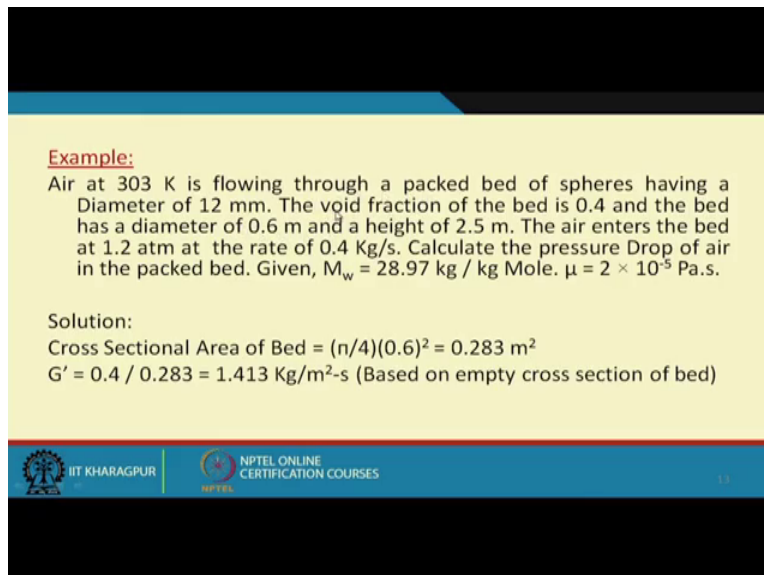


Course on Momentum Transfer in Process Engineering
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Lecture 50
Module 10
Solving problems on Ergun's equation

Yeah, so we have done Blake Kozeny, Burke Plummer equation, right? And there we have seen that whenever are using this and whenever we are coming to the final form of the equation known as Ergun's equation this Ergun's equation is very very useful when you are applying it for packed bed, right? So know and I also said and I am telling you repeatedly that if you can solve problems then you can think that yes you have understood the problem or you have understood the theory whatever has been taught, right? So now let us go and find out a solution of such problem say one typical problem as it is given here, right?

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Example:
Air at 303 K is flowing through a packed bed of spheres having a Diameter of 12 mm. The void fraction of the bed is 0.4 and the bed has a diameter of 0.6 m and a height of 2.5 m. The air enters the bed at 1.2 atm at the rate of 0.4 Kg/s. Calculate the pressure Drop of air in the packed bed. Given, $M_w = 28.97 \text{ kg / kg Mole}$. $\mu = 2 \times 10^{-5} \text{ Pa.s}$.

Solution:
Cross Sectional Area of Bed = $(\pi/4)(0.6)^2 = 0.283 \text{ m}^2$
 $G' = 0.4 / 0.283 = 1.413 \text{ Kg/m}^2\text{-s}$ (Based on empty cross section of bed)

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For example, if we say air at 303 Kelvin is flowing through a packed bed of spheres having a diameter of 12 millimeter. The void fraction of the bed is 0.4 and the bed has a diameter of 0.6 meter and a height of 2.5 meter. The air enters the bed at 1.2 atmosphere at the rate of 0.4 kg per second. Calculate the pressure drop of air in the packed bed. Given molecular weight of air to be 28.97 kg per kg mole and viscosity to be 2 into 10 to the power minus 5 Pascal second. So let us solve it but before solving let us read once more.

Air at 303 Kelvin is flowing through a packed bed of spheres having a diameter of 12 millimeter. The void fraction of the bed is 0.4 and the bed has a diameter of 0.6 meter and a height of 2.5 meter. The air enters the bed at 1.2 atmosphere at the rate of 0.4 kg per second. Calculate the pressure drop of air in the packed bed. Given molecular weight of air is 28.97 kg per kg mole and viscosity of air to be 2×10^{-5} Pascal second, right?

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Solⁿ

$T = 303 \text{ K}$
 $D_p = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$
 $\epsilon = 0.4$
 $D = 0.6 \text{ m}$
 $H = 2.5 \text{ m}$
 $P_m = 1.2 \text{ atm}$
 $\dot{m} = 0.4 \text{ kg/s}$
 $M_{air} = 28.92 \text{ kg/kg mole}$
 $\mu = 2 \times 10^{-5} \text{ Pa.s.}$

Ergun's eqⁿ.

$$\frac{\Delta P}{(G')^2 L} \left(\frac{\phi_s D_p}{0.2} \right) \left(\frac{\epsilon^3}{1-\epsilon} \right) = \frac{150}{N_{Re}} + 1.75$$

Cross sectional area of the bed $= \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.6)^2 \text{ m}^2$

$$G' = 0.4 \left(\frac{\text{kg}}{\text{s}} \right) \times \frac{1}{0.5 \text{ m}^2} = \frac{0.4}{0.5} \frac{\text{kg}}{\text{m}^2 \cdot \text{s.}}$$

based on the supply cross sectional area.

