + 150 \mu^3$ into $1 - \epsilon \mu^2$ divided by ϵ this $150 \mu^3$ by Φ^3 $1 - \epsilon \mu^2$ by $\epsilon \mu^3 N Re$.

This $- D^3 \rho$ by μ^2 into $\rho P - \rho g$ this is equal to 0. So we have found out that the relation using Ergun's equation with $N Re$ is a quadratic equation $N Re^2$ and the (ΔP) so $B^2 - 4AC + -4AC + C^2$ so this we can $V^2 - 4AB + A^2$ this we can use $\pm B$ that finding out fluidise solution and get the correct value okay thank you.