Now let us solve this problem, so if we solve this problem what is given we have a packed bed, right? So bed is like this and there some spherical materials like this are there, right? And air is passing through this, right? So we have been given that temperature T is equals to 303 Kelvin, right? Packed bed of spheres so D_p is equals to 12 millimeter is equal to 12 into 10 to the power minus 3 meter, then we had been given that the void fraction epsilon is 0.4 bed has a diameter of 0.6 so this diameter of the bed D is equals to 0.6 meter, height of the bed that is this height H is say given here as 2.5 meter, right?

And bed it enters our bed at 1.2 atmosphere, so pressure at the bed inlet is equals to 1.2 atmosphere, right? And the flow rate is given in this way that the it is the G prime that is mass velocity is 0.4 kg per second, right? Or we can say not mass velocity we can say this to be the mass flow rate \dot{m} to be 0.4 kg per second. Other things which are given are molecular weight of air M air to be 28.97 kg per kg mole, right? And viscosity μ is equal to 2×10^{-5} Pascal second, right?

Now if we remember that Ergun's equation which we said that was delta P is equal to not delta P it was delta P rho by G prime square phi s Dp over delta L epsilon cube over 1 minus epsilon this was equals to 150 over Nre plus 1.75, right? So first let us then find out the sectional through which it is flowing, so cross sectional area of the bed this we can write to be pi by 4 D square is equals to pi by 4 into 0.6 square so much meter square, right? Therefore we can write that G prime that is mass velocity was 0.4 was kg per second, right?

So kg per second divided by the area into 1 by area or CS cross sectional area if we write, so this is in terms of meter square, right? So that becomes equals to 0.4 divided by pi by 4 0.6 square, right? So so much kg per meter square per second, right? So we can say that this is based on the empty cross sectional area, right? So how much it comes then 0.4 by 584 by 0.6 square so let us look into this, so it was 0.4 divided by pi by 4 is equal to this, right? Divided by 0.6 square is equal to 1.414 kg per meter square second, right? So G prime square we got to be so much, right?

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Calculate ΔP ?
 Let us assume $\Delta P = 0.05 \times 10^5 \text{ Pa}$ ✓ $p_1 - p_2 = \Delta P$
 $p_1 = 1.2 \text{ atm} = 1.2 \times 1.01325 \times 10^5 = 1.2159 \times 10^5 \text{ Pa}$
 $p_2 = 1.2159 \times 10^5 \text{ Pa} - 0.05 \times 10^5 \text{ Pa} = 1.1659 \times 10^5 \text{ Pa}$
 $\rho_{av} = 1.1909 \times 10^5 \text{ Pa}$
 $\frac{\rho_{av} M}{RT} = \frac{1.1909 \times 10^5 \times 28.97}{8314 \times 303} = 1.3695 \text{ kg/m}^3$
 $N_{Re} = \frac{D_p G'}{\mu (1-\epsilon)} = \frac{12 \times 10^{-3} \times 1.414}{2 \times 10^{-5} (1-0.4)} = 1414$
 $\left(\frac{\Delta P}{(G')^2}\right) \left(\frac{\phi_s D_p}{\epsilon^3}\right) = \frac{150}{N_{Re}} + 1.75$
 $\left(\frac{\Delta P \times 1.3695}{(1.414)^2}\right) \left(\frac{0.012}{2.5}\right) \left(\frac{(0.4)^3}{(1-0.4)}\right) = \frac{150}{1414} + 1.75$
 $(\Delta P \times 0.6659) (0.0006) (0.106) = 0.106 + 1.75 = 1.856$
 $\therefore \Delta P = 1.856 / (0.6659 \times 0.0006 \times 0.106) = 0.05 \times 10^5 \text{ Pa} = 5326 \text{ Pa} = 5326 \times 10^2 \text{ Pa}$

Then we have to find out what is the calculate delta P, right? So we have to find out delta P so now let us assume delta P to be say as small as possible say 0.05 10 to the power 5 Pascal, right? We have already been given p1 equal to we have been given 1.2 atmosphere so this is equals to 1.2 into 1.01325 into 10 to the power 5 is equals to, how much? 1.2 into 1.01325 is equals to 1.2159 into 10 to the power 5 Pascal 1.2159 into 10 to the power 5 Pascal, right?

So this we have seen, so we can say that if P_1 so much then P_2 is equals to $1.2159 \cdot 10^5$ Pascal plus ΔP uhh this P_2 will be minus so P_1 minus P_2 is equals to ΔP so P_1 or P_2 is equals to P_1 minus ΔP so minus $0.05 \cdot 10^5$ Pascal is equals to this becomes equal to 1.159 minus 0.05 is equal to 1.1659 , $1.1659 \cdot 10^5$ Pascal, so that is our P_2 , right? So we got P_1 and P_2 so then P average is equals to this 1.1619 plus 1.2159 is equals to by 2 is equal to $1.1909 \cdot 10^5$ Pascal, right?

So we got P average so if we get P average, then we can also say the ρ average is equals to P average, right? P average into M by RT is equals to $1.1909 \cdot 10^5$ Pascal M is molecular weight that is 28 point it was given 28.97, right? R is 8314 into T is 303, right? So then ρ average then comes to be equal to 1.1 this into 10^3 to the power 5 is equals to this much into 28.97 is equal to this divided by 8314 is equals to this divided by 303 is equal to 1.3695, right? Is 1.3695 kg per meter cube, right? So now we have G average G prime we have $(\rho) \cdot (18:25)$ average, right? And now we can say that 1.3695, okay so we can say now N_{re} this is equal to Dg , right? D into g that is okay D particle G prime by μ into 1 minus ϵ , right?

So D particle was 12 millimeter $12 \cdot 10^{-3}$ meter, right? Into this G prime is 1.3695, right? And μ given was $2 \cdot 10^{-5}$ into ϵ given was 1 minus 0.4 , right? This becomes equal to $12 \cdot 10^3$ to the power 3 plus minus equals to so much into 1.3695 no G prime was 1.414 sorry 1.3695 is ρ average, right? 1.3695 is ρ average, right? So we can say that it was 1.414, right? Is equals to this divided by 2 is equal to this divided by 10^5 to the power minus 5 10^3 to the power y 5 plus minus this, right? Divided by 1 minus 0.4 is 0.6 equals to 1414, right? So N_{re} has become equal to 1414, right?

So now we use that Ergun's equation which we had shown earlier that was ΔP rho by G prime square ΔP rho by G prime square into $\phi_s D_p$ by ΔL , right? So we have taken a sphere, right? So ϕ_s is 1 so ΔP by $\phi_s \Delta p$ by ΔL , okay this is this another epsilon cube by 1 minus ϵ , right? So this three term into this is equals to 150 by N_{re} plus 1.75. So out of this if we say that ΔP is now not known, then we write ΔP into rho already we have found out 1.3695, right? Divided by G prime we have found out 1.414 this square, right? Times $\Delta \phi_s$ is 1 so D_p is given as 0.012, right? $D_p \Delta L$ that is height in this case given as 2.5 meter, right? ϵ given here is 0.4 cube divided by 1 minus 0.4 , right? This is equals to 150 by N_{re} we have found out 1414 plus 1.75, right?

Now, let us find out delta P times that 1.13 this value 1.3695 divided by 1.414 square is equal to this, that is 0.6849, okay 0.6849 so this times 0.012 by 2.5 is equal to 0.0048, right? Into 0.4 cube is this divided by 1 minus 0.4 is 0.6 so this becomes 0.106, right? So this is equals to 150 by 1414, 150 divided by 1414 is equal to 0.106 plus 1.75, right? Or delta P then becomes equals to this is equals to 1.75 plus 0.106 is equals to 1.856, right? So then delta P becomes equals to 1.856 divided by 0.6849 into 0.0048 into 0.106, right?

So 1.856 divided by 0.6849 into 0.0048 into 0.106 so this becomes equal to 5326, right? Is equals to 5326 Pascal now that can be written as equal to 0.05 into is equals to 5 into 326 into 10 to the power 3 Pascal, right? So when we made it this 0.05 so into 10 to the power minus 5 Pascal, right? So this means that we have shown that delta P which we assumed to be 0.05 here also by calculation we have made that to be equal to 0.05, right? So here also we have shown that this delta P he has begun 5326 Pascal meaning 0.05 10 to the power minus 5, 10 to the power 5 sorry not minus sorry sorry, 10 to the power 5 Pascal 0.05 10 to the power 5 Pascal, right?

So which was our original assumption, so that means what we have assumed is correct, right? So this way this is one way how we can use Ergun's equation, right? Do some calculations some problem solving then it become more and more (30:12) with this, okay. We will try to solve if we get some more in future maybe in the following class, okay thank you